Axial anomaly and Schwinger terms in two-dimensional general quantum field theory

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We compute the vacuum expectation value $(\Omega, j^\mu(x) j^\nu(y)\Omega) \equiv J^{\mu\nu}(x-y)$ for a relativistic, Hermitian, not necessarily local vector field in two-dimensional space-time when $\partial_\mu J^{\mu\nu} = \partial_\mu J^{\nu\mu} = \partial_\mu \epsilon^\mu{}_\alpha J^{\alpha\nu} = \partial_\mu \epsilon^\mu{}_\alpha J^{\nu\alpha} = 0$ holds in $S'(\mathbb{R}^2)$, the space of tempered distributions. As the space-time symmetry group of the model we take the inhomogeneous proper orthochronous Lorentz group $P_+^\uparrow$. $(\Omega, Tj^\mu(x)j^\nu(y)\Omega)$ is verified to have anomalous (axial-)vector Ward identities due to nonvanishing equal-time current-current commutators. *Additional* conditions on j^μ are specified which imply that j^μ is a free zero-mass Wightman field with $\partial_\mu j^\mu = \partial_\mu \epsilon^\mu{}_\nu j^\nu = 0$.

I. INTRODUCTION AND CONCLUSIONS

Chiral anomalies of two-dimensional (2D) models have been inferred first from the current's two-point distribution of zero mass QED_2.[1] A strong covariant operator solution for this model exhibits the axial-vector-current nonconservation explicitly.[2] This nonconservation implies anomalous Ward identities for the time-ordered vacuum expectation values of field products containing the axial-vector current. More generally, there exists, independently of any specific dynamics, an argument which demonstrates that anomalous Ward identities of the time-ordered current-current vacuum expectation value are inevitable in 2D Poincaré-invariant Dirac quantum field models.[3]

However, the anomalous Ward identities are not necessarily due to current nonconservation, but they can be caused by nonvanishing equal-time (ET) commutators of the current with the remaining fields in the T product,[4] the Schwinger terms.[5] This is the mechanism which makes the time-ordered two-point distribution of the normal-ordered current, which is associated with the free zero-mass Fock-represented Dirac field, anomalous. This can be seen as follows.

A separation between the two possible sources of anomalous Ward identities is feasible by considering in addition to the time-ordered the "unordered" vacuum expectation values, the Wightman distributions.[6] Let $j_V^\mu(x) \equiv :\bar\psi\gamma^\mu\psi:(x)$ and $j_A^\mu(x) \equiv :\bar\psi\gamma_5\gamma^\mu\psi:(x)$ be the free zero-mass Fock-represented Dirac field vector and axial-vector currents. With $\gamma_5 \equiv \gamma^0\gamma^1$ and the antisymmetric $\epsilon^{\mu\nu}$, $\epsilon^{01} \equiv 1$, one gets the relation $j_V^\mu(x) = \epsilon^\mu_\alpha j_A^\alpha(x)$ between the spinor field currents. Then the following formulas hold in $S'(\mathbb{R}^2)$, the space of tempered distributions over $\mathbb{R}^2$:

$$J^{\mu\nu}(x) \equiv (\Omega, j_V^\mu(x) j_V^\nu(0)\Omega) = (2\pi)^{-2}\partial^\mu\partial^\nu \ln(-x^2 + i\epsilon x^0)\,|_{\epsilon\downarrow 0}\ , \tag{1.1}$$

$$\begin{aligned} TJ^{\mu\nu}(x) &\equiv (\Omega, Tj_V^\mu(x) j_V^\nu(0)\Omega) \\ &\equiv \Theta(x^0)J^{\mu\nu}(x) + \Theta(-x^0)J^{\nu\mu}(-x) \\ &= (2\pi)^{-2}\partial^\mu\partial^\nu \ln(-x^2+i\epsilon)\,|_{\epsilon\downarrow 0} - \frac{i}{\pi}\eta_0^\mu\eta_0^\nu\delta^2(x)\ . \end{aligned} \tag{1.2}$$

Note that the computation of $(\Omega, j_V^\mu(x) j_V^\nu(0)\Omega)$ is free of ambiguities (e.g., infinities) since Wightman distributions can be multiplied with each other. (This follows from the fact that their Fourier transforms have their support within the upper light cone.) A direct computation of $(\Omega, Tj_V^\mu(x) j_V^\nu(0)\Omega)$ by Wick's theorem is ambiguous due to the ill-defined product of Feynman propagators. This is the well-known logarithmic singularity of the one-loop vacuum-polarization graph of QED_2. The definition for $(\Omega, Tj_V^\mu(x) j_V^\nu(0)\Omega)$ given in (1.2) is free of ill-defined products as will be discussed briefly in Sec. III.

The conservation laws $\partial_\mu j_A^\mu = 0$ and $\partial_\mu j_V^\mu = 0$ (Ref. 7), which imply $\Box j_A^\mu = \Box j_V^\mu = 0$, manifest themselves in

$$\partial_\mu J^{\mu\nu} = \partial_\mu \epsilon^\mu{}_\alpha J^{\alpha\nu} = \partial_\mu J^{\nu\mu} = \partial_\mu \epsilon^\mu{}_\alpha J^{\nu\alpha} = 0 \tag{1.3}$$

while the current commutator

$$\delta(x^0)[j_V^0(x), j_V^\nu(0)] = -\frac{i}{\pi}\epsilon^{0\nu}\delta(x^0)\delta'(x^1)$$

(Ref. 7), leads to

$$\partial_\mu TJ^{\mu\nu}(x) = -\frac{i}{\pi}\epsilon^{0\nu}\delta(x^0)\delta'(x^1)\ .$$

These relations can be easily checked on (1.1) and (1.2) as

$$\ln(-x^2+i\epsilon x^0)\,|_{\epsilon\downarrow 0} = \ln|x^2| + i\pi\Theta(x^2)\mathrm{sgn}(x^0)\ ,$$

$$\Box \ln(-x^2+i\epsilon x^0)\,|_{\epsilon\downarrow 0} = 0\ ,$$

and

$$\Box \ln(-x^2+i\epsilon)\,|_{\epsilon\downarrow 0} = 4\pi i\delta^2(x)$$

hold. [We take the ln with its cut on $\mathbb{R}^-$ and $x^2 \equiv (x^0)^2 - (x^1)^2$.]

In this paper we investigate whether other models exist with the inevitable anomaly in their time-ordered current two-point distributions being exclusively due to nonvanishing ET commutators. To this end we compute the set of $L_+^\uparrow$-invariant tempered distributions with vanishing (axial) divergences. ($L_+^\uparrow$ and $L^\uparrow$ denote the proper orthochronous and the orthochronous Lorentz groups.) In this set we then isolate those elements with the properties of the two-point distribution of a Hermitian, not necessarily local vector field j^μ within the framework of

Wightman's axioms.[8,9] We find the above expression (1.1) times an arbitrary positive real constant as the only $L^{\uparrow}$-invariant solutions. (The general $L_+^{\uparrow}$-invariant element also contains a parity-odd part.) Our conclusion for the $L^{\uparrow}$-invariant case is that the anomalous Ward identities are due exclusively to the Schwinger terms if and only if the two-point distribution agrees with (1.1) (up to a multiplicative constant). If the two-point distribution agrees with (1.1) one might suspect that j^μ obeys $\partial_\mu j^\mu = \partial_\mu \epsilon^\mu{}_\nu j^\nu = 0$. This is indeed the case under somewhat stronger assumptions on j^μ than we have to impose.[10]

II. THE CURRENT-CURRENT VACUUM EXPECTATION VALUE

We shall now compute the two-point distribution of a Hermitian vector field in Wightman's sense[8,9] which obeys (1.3). The list of assumptions about the model is as follows. Suppose we are given linear operator-valued distributions j^μ on $S(\mathbb{R}^2)$ with $\mu=0$ or 1. $j^\mu[f]^\dagger = j^\mu[f^*]$ hold on a dense domain in the (positive-metric) Hilbert space $\mathcal{H}$ which carry a unitary representation $U(a,\Lambda)$ of the proper orthochronous Poincaré group. $U(a,\Lambda)$ implement the Poincaré transformations on j^μ:

$$U(a,\Lambda) j^\mu(x) U(a,\Lambda)^\dagger = (\Lambda^{-1})^\mu{}_\nu j^\nu(\Lambda x + a) \quad \forall \Lambda \in L_+^{\uparrow} \text{ and } \forall a \in \mathbb{R}^2 . \tag{2.1}$$

Let there be an, up to a factor unique, vector $\Omega \in \mathcal{H}$, which is invariant under all $U(a,\Lambda)$, the vacuum. Therefore, the current-current vacuum expectation value defines a $L_+^{\uparrow}$-invariant tempered distribution $J^{\mu\nu}(x-y) \equiv (\Omega, j^\mu(x) j^\nu(y)\Omega)$:

$$J^{\mu\nu}(\Lambda x) = \Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma J^{\rho\sigma}(x) \quad \forall \Lambda \in L_+^{\uparrow} . \tag{2.2}$$

In view of the unphysical negative-energy degrees of freedom, which are present in gauge theories, we relax the spectral condition for the translational generators

$$P^\mu \equiv -i \frac{\partial}{\partial a_\mu} U(a, Id) \mid_{a=0} .$$

We only assume that the restriction of the operators P^μ to the set of vectors, which are generated from the vacuum by the application of the smeared current components, have their joint spectra in the forward light cone:

$$\mathrm{Spec} P^\mu \mid_{j\Omega} \subseteq \bar{V}_+ \equiv \{p : p^2 \geq 0, p^0 \geq 0\} . \tag{2.3}$$

The conservation laws (1.3) imply for

$$(2\pi)^2 \Pi^{\mu\nu}(p) \equiv \int_{\mathbb{R}^2} d^2x \, e^{ipx} J^{\mu\nu}(x) ,$$

the equations (2.4) and $L_+^{\uparrow}$ invariance carries over to $\Pi^{\mu\nu}$:

$$p_\mu \Pi^{\mu\nu}(p) = p_\mu \Pi^{\nu\mu}(p) = p_\mu \epsilon^\mu{}_\alpha \Pi^{\alpha\nu} = p_\mu \epsilon^\mu{}_\alpha \Pi^{\nu\alpha} = 0 , \tag{2.4}$$

$$\Pi^{\mu\nu}(\Lambda p) = \Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma \Pi^{\rho\sigma}(p) \quad \forall \Lambda \in L_+^{\uparrow} . \tag{2.5}$$

In order to determine the set of solutions in $S'(\mathbb{R}^2)$ to Eq. (2.4) we transform these into light-cone coordinates $q^\alpha(p) \equiv p^0 + \alpha p^1$, $\alpha = \pm 1$. The tensor components of Π in the associated coordinate frame are given in Eq. (2.6):

$$Q^{\alpha\beta}(q[p]) = \Pi^{00}(p) + \alpha \Pi^{10}(p) + \beta \Pi^{01}(p) + \alpha\beta \Pi^{11}(p) . \tag{2.6}$$

Equation (2.4) transforms into

$$q^+ Q^{-,\alpha}(q) + q^- Q^{+,\alpha}(q) = q^+ Q^{\alpha,-}(q) + q^- Q^{\alpha,+}(q) = 0 , \tag{2.7a}$$

$$q^+ Q^{-,\alpha}(q) - q^- Q^{+,\alpha}(q) = q^+ Q^{\alpha,-}(q) - q^- Q^{\alpha,+}(q) = 0 . \tag{2.7b}$$

They are equivalent to

$$q^- Q^{++}(q) = q^+ Q^{--}(q) = 0 , \tag{2.8a}$$

$$q^- Q^{-+}(q) = q^+ Q^{-+}(q) = q^- Q^{+-}(q) = q^+ Q^{+-}(q) = 0 . \tag{2.8b}$$

The set of solutions to (2.8) in $S'(\mathbb{R}^2)$ is exhausted by the distributions

$$\begin{bmatrix} Q^{++} & Q^{+-} \\ Q^{-+} & Q^{--} \end{bmatrix}(q) = \begin{bmatrix} c^{++}(q^+)\delta(q^-) & c^{+-}\delta(q^-)\delta(q^+) \\ c^{-+}\delta(q^-)\delta(q^+) & c^{--}(q^-)\delta(q^+) \end{bmatrix} . \tag{2.9}$$

Here c^{++} and c^{--} vary through $S'(\mathbb{R})$ and c^{+-}, c^{-+} through $\mathbb{C}$. The $L_+^{\uparrow}$ invariance of Π translates into

$$Q^{\alpha\beta}(e^\chi q^+, e^{-\chi} q^-) = e^{(\alpha+\beta)\chi} Q^{\alpha\beta}(q^+, q^-) \quad \forall \chi \in \mathbb{R} . \tag{2.10}$$

The $L_+^{\uparrow}$-invariant solutions to (2.8) therefore are those with $c^{\alpha\alpha}$ being homogeneous distributions of degree one. These are given by (2.11) with c_α^α varying through $\mathbb{C}$:[11]

$$\begin{aligned} c^{++}(x) &= c_+^+ \Theta(x) x + c_-^+ \Theta(-x) x , \\ c^{--}(x) &= c_+^- \Theta(x) x + c_-^- \Theta(-x) x . \end{aligned} \tag{2.11}$$

Transforming $Q^{\alpha\beta}$ of the equations (2.9) and (2.11) back into Minkowski coordinates yields the distributions $\Pi^{\mu\nu}$:

$$\begin{aligned} \Pi^{\mu\nu}(p) &= p^\mu p^\nu \{ [c_+^+ (p^0+p^1)_+^{-1} + c_-^+ (p^0+p^1)_-^{-1}] \delta(p^0-p^1) + [c_+^- (p^0-p^1)_+^{-1} + c_-^- (p^0-p^1)_-^{-1}] \delta(p^0+p^1) \} \\ &\quad + \tfrac{1}{8} \delta^2(p) [(c^{+-}+c^{-+}) \eta^{\mu\nu} - (c^{+-}-c^{-+}) \epsilon^{\mu\nu}] , \\ (x)_\pm^{-1} &\equiv \frac{d}{dx} [\Theta(\pm x) \ln |x|], \quad c^{\alpha\beta} \text{ and } c_\beta^\alpha \in \mathbb{C} . \end{aligned} \tag{2.12}$$

Computation of the Fourier transform of $\Pi^{\mu\nu}$ is facilitated by the formula

$$\int_{-\infty}^{+\infty} dk\, e^{-ikx}\frac{d}{dk}[\Theta(\pm k)\ln|k|] = -i\frac{\pi}{2}\mathrm{sgn}(x)\mp(\ln|x|+\gamma)\ . \tag{2.13}$$

Here γ is Euler's contant $\gamma=0.5772\ldots$. $J^{\mu\nu}$ is then given by

$$J^{\mu\nu}(x)=\int_{\mathbb{R}^2} d^2p\, e^{-ipx}\Pi^{\mu\nu}(p)\ ,$$

$$J^{\mu\nu}(x)=-\partial^\mu\partial^\nu\frac{1}{2}\left[(c_+^+ + c_-^+)\left[-i\frac{\pi}{2}\right]\mathrm{sgn}(x^0-x^1)-(c_+^+ - c_-^+)\ln|x^0-x^1| + (c_+^- + c_-^-)\left[-i\frac{\pi}{2}\right]\mathrm{sgn}(x^0+x^1) - (c_+^- - c_-^-)\ln|x^0+x^1|\right] + \tfrac{1}{8}[(c^{+-}+c^{-+})\eta^{\mu\nu}-(c^{+-}-c^{-+})\epsilon^{\mu\nu}]\ . \tag{2.14}$$

This completes the proof of proposition (2.1).

Proposition (2.1)

The set of solutions $\Pi^{\mu\nu}$ to the equations

$$p_\mu\Pi^{\mu\nu}=p_\mu\epsilon^\mu{}_\alpha\Pi^{\alpha\nu}=p_\mu\Pi^{\nu\mu}=p_\mu\epsilon^\mu{}_\alpha\Pi^{\nu\alpha}=0$$

and

$$\Pi^{\mu\nu}(\Lambda p)=\Lambda^\mu{}_\rho\Lambda^\nu{}_\sigma\Pi^{\rho\sigma}(p)\quad \forall\Lambda\in L_+^\uparrow$$

in $S'(\mathbb{R}^2)$ is generated by formula (2.12) with the constants $c^{\alpha\beta}$ and c_β^α varying through $\mathbb{C}$. The Fourier transform of any of the $\Pi^{\mu\nu}$'s is given by formula (2.14).

The current's Hermiticity implies the following equivalent relations:

$$J^{\mu\nu}(x)=J^{\nu\mu}(-x)^* \Longleftrightarrow \Pi^{\mu\nu}(p)=\Pi^{\nu\mu}(p)^* \Longleftrightarrow c_\beta^\alpha\in\mathbb{R}$$

and

$$c^{-+}=(c^{+-})^*\ .$$

Positivity of the scalar product implies

$$\begin{aligned}0 &\le (j[f]\Omega, j[f]\Omega)\\ &=\int d^2x\, d^2y\, f_\mu(x)^* J^{\mu\nu}(x-y) f_\nu(y)\quad \forall f_\mu\in S(\mathbb{R}^2)\\ &\Longleftrightarrow \int d^2p f_\mu(p)^*\Pi^{\mu\nu}(p)f_\nu(p)\ge 0\quad \forall f_\mu\in S(\mathbb{R}^2)\\ &\Longleftrightarrow 0\le c_+^\alpha\in\mathbb{R},\ \ 0\ge c_-^\alpha\in\mathbb{R},\ \ c^{+-}=c^{-+}=0\ .\end{aligned}$$

The spectral condition (2.3) implies

$$\mathrm{supp}\Pi^{\mu\nu}\subseteq\bar{V}_+ \Longrightarrow c_-^\alpha=0\ .$$

Combining these conditions one derives by use of

$$\mathrm{sgn}(x^0-x^1)+\mathrm{sgn}(x^0+x^1)=2\,\mathrm{sgn}(x^0)\Theta(x^2)$$

and

$$\partial^\mu[f(x^0+x^1)-f(x^0-x^1)] =\epsilon^\mu{}_\alpha\partial^\alpha[f(x^0+x^1)+f(x^0-x^1)]$$

the proposition (2.2).

Proposition (2.2)

Let j^μ be Hermitian, linear operator-valued distributions on $S(\mathbb{R}^2)$ acting on a Hilbert space $\mathcal{H}$ which carry a unitary representation of $L_+^\uparrow$ with Eq. (2.1) and the spectral condition (2.3) being true. $J^{\mu\nu}(x)\equiv(\Omega, j^\mu(x)j^\nu(0)\Omega)$ obey Eq. (1.3). This implies $\Longrightarrow \exists c^\alpha\in\mathbb{R}$, $c^\alpha\ge 0$ with

$$\begin{aligned}J^{\mu\nu}(x)&=(c^+ + c^-)J_+^{\mu\nu}(x)-(c^+ - c^-)J_-^{\mu\nu}(x)\ ,\\ J_+^{\mu\nu}(x)&\equiv\partial^\mu\partial^\nu[i\pi\Theta(x^2)\mathrm{sgn}(x^0)+\ln|x^2|]\\ &=\partial^\mu\partial^\nu\ln(-x^2+i\epsilon x^0)|_{\epsilon\downarrow 0}\ ,\\ J_-^{\mu\nu}(x)&\equiv\tfrac{1}{2}[\epsilon^\mu{}_\alpha J^{\alpha\nu}(x)+\epsilon^\nu{}_\alpha J^{\alpha\mu}(x)]\ .\end{aligned} \tag{2.15}$$

Additional space-inversion invariance

$$J^{\mu\nu}(\pi x)=\pi^\mu{}_\rho\pi^\nu{}_\sigma J^{\rho\sigma}(x),\quad \pi x\equiv(x^0,-x^1)\ ,$$

implies $c^+ - c^- = 0$ and time-inversion invariance

$$J^{\mu\nu}(\tau x)^*=\tau^\mu{}_\rho\tau^\nu{}_\sigma J^{\rho\sigma}(x),\quad \tau x\equiv(-x^0,x^1)\ .$$

Note that $J^{\mu\nu}$ in (2.15) obeys $J^{\mu\nu}(x)=J^{\nu\mu}(-x)$ $\forall x^2<0$, a condition which would have followed from local commutativity of the current $[j^\mu(x),j^\nu(y)]=0$ $\forall x^2<0$, even though this axiom has not been imposed. Proposition (2.2) demonstrates for any model with (axially) conserved current two-point distributions for conserved current-two-point distributions that these distributions are trivial. Thus in a nontrivial model the anomalous Ward identities are at least partially due to the necessarily nonconserved (axial-)vector currents and not only caused by Schwinger terms.

In case

$$(\Omega, j^\mu(x)j^\nu(y)\Omega)=cJ_+^{\mu\nu}(x-y),\ \ c>0$$

holds, the equations $\partial_\mu j^\mu=\partial_\mu\epsilon^\mu{}_\alpha j^\alpha=0$ (and thus $\Box j^\mu=0$) now follow under somewhat stronger assumptions according to the lemma in Ref. 10. The additional assumptions are $\mathrm{Spec}P^\mu\subseteq\bar{V}_+$, j^μ be local and local relative to a set of fields for which the (unique) vacuum Ω is cyclic. (This is the zero-gap analog to the theorem of Jost and Schroer.) Note that in the example of Eq. (1.1) this lemma is operative.

III. THE TIME-ORDERED CURRENT-CURRENT VACUUM EXPECTATION VALUE

We shall compute now the time-ordered vacuum expectation value that corresponds to $J^{\mu\nu}$ of Eq. (2.15). The relation

$$\begin{aligned}&i\pi\Theta(x^2)\mathrm{sgn}(x^0)+\ln|x^2|\\ &\quad=i\frac{\pi}{2}[\mathrm{sgn}(x^0-x^1)+\mathrm{sgn}(x^0+x^1)]\\ &\qquad+\ln|x^0-x^1|+\ln|x^0+x^1|\end{aligned}$$

demonstrates that $J^{\mu\nu}$ is, for any fixed value x^0, an element of $S'(\mathbb{R})$. x^0 enters as a translational parameter of $\text{sgn}(x^1)$ and $\ln|x^1|$. This shows that the mapping $\mathbb{R}\to S'(\mathbb{R})$, $x^0\mapsto J^{\mu\nu}(x^0,-)$ is of C^∞ type. Therefore the product $\Theta(x^0)J^{\mu\nu}(x)$ is well defined without any specific regularization prescription. By use of the formula

$$\partial^\mu\partial^\nu\Theta(x^2)=\text{sgn}(x^0)\partial^\mu\partial^\nu[\text{sgn}(x^0)\Theta(x^2)] + 4\eta_0^\mu\eta_0^\nu\delta^2(x)$$

one derives proposition (3.1).

Proposition (3.1)

Let $J^{\mu\nu}$ be given by (2.15). This implies

$$\begin{aligned}(TJ^{\mu\nu})(x)&\equiv\Theta(x^0)J^{\mu\nu}(x)+\Theta(-x^0)J^{\nu\mu}(-x)\\&=(c^++c^-)(TJ_+^{\mu\nu})(x)\\&\quad-(c^+-c^-)(TJ_-^{\mu\nu})(x)\ ,\\(TJ_+^{\mu\nu})(x)&\equiv\Theta(x^0)J_+^{\mu\nu}(x)+\Theta(-x^0)J_+^{\nu\mu}(-x)\\&=\partial^\mu\partial^\nu\ln(-x^2+i\epsilon)|_{\epsilon\downarrow 0}-4\pi i\eta_0^\mu\eta_0^\nu\delta^2(x)\ ,\\(TJ_-^{\mu\nu})(x)&\equiv\Theta(x^0)J_-^{\mu\nu}(x)+\Theta(-x^0)J_-^{\nu\mu}(-x)\\&=\tfrac{1}{2}(\epsilon^\mu{}_\alpha\partial^\alpha\partial^\nu+\epsilon^\nu{}_\alpha\partial^\alpha\partial^\mu)\ln(-x^2+i\epsilon)/_{\epsilon\downarrow 0}\\&\quad-2\pi i(\epsilon^\mu{}_0\eta^\nu{}_0+\epsilon^\nu{}_0\eta^\mu{}_0)\delta^2(x)\ .\end{aligned}\tag{3.1}$$

The relation

$$\Box\ln(-x^2+i\epsilon)|_{\epsilon\downarrow 0}=4\pi i\delta^2(x)$$

implies

$$\partial_\mu(TJ_+^{\mu\nu})(x)=-4\pi i\epsilon^{0\nu}\delta(x^0)\delta'(x^1)\ ,$$

$$\partial_\mu(TJ_-^{\mu\nu})(x)=4\pi i\eta^{0\nu}\delta(x^0)\delta'(x^1)\ ,$$

$$\delta'(x)\equiv\frac{d}{dx}\delta(x)\ ,$$

thereby illustrating anomalous Ward identities caused by nonvanishing equal-time current commutators. The Lorentz noninvariance of the vector field's T product is explicit in (3.1).

Finally we verify that it is impossible to modify the definition for $TJ^{\mu\nu}$ in (3.1) so that the modified Green distribution $\tilde{T}J^{\mu\nu}$ is $L_+^\uparrow$ invariant, the conditions

$$(\tilde{T}J^{\mu\nu})(x)-(\tilde{T}J^{\nu\mu})(-x)=0\ ,$$

$$\text{supp}[(\tilde{T}J^{\mu\nu})(x)-(TJ^{\mu\nu})(x)]=\{x:x^0=0\}$$

are valid and normal (axial-)vector Ward identities hold. Ward identities together with $L_+^\uparrow$ invariance enforce that $(\tilde{T}J^{\mu\nu})(x)$ is of the type given in Eq. (2.14). The support condition implies $0=[(\tilde{T}J^{\mu\nu})-J^{\mu\nu}]|_{x^0>0}$ so that the constants, which multiply the $\text{sgn}(x^0\pm x^1)$ and the $\ln|x^0\pm x^1|$ terms of the two expressions, coincide and the $\eta^{\mu\nu},\epsilon^{\mu\nu}$ terms of $(\tilde{T}J^{\mu\nu})$ vanish. Then the symmetry condition for $\tilde{T}J^{\mu\nu}$ makes the (odd) $\text{sgn}(x^0\pm x^1)$ terms vanish so that $\tilde{T}J^{\mu\nu}=TJ^{\mu\nu}=0$ holds. Thus it is impossible to modify the time-ordered Green distribution (3.1) by adding a distribution with support at $x^0=0$ in order to evade both anomalous vector *and* axial-vector Ward identities in accordance with Jackiw's[3] argument. This parallels the situation in the 4D σ model. There, the anomaly shows up in the time-ordered axial-vector–vector–vector current three-point distribution and cannot be eliminated by the definition of a modified T product.[12]

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