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Price-Directed Search, Product Differentiation and Competition*

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Abstract

Especially in many online markets, consumers can readily observe prices, but may need to further inspect products to assess their suitability. We study the effects of product differentiation and search costs on competition and market outcomes in a tractable model of price-directed consumer search. We find that (i) firms' equilibrium pricing always induces efficient search behavior, (ii) for relatively large product differentiation, welfare distortions still occur because some consumers (may) forgo consumption, and (iii) lower search costs lead to stochastically higher prices, increasing firms' expected profits and decreasing their frequency of sales. Consumer surplus often falls when search costs decrease.

Keywords: Consumer Search, Price-Directed Search, Product Differentiation, Price Competition, Mixed-Strategy Pricing, Search Costs

JEL Classification: D43, D83, L13

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1 Introduction

The advent of the Internet has drastically improved consumers' ability to shop around and compare different offerings. Access to price comparison websites and product search engines enables consumers to quickly obtain price quotes from many different sellers. Yet, while the Internet has substantially reduced the search frictions consumers face, finding the most suitable product is in many cases still not a trivial task due to the costs involved in evaluating different options and the large product variety that is often available.

Models of price-directed search, where consumers can freely observe prices but need to engage in costly search to find out how much they like individual products – their so-called *match values* – seem particularly well suited to describe competition in online markets for differentiated products. However, solving these models can be quite intricate. This is especially the case when there is no built-in ex-ante product differentiation, such that a pure-strategy price equilibrium fails to exist.[1] Two recent articles, Armstrong and Zhou (2011) and Ding and Zhang (2018), have managed this task by simplifying the market setting to gain enough tractability to allow for a characterization of the mixed-strategy equilibrium. But consumers' match values are perfectly negatively correlated in the former and all-or-nothing in the latter, which undermines an understanding of the role of product differentiation on market outcomes. In particular, two features are implicitly ruled out in these models: consumers never return to previously sampled firms – so-called *returning demand* does not exist – and they never optimally forgo consumption *after* finding a product that they value above marginal cost. In our view, the implications for competition and market outcomes stemming from the first neglected factor, as well as the classic allocative losses stemming from the second neglected factor, are however important aspects for a realistic depiction of markets with price-directed search.

Our contribution is hence to set up a tractable model of price-directed search that does not preclude these two elements and allows for an analysis of the effect of product differentiation on market outcomes. In our model, for each firm's single product, a consumer's

[1] The main issue is that when consumers' match values are drawn from a continuous distribution, such as in the workhorse model for analyzing search in differentiated-goods markets by Wolinsky (1986), and prices are observable before search, the demand faced by any given seller depends both discontinuously on her price rank *and*, for a given rank, continuously on her price differences to other sellers. As it turns out, the characterization of the resulting mixed-strategy equilibrium is extremely difficult.

valuation can either be high, $v = v_H$ – a *full match* – or low, $v = v_L < v_H$ – a *partial match* – following an exogenous two-point distribution. Importantly, consumers' valuation for non-fully matched products can take on any positive value up to v_H, and may therefore exceed firms' constant marginal cost of production c. This seems to be satisfied in many relevant product markets,[2] and differentiates our work from the most closely related model of price-directed search in Ding and Zhang (2018) where product matches are all-or-nothing.[3]

We study the outlined setup to solve for consumers' optimal search procedure and characterize firms' equilibrium pricing. We find that, depending on the degree of product differentiation, one of four types of unique symmetric pricing equilibria emerges. To start with, if product differentiation is very low such that v_L is very close to v_H, firms deterministically price at marginal cost, while consumers search exactly one random firm, buying there no matter whether a full or partial match is found. Marginal-cost pricing occurs because under the described circumstances, no consumer ever searches on after starting at (one of) the lowest-priced firm(s), giving rise to Bertrand-type competition.

If instead product differentiation is not so low, firms can sustain positive profits in equilibrium. Due to undercutting incentives to be searched earlier, firms draw prices randomly from an atomless distribution bounded away from marginal cost, and consumers search products in ascending order of prices according to their optimal search rule. As we show, this mixed-strategy equilibrium comes in three different subtypes, which we now describe successively.

First, for relatively large product differentiation, v_L/v_H low, a "high-price equilibrium" emerges in which all firms always price above v_L. This happens because firms do not find it worthwhile to reduce their prices so much as to be able to serve only partially matched consumers. Variants of this equilibrium have surfaced before in the literature, arising in models where product matches are all-or-nothing, and we show under which condition it prevails when allowing for partial product matches. In this equilibrium, consumers keep searching until a full match is found, which is the socially efficient thing to do. Yet, when $v_L > c$, a deterministic welfare loss occurs, as a share of consumers do not find a full match

[2]For example, consider a consumer who is looking to replace a defective toaster. While this consumer may have a full match only for red toasters with a warming rack, it is very likely that she will also have a valuation for other toasters that exceeds their marginal cost of production.

[3]See the literature discussion below for a more detailed delineation.

at any firm and hence optimally forgo consumption after their search process has concluded. This is clearly inefficient, as for $v_L > c$, the valuation of partial matches exceeds firms' marginal cost of production.

Second, for intermediate product differentiation, v_L/v_H intermediate, the high-price equilibrium cannot be sustained anymore. This is because firms would have an incentive to reduce their price to the (now moderately high) partial valuation v_L to sell to the share of consumers without a full match at any firm. Instead, a novel "gap equilibrium" is played in which the firms randomize between pricing in a low range weakly below v_L or in a high range strictly above it, with a gap in between. In a way, firms optimally hedge their bets between choosing high prices targeted at only fully matched consumers and fighting for the share of returning consumers without a full match at any firm.

Interestingly, the gap equilibrium price distribution has the property that even for the highest possible spread of prices, consumers keep searching until a full match is found: On the equilibrium path, they never settle for a partially matched product before having searched all products, and only those consumers who do not find a full match at any firm return to the lowest-priced firm, provided that its price does not exceed v_L. This search behavior is again efficient from a social point of view. However, an inefficiency may still arise: In case all firms price above v_L, the share of consumers without a full match at any firm eventually drop out of the market, even though they should buy from a social perspective. In other words, a probabilistic deadweight loss occurs.

A gap equilibrium with non-convex pricing support may also emerge in the closely related model of price-directed search in Ding and Zhang (2018), but for quite a different reason and with a different structure. In our work, v_L and $v_L + \varepsilon$ cannot both lie in firms' equilibrium price distribution, as the marginal price increase starting from v_L would lead to a discrete loss of expected demand from returning consumers. On the other hand, Ding and Zhang (2018) allows for a share of *informed consumers* who do not have to pay any search costs and know all match values. A gap equilibrium then arises when the share of these consumers is sufficiently large such that firms find it optimal to (also) set very high prices that are addressed at informed consumers only.

Finally, for relatively low product differentiation in our model, v_L/v_H moderately high, another novel "low-price equilibrium" emerges in which the firms always price below v_L.

Same as for the gap equilibrium described above, the equilibrium price distribution is such that the consumers keep searching until they find a full match, which is once more the efficient thing to do. But there is no deterministic or probabilistic welfare loss anymore, as those consumers without a full match at any firm now deterministically return to purchase from the lowest-priced firm.

We also use our model to analyze the effects of lower search costs – such as caused by the ongoing advancement of information technologies – on market outcomes. We first show that lower search costs have a rather perverse effect on firms' pricing: they lead to stochastically higher prices, resulting in higher expected prices and profits and a weakly lower probability that firms engage in sales (which we define as prices below v_L that may enable firms to sell to partially matched consumers). The intuition is that lower search costs make consumers willing to search higher-priced products when having found only partial matches so far, both when holding a purchase option (which is the case when the lowest price lies below v_L) and when not. In either case, this relaxes price competition. The monotone-increasing effect of lower search costs on firms' pricing is in line with other contributions on price-directed search (Armstrong and Zhou (2011), Choi et al. (2018), Haan et al. (2018)), and it differs from the closely related work by Ding and Zhang (2018) where lower search costs stochastically decrease prices in their most interesting (gap) equilibrium.

In terms of welfare, there are two conflicting effects of lower search costs: while they directly reduce the aggregate search friction incurred by consumers, they also lead to stochastically higher prices, which may imply a lower probability that at least one firm makes a sale – such that consumers who are everywhere only partially matched may drop out of the market more often. We however show that the former effect almost always dominates (the only exception being specific parameter combinations under duopoly), such that lower search costs are generally good for society. This is quite different for consumers, however: due to the price-increasing effect, we show that consumers are often harmed as search costs decrease.

We finally also compare market outcomes to a model variant with unobservable prices, such that consumers must search in an essentially random order. While a full equilibrium characterization of the setting with random search is difficult, we provide at least some suggestive evidence that unobservability of prices should tend to decrease market performance.

The remainder of this article is structured as follows. In Section 2 we briefly summarize the related literature. Section 3 introduces the model, while in Section 4 we provide a full equilibrium characterization. In Section 5 we study the effects of a decrease in search costs. The model variant with random consumer search is discussed in Section 6. Section 7 concludes. Several technical proofs are relegated to the Appendix.

2 Related Literature

Our paper joins an extensive literature on costly consumer search, studying the effects of frictions and incomplete information about product characteristics and/or prices on market outcomes.[4] In early work which relates to our model, such as the seminal papers by Wolinsky (1986), Stahl (1989) and Anderson and Renault (1999), prices are unobservable and consumer search is random. Departing from models of random search, there have been efforts to describe environments in which consumers search firms according to some order. The first papers in this vein focused on predetermined orders, arising naturally e.g. when thinking about geographical distance (see Arbatskaya (2007) for homogeneous products, Armstrong et al. (2009) for differentiated products with a "prominent" firm[5], or Zhou (2011) for a general analysis with differentiated products). In Athey and Ellison (2011) and Chen and He (2011), firms bid for positions along consumers' search path, while in Haan and Moraga-González (2011), consumers' search order is influenced by firms' advertising intensities. However, in these models, prices do not affect the order of search. Armstrong (2017) outlines a setting in which the order of search is chosen endogenously by consumers forming *expectations* about prices and firms acting according to their beliefs in equilibrium.

One of the first attempts to model *observable* prices as important strategic variables for directing search can be found in Armstrong and Zhou (2011, Section 2), where firms advertise the price of their differentiated product on a price-comparison website. Consumers' optimal search path is then guided by those advertised prices. To keep the model tractable, Armstrong and Zhou introduce a specific (Hotelling duopoly) structure in which consumers'

[4]For comprehensive literature reviews see Anderson and Renault (2018) and Baye et al. (2006), or, for the case of digital markets, Moraga-González (2018).

[5]That is, one firm is exogenously searched first by all consumers, while the remaining firms are searched in random order.

match values are perfectly negatively correlated.[6] A main finding is that the competition among firms to receive a larger market share by being sampled first drives down retail prices, relative to a benchmark model without price advertising, and that this effect is stronger when search frictions increase.

As hinted at in the Introduction, tractability is generally a major issue when it comes to solving models of price-directed search. For example, even a duopoly version of the standard differentiated-products framework by Wolinsky (1986) with independently distributed match values becomes essentially intractable with observable prices. Choi et al. (2018) and Haan et al. (2018) circumvent this problem by incorporating sufficiently strong ex-ante differentiation into Wolinsky's framework with observable prices.[7] This restores existence of a pure-strategy equilibrium that can be characterized. However, there are two problems with this approach. The first is that the pure-strategy equilibrium candidate breaks down when firms' ex-ante differentiation becomes relatively weak, as then non-local deviations become profitable. The second is that a pure-strategy price equilibrium and continuous demand around price-rank changes is hard to reconcile with the empirical findings in many online markets.[8] By considering a two-point distribution of match values, we obtain tractability without introducing any exogenous ex-ante differentiation.

The most closely related article is Ding and Zhang (2018). The paper both extends Stahl's 1989 model of random search for (originally) homogeneous products to incorporate binary all-or-nothing consumer product valuations, and also studies the same setting with observable prices – as well as carrying out a comparison between the two setups. Their latter model of price-directed search differs in two major aspects from our contribution. First, and most importantly, we allow for a variable degree of product differentiation. While in Ding and Zhang consumers either fully value a product or not at all, in our setting they may

[6] More concretely, upon inspecting the lower-priced product first, consumers learn its match value and can then perfectly deduce the match value offered by the other firm.

[7] See also Shen (2015) for a related analysis in a Hotelling context.

[8] For evidence that firms resort to mixed-strategy pricing, see e.g. Baye et al. (2004a,b), Bachis and Piga (2011) and Seim and Sinkinson (2016). Baye et al. (2009) document that the number of clicks received by online retailers is highly dependent on their price rank. Examining a large price-comparison site at the time, they find that the lowest-priced retailers for a given product received on average 60% more clicks than higher-priced competitors. Relatedly, Ellison and Ellison (2009) establish that the price transparency provided by a price search engine tended to make demand (for low-quality computer memory modules, a relatively homogeneous good) extremely elastic, even though this was counteracted by obfuscation attempts by some of the examined online retailers.

also have a positive valuation (exceeding firms' marginal cost of production) for non-fully matched products. If this partial valuation is not very low, this directly affects competition by influencing consumers' search behavior: the highest price they are willing to search may now depend on the price of the lowest-priced product, and some consumers may optimally return to purchase this product after their search process has ended. Moreover, classic deadweight losses occur when not all consumers purchase eventually.

Second, we do not include informed consumers who costlessly observe all match values, which is however crucial to generate most interesting results in Ding and Zhang (2018). In particular, their "gap equilibrium" with non-convex pricing support and resulting welfare losses only arises when their share of informed consumers is quite large. But especially for online product markets where many consumers are casual first-time buyers, consumers who know their match values in advance (or can search them for free) will arguably often constitute a small minority. For simplicity and to highlight a different channel, we set their number to zero in our model.[9][10]

3 Model Setup

We study the following market. There are $N \geq 2$ risk-neutral firms $i = 1, \ldots, N$ that compete in prices p_i. Each firm offers a single differentiated product of which an arbitrary amount can be sold at common and constant marginal cost of production $c \geq 0$.

There is a unit mass of risk-neutral consumers with unit demand and an outside-option value that is normalized to zero. Each consumer freely observes the prices of all products. However, consumers are initially unaware whether any given product will be a full or partial match for them. Precisely, product i perfectly suits a consumer's needs (the product is "a full match") with probability $\theta \in (0,1)$. In case of a full match, consumers' willingness to pay is given by $v_i = v_H > c$. With complementary probability $1 - \theta$, product i is only "a partial

[9]For a sufficiently small fraction of informed consumers, we can show that our equilibrium characterization would remain completely unaffected.

[10]As a third distinction to Ding and Zhang (2018), consumers' first search is not costless in our model. While we anyway consider a positive search cost for every sampled product to be more realistic, the equilibrium characterization of our model would remain virtually unchanged with costless first search: just the parameter region where the market is inactive would vanish. Of course, our analysis of the welfare effects of changes in search costs would have to be adapted.

match", for which consumers' willingness to pay is given by $v_i = v_L \in [c, v_H)$.[11] We assume that the match values v_i are identically and independently distributed across each consumer-firm pair, and that the firms are unable to identify which product(s) will be a match for any individual consumer, ruling out price discrimination.

In order to find out their match values, consumers have to incur a search cost $s \geq 0$ per product that they sample. It is assumed that they cannot purchase any product before searching it first. Consumers engage in optimal sequential search with free recall and maximize their expected consumption utility, where consumption utility is given by

$$u_i \equiv v_i - p_i - ks, \qquad \text{with } v_i \in \{v_L, v_H\} \tag{1}$$

when buying product i (which can either be a full or partial match) after having searched $k \in \{1, \ldots, N\}$ products, and $u_0 = -ks$ when taking their outside option after having searched $k \in \{0, \ldots, N\}$ products. All market parameters are common knowledge.

The timing of the game is as follows. First, firms simultaneously set prices p_i. Second, consumers observe these prices, and engage in optimal sequential search. Third, payoffs realize.

In order to make the problem interesting, we finally assume that the search cost is not too large, $s \leq \theta v_H + (1-\theta)v_L - c$. Otherwise, the market collapses, as no firm could offer a non-negative expected surplus to consumers even when setting $p_i = c$.

4 Equilibrium Analysis

Optimal Search. Since, apart from their prices, firms' products appear ex-ante identical, consumers will clearly find it optimal to search firms in ascending order of their prices.[12] Without loss of generality, we index firms such that $p_1 \leq p_2 \leq \ldots \leq p_{N-1} \leq p_N$. Given a consumer started at firm 1 and found a full match, the consumer optimally purchases, since there can be no gain from searching on. However, if only a partial match is found at firm 1,

[11] For $v_L < c$, since firms never optimally price below their marginal cost, our setup collapses to one with all-or-nothing product matches, as studied, for example, by Ding and Zhang (2018).

[12] In case of ties, consumers are assumed to randomize with equal probability between firms, which is however inconsequential for our results. We moreover assume that whenever a consumer is indifferent between purchasing directly and searching on, the consumer searches on, and whenever a consumer is indifferent between buying and not buying after their search process has ended, the consumer buys.

the consumer might want to continue to search firm 2, and so on. Consumers' optimal search behavior now crucially depends on whether $p_1 > v_L$ or $p_1 \leq v_L$, as only in the latter case, consumers may want to return to purchase at firm 1 in the course of their search process. The following lemma fully characterizes consumers' optimal search behavior.

Lemma 1. *Optimal Search:*

- *If $p_1 > v_L$, search, in increasing order of prices, all firms $i = 1,\ldots,N$ for which $p_i \leq v_H - \frac{s}{\theta}$. Purchase immediately if a full match is found, and search on if not. If no full match is found at any suitable firm, take the outside option.*
- *If $p_1 \leq v_L$, start search at firm 1 if $p_1 \leq \theta v_H + (1-\theta)v_L - s$, and otherwise take the outside option. Given firm 1 is searched and a full match is found, purchase there immediately. If not, search, in increasing order of prices, all firms $i = 2,\ldots,N$ for which $p_i \leq p_1 + (v_H - v_L - \frac{s}{\theta})$. Purchase immediately if a full match is found, and search on if not. If no full match is found at any suitable firm, purchase at firm 1.*

Proof. The first part is straightforward: Given that all prices exceed v_L, consumers will only buy from a firm if it provides a full match, and as long as no full match has been found, consumers hold a utility of zero. Hence, provided that no full match has been found yet, the expected one-shot gains from searching any firm i are given by $\theta(v_H - p_i) - s$, which is non-negative if and only if $p_i \leq v_H - \frac{s}{\theta}$. It is therefore optimal to search, in increasing order of prices, all firms for which this holds, and purchase immediately if a full match is found. If no full match is found at any firm which satisfies $p_i \leq v_H - \frac{s}{\theta}$, a consumer optimally takes the outside option.

If, on the other hand, $p_1 \leq v_L$, the expected one-shot gains of searching any firm are clearly largest for firm 1 and if no other firm has been searched yet. Hence, a consumer should only start to search (at firm 1) if the expected one-shot gains of doing so, $\theta(v_H - p_1) + (1-\theta)(v_L - p_1) - s$, are non-negative. This transforms to $p_1 \leq \theta v_H + (1-\theta)v_L - s$. If this holds and it is therefore optimal to search firm 1, consumers should clearly purchase there immediately if a full match is found. If a partial match is found, a consumer holds a purchase option of value $v_L - p_1 \geq 0$, which remains true as long as only partial matches have been found at every searched firm. Hence, provided that only partial matches have been found so far, the expected one-shot gains from searching any firm $i = 2,\ldots,N$ are given by $\theta((v_H - p_i) - (v_L - p_1)) - s$. This is non-negative for all firms i which satisfy $p_i \leq p_1 + (v_H - v_L - \frac{s}{\theta})$. It is therefore optimal to search these firms in increasing order of their prices and purchase immediately if a full match is found. If no full match is found at any firm which satisfies $p_i \leq p_1 + (v_H - v_L - \frac{s}{\theta})$, a consumer optimally returns to purchase from firm 1. □

Preliminary Equilibrium Results. Having characterized consumers' optimal search behavior, one may first note that for very high search costs, $s \geq \theta(v_H - v_L)$, the binding condition for consumers to start searching is $p_1 \leq \theta v_H + (1-\theta)v_L - s$ $(\leq v_L)$; moreover, consumers will never search firms that are not among the lowest-priced. The reason is that in this case, after obtaining a partial match at (one of) the lowest-priced firm(s), the expected gains from searching are too low for any higher-priced firms. Then, the property that consumers will only search firms which are among the lowest-priced immediately implies the following.

Proposition 1. *If $s \geq \theta(v_H - v_L)$, or equivalently*

$$\frac{v_L}{v_H} \geq \bar{\gamma} \equiv 1 - \frac{s}{\theta v_H}, \tag{2}$$

then in the unique symmetric equilibrium each firm chooses $p^* = c$ and earns zero profit. On the equilibrium path, each consumer searches exactly one random firm and buys there immediately, independent of whether a full or partial match is found.[13]

Proof. See the argument above. Given $p^* = c$, consumers indeed find it optimal to search one random firm due to the parameter assumption of $s \leq \theta v_H + (1-\theta)v_L - c$. □

We will subsequently refer to the parameter region where Proposition 1 holds as "Bertrand region", since intense price competition drives firms to price at marginal cost. It is worth noting that consumers' search is efficient from a social point of view in this region: given that $s \in [\theta(v_H - v_L), \theta v_H + (1-\theta)v_L - c]$, welfare is maximized precisely if consumers search one firm and buy there immediately.

As we show next, the market outcome is decisively different for lower search costs.

[13]In the borderline case where $s = \theta(v_H - v_L)$, given that $p_i = c$ for all firms, consumers are actually indifferent between buying immediately after obtaining a partial match or searching on. This is however inconsequential for the equilibrium outcome.

Lemma 2. *If $s < \theta(v_H - v_L)$, or equivalently $v_L/v_H < \overline{\gamma}$, there exists no symmetric pure-strategy equilibrium. In a symmetric mixed-strategy equilibrium, firms make positive expected profits and draw prices from an atomless CDF bounded away from marginal cost.*

Proof. A symmetric pure strategy-equilibrium at any positive price level cannot exist because firms would have an incentive to marginally undercut to be searched first by all consumers, rather than just by $1/N$ of the consumers. However, unlike the case where $s \geq \theta(v_H - v_L)$, it is also no equilibrium that every firm prices at marginal cost (i.e., c). This is because, when all rival firms price at c, setting a price in the non-empty range $(c, c + v_H - v_L - \frac{s}{\theta}]$ guarantees a firm to be searched (by those consumers who do not find a full match at any rival firm; compare with Lemma 1) and make a positive profit. Hence, any symmetric equilibrium must be in mixed strategies. The respective equilibrium pricing CDF must be bounded away from marginal cost because firms can guarantee a positive profit. It must be atomless because otherwise, transferring probability mass from the atom(s) to prices marginally below would pay because this avoids ties. □

Preview of Mixed-Strategy Equilibria. It turns out that the symmetric mixed-strategy equilibrium for $s < \theta(v_H - v_L)$ – that is, equivalently, $\frac{v_L}{v_H} < \overline{\gamma}$ – comes in three qualitatively different subtypes, depending on the degree of product differentiation (which is inversely related to v_L/v_H) in combination with the other market parameters.

In particular, as mentioned in the Introduction, either a "high-price equilibrium" (high differentiation, with $v_L/v_H \leq \underline{\gamma}$), a "gap equilibrium" (intermediate differentiation, with $v_L/v_H \in (\underline{\gamma}, \tilde{\gamma})$), or a "low-price equilibrium" (relatively low differentiation, with $v_L/v_H \in [\tilde{\gamma}, \overline{\gamma})$) emerges as the unique equilibrium. We will now characterize these equilibria in turn. Figure 1 previews the various equilibrium regions in $(s/v_H, v_L/v_H)$-space for an exemplary combination of the probability of full matches, number of firms, and their constant marginal costs of production relative to v_H. In the region to the right of the dotted line, where $s \geq \theta(v_H - c)$, the Bertrand equilibrium is played whenever our parameter assumption of $s \leq \theta v_H + (1-\theta)v_L - c$ holds.[14] The light-gray dashed line ($\check{\gamma}$) will be relevant for the discussion of random search in Section 6.

[14]For $v_L < c$, the high-price equilibrium is played whenever $s < \theta(v_H - c)$; otherwise, the market breaks down.

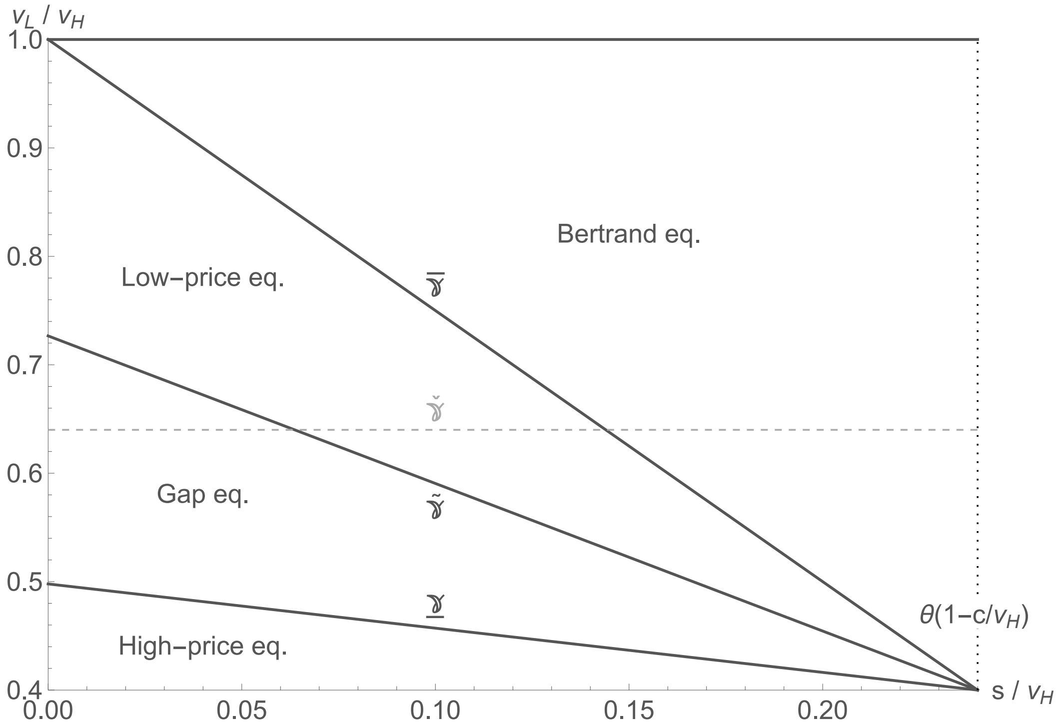

Figure 1: Depiction of equilibrium regions for $\theta = 0.4$, $N = 4$ and $c/v_H = 0.4$.

4.1 High-Price Equilibrium

We show first that if product differentiation is relatively large, a "high-price equilibrium" emerges in which firms draw prices from a convex support that lies strictly above v_L and extends up to consumers' "threshold price" (i.e., the highest price they are ever willing to search) $v_H - \frac{s}{\theta}$. Clearly, in this equilibrium, a firm cannot attract any "returning" demand: consumers either buy immediately after having searched some firm or never return. Proposition 2 provides the full characterization.

Proposition 2. *Suppose that $v_L/v_H \leq \underline{\gamma}$, where*

$$\underline{\gamma} \equiv \frac{c}{v_H} + \left(1 - \frac{s}{\theta v_H} - \frac{c}{v_H}\right) \frac{\theta(1-\theta)^{N-1}}{\theta + (1-\theta)^N}. \tag{3}$$

Then in the unique symmetric equilibrium each firm samples prices continuously from the interval $[\underline{p}_H, \overline{p}_H]$ following the atomless CDF

$$F_H(p) \equiv \frac{1}{\theta}\left[1 - (1-\theta)\left(\frac{v_H - \frac{s}{\theta} - c}{p - c}\right)^{\frac{1}{N-1}}\right], \tag{4}$$

with

$$\underline{p}_H \equiv c + \left(v_H - \frac{s}{\theta} - c\right)(1-\theta)^{N-1} > v_L \tag{5}$$

and

$$\overline{p}_H \equiv v_H - \frac{s}{\theta}. \tag{6}$$

Each firm makes an expected profit of

$$\pi_H^* \equiv \left(v_H - \frac{s}{\theta} - c\right)\theta(1-\theta)^{N-1}. \tag{7}$$

On the equilibrium path, each consumer keeps searching (in increasing order of prices) until a full match is found, and takes the outside option if no full match is found at any firm.

Proof. See Appendix. □

It may be noted that various versions of the above pricing equilibrium have appeared before in the literature, where it was generally assumed that $v_L = 0$. In particular, setting $v_L = 0$ and $c = 0$, it is easy to see that we nest the model of price-directed search by Ding and Zhang (2018) for the case in which there are no informed consumers ($\mu = 0$ in their notation).[15][16] We extend their findings by showing that even when consumers have a positive valuation for non-fully matched products, their price equilibrium prevails, provided that this valuation is not too large ($v_L/v_H \leq \underline{\gamma}$).[17] The economic ratio is that when v_L is low, firms have no incentive to cater to only partially matched consumers, as they would have to

[15]To be precise, consider Ding and Zhang (2018, Proposition 2) for $\mu = 0$, and let $V \equiv v_H$. Then $r = v_H - \frac{s}{\theta}$ (compare with their equation (2)), and their threshold value s_1' equals θv_H such that part (i) of their Proposition 2 applies. It is then immediate that their equilibrium CDF $R(p)$ coincides with our equilibrium CDF $F_H(p)$ in the high-price equilibrium (and of course, also the equilibrium expected profits are identical).

[16]Letting $s = 0$ and $N = 2$, we also nest a duopoly version of Varian (1980) with inelastic demand up to a maximum valuation of v_H (with a fraction $\lambda = \frac{\theta^2}{1-(1-\theta)^2} = \frac{\theta}{2-\theta}$ of fully-informed "shoppers"). For $s = 0$ and arbitrary $N \geq 2$, our setup is moreover identical to the second stage of Ireland (1993) when his "information shares" s_i (i.e., the share of consumers who know about the existence of firm i) satisfy $s_i = \theta$ for all $i = 1, \ldots, N$ (and $v_H = 1$ to match his normalization). For $N = 2$ and $s_1 = s_2 = \theta$, it is then straightforward to check that Ireland's second-stage solution coincides with ours (compare with (Ireland, 1993, p.66)). For $N > 2$, this should also be the case, but due to his focus on asymmetric information shares, the comparison of equilibria is less obvious.

[17]On top of that, the characterized high-price equilibrium is also robust to introducing shoppers to our model, given that their share in the population is not too large. Indeed, with a fraction μ of shoppers having zero search cost (or alternatively, knowing all match values), the best possible deviation price above $v_H - \frac{s}{\theta}$ is simply v_H. At this price, a deviating firm's profit is $\mu(v_H - c)(1-\theta)^{N-1}\theta$, which does not exceed the candidate equilibrium profit whenever $\mu \leq \frac{v_H - \frac{s}{\theta} - c}{v_H - c}$.

choose too low prices to make it worthwhile. Hence, firms only compete for and sell to fully matched consumers.

Consumers' search is clearly efficient from a social point of view in the characterized equilibrium: they keep searching until a full match is found, which is socially optimal since $\theta(v_H - v_L) > s$. Still, when v_L exceeds firms' marginal cost, the high-price equilibrium is inefficient: there is a mass $(1-\theta)^N$ of consumers who don't have a full match anywhere and eventually drop out of the market, even though from an allocative perspective, they *should* buy. Hence, a classic deadweight loss arises.

4.2 Gap Equilibrium

When product differentiation is not too large such that $v_L/v_H > \underline{\gamma}$, the high-price equilibrium characterized above breaks down. This is because, even when still $\underline{p}_H > v_L$ in a candidate high-price equilibrium, firms have an incentive to reduce their price to v_L. Doing so, they would be able to sell to the segment $(1-\theta)^N$ of consumers without a full match anywhere, who would eventually return to the deviating firm.[18] If this is the case but still v_L is not too close to v_H, we show that a pricing equilibrium with non-convex support arises. In this "gap equilibrium", firms optimally randomize between pricing in a high range strictly above v_L, selling only to fully matched consumers that have not found a full match at any lower-priced firm, and in a low range that extends up to v_L, with a gap in between. Pricing in the low range gives firms a chance to sell to returning consumers that have not found a full match anywhere, which happens when a firm offers the best deal in the market. Proposition 3 gives the detailed characterization. An example equilibrium CDF is depicted in Figure 2.

Proposition 3. *Suppose that* $v_L/v_H \in (\underline{\gamma}, \tilde{\gamma})$*, where*

$$\tilde{\gamma} \equiv \frac{c}{v_H} + \left(1 - \frac{s}{\theta v_H} - \frac{c}{v_H}\right) \frac{\theta + (1-\theta)^N}{2\left[\theta + (1-\theta)^N\right] - \theta(1-\theta)^{N-1}}. \tag{8}$$

[18] As is shown in the proof of Proposition 2, this is indeed the best deviation from the high-price equilibrium, even though firms could further boost their demand by pricing strictly below v_L and probabilistically blocking some rival firms from being searched.

Then in the unique symmetric equilibrium each firm samples prices from two disconnected intervals $[\underline{p}_M, v_L] \cup [\underline{p}'_M, \overline{p}_M]$, *with* $\underline{p}'_M > v_L$. *In the lower interval, firms draw prices from an atomless CDF* $F_{M_1}(p)$ *implicitly defined by*

$$(p-c)\left[\theta(1-\theta F_{M_1}(p))^{N-1} + (1-F_{M_1}(p))^{N-1}(1-\theta)^N\right] = \pi^*_M, \tag{9}$$

where

$$\pi^*_M \equiv \frac{\left(v_H - v_L - \frac{s}{\theta}\right)[\theta + (1-\theta)^N]\theta(1-\theta)^{N-1}}{\theta + (1-\theta)^N - \theta(1-\theta)^{N-1}} \tag{10}$$

denotes firms' equilibrium expected profit and

$$\underline{p}_M \equiv c + \frac{\left(v_H - v_L - \frac{s}{\theta}\right)\theta(1-\theta)^{N-1}}{\theta + (1-\theta)^N - \theta(1-\theta)^{N-1}}. \tag{11}$$

In the upper interval, firms draw prices from the atomless CDF

$$F_{M_2}(p) \equiv \frac{1}{\theta}\left[1 - \left(\frac{\pi^*_M}{\theta(p-c)}\right)^{\frac{1}{N-1}}\right], \tag{12}$$

where

$$\underline{p}'_M \equiv c + \frac{\pi^*_M}{\theta(1-\theta\kappa)^{N-1}}, \tag{13}$$

$$\overline{p}_M \equiv c + \frac{\left(v_H - v_L - \frac{s}{\theta}\right)[\theta + (1-\theta)^N]}{\theta + (1-\theta)^N - \theta(1-\theta)^{N-1}}, \tag{14}$$

and $\kappa \equiv F_{M_1}(v_L)$ *is implicitly defined by*

$$(v_L - c)\left[\theta(1-\theta\kappa)^{N-1} + (1-\kappa)^{N-1}(1-\theta)^N\right] - \pi^*_M = 0. \tag{15}$$

On the equilibrium path, each consumer keeps searching (in increasing order of prices) until a full match is found, and returns to purchase at the lowest-priced firm if $p_1 \leq v_L$ *and no full match is found at any firm.*

Proof. See Appendix. □

Remarkably, firms' pricing strategies in the gap equilibrium are still such that consumers' optimal search behavior is always efficient from a social point of view: Since

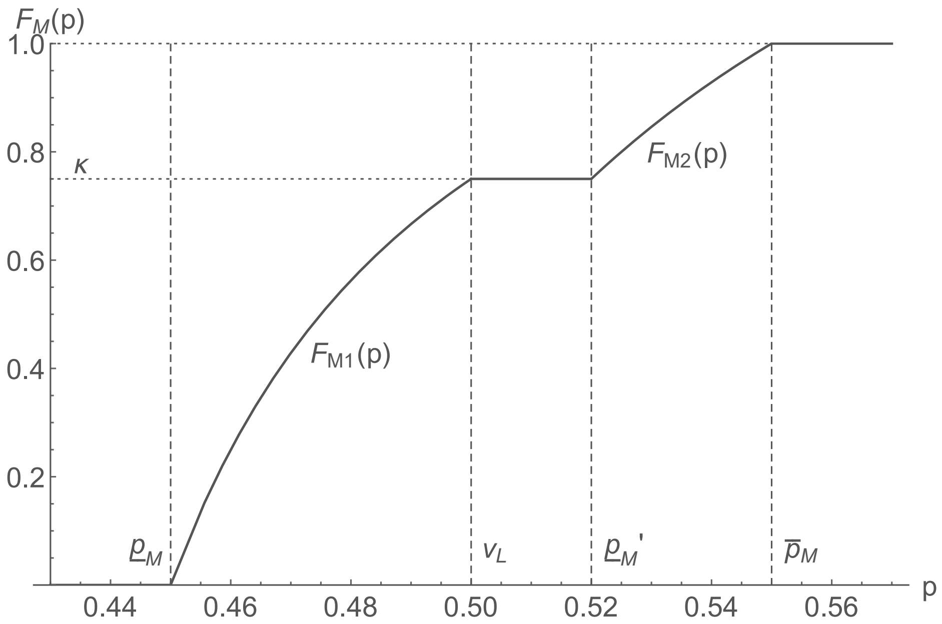

Figure 2: Example equilibrium CDF in the gap equilibrium. The parameters used are $v_H = 1$, $v_L = 0.5$, $s = 0.2$, $c = 0.4$, $\theta = 0.5$, $N = 2$.

$\overline{p}_M - \underline{p}_M = v_H - v_L - \frac{s}{\theta}$, the price difference between the lowest-priced firm and any higher-priced rival is never so large to stop non-fully matched consumers from searching on, even when $p_1 < v_L$ (compare with Lemma 1). This search behavior is again clearly optimal, since $s < \theta(v_H - v_L)$. However, even though consumers' search is efficient, it is also immediate that an avoidable welfare loss may still occur. This is because with probability $(1-\kappa)^N$, each firm chooses a price in the high range strictly above v_L, in which case the mass $(1-\theta)^N$ of consumers without a full match anywhere optimally forgo consumption after their search process has concluded – a probabilistic deadweight loss. If this welfare loss occurs, it is particularly pronounced when full matches are relatively rare.

Another interesting finding is that firms' equilibrium profits are independent of their (common) marginal costs of production in the gap equilibrium. The reason is that higher marginal costs of production have a similar effect on firms' incentive to compete as a higher degree of product differentiation (lower v_L/v_H): it becomes relatively more attractive to choose high prices aimed at fully matched consumer rather than to try to have the lowest price in the market and also be able to serve consumers who do not have a full match anywhere. As a result, competition relaxes by moving up the equilibrium pricing support one to one, maintaining the same level of equilibrium profits.

It should be noted that a gap equilibrium in a model of price-directed search has already been documented in the closely related work by Ding and Zhang (2018). However, compared to their model, the reasons why such an equilibrium may occur and why it entails a welfare loss, as well as the magnitude of this loss, are fundamentally different. In Ding and Zhang (2018), product matches are all-or-nothing, which translates to $v_L = 0$ in our setting. On the other hand, next to "uninformed consumers" who need to engage in costly search, there is also a share μ of "informed consumers" who don't face any search cost and know all match values. When μ is sufficiently large, a gap equilibrium arises in which firms optimally swing between choosing low prices catering to all consumers and high prices catering to informed consumers only. More precisely, firms either price in a low range up to the highest ("threshold") price uninformed consumers would ever search, $v_H - \frac{s}{\theta}$ in our notation, or in a high range that starts somewhere strictly above this threshold price and extends up to v_H. There is a hole in between because marginally increasing one's price starting from $p_i = v_H - \frac{s}{\theta}$ entails a discrete loss of demand by losing all search traffic from uninformed consumers. In contrast, in our model, a gap above v_L arises because marginally increasing one's price starting from v_L implies a probabilistic loss of demand from the mass $(1-\theta)^N$ of consumers who have no full match anywhere: in the event that all other firms price in the high range above v_L, a firm would sell to these consumers with $p_i = v_L$ (in which case they would return) but not with $p_i = v_L + \varepsilon$.

Regarding welfare losses, in the gap equilibrium in Ding and Zhang (2018), firms' equilibrium pricing is such that the uninformed consumers' search behavior may be inefficient: when having obtained no match at any firm pricing below $v_H - \frac{s}{\theta}$, these consumers leave the market, even though from a social perspective, they should also search higher-priced firms (since by assumption $s < \theta(v_H - c)$). In our model consumers' *search* is instead always efficient, while on the other hand a deadweight loss occurs if all firms price above v_L. The magnitude of the expected welfare losses in the two models' gap equilibria is also clearly different: In Ding and Zhang (2018), every single firm that prices above $v_H - \frac{s}{\theta}$ causes a welfare loss, as some uninformed consumers that would have a match at the respective firm are not served. In our setting, a welfare loss only occurs if all firms price above v_L, in which case the mass $(1-\theta)^N$ of consumers without a full match at any firm inefficiently forgo consumption *after* their search process has terminated.

4.3 Low-Price Equilibrium

As last type of equilibrium, we show that when product differentiation is relatively low (but not so low to imply Bertrand competition), $v_L/v_H \in [\tilde{\gamma}, \overline{\gamma})$, firms again resort to choosing prices from a convex support. However, in contrast to the high-price equilibrium of Proposition 2, the prices drawn now never exceed v_L.

Proposition 4. *Suppose that $v_L/v_H \in [\tilde{\gamma}, \overline{\gamma})$. Then in the unique symmetric equilibrium each firm samples prices continuously from the interval $[\underline{p}_M, \overline{p}_M]$, with $\overline{p}_M \leq v_L$, following the atomless CDF $F_{M_1}(p)$ and making an expected profit of π_M^*, where $\underline{p}_M$, $\overline{p}_M$, $F_{M_1}(p)$ and π_M^* are defined in Proposition 3. On the equilibrium path, each consumer keeps searching (in increasing order of prices) until a full match is found, and returns to purchase from the lowest-priced firm if no full match is found at any firm.*

Proof. See Appendix. □

The lower and upper pricing support bounds (as well as the equilibrium profit) in the low-price equilibrium have the same functional form as in the gap equilibrium characterized above. Since $\overline{p}_M - \underline{p}_M = v_H - v_L - \frac{s}{\theta}$ such that consumers keep searching until they find a full match, while still $s < \theta(v_H - v_L)$, this again implies that consumers' search process is efficient from a social point of view. Moreover, since now competition is so strong that $\overline{p}_M \leq v_L$, no deterministic or probabilistic deadweight loss occurs: eventually, all consumers buy – and all consumers who have a full match at at least one firm also end up with a fully matched product. This means that, same as in the Bertrand equilibrium arising for very high search costs, there is no welfare distortion in the low-price equilibrium. Note finally that, as for the gap equilibrium discussed in the previous subsection, firms' equilibrium profits are still independent of their (common) marginal costs, for the same reason as outlined there.

4.4 Welfare

We finish the equilibrium characterization by deriving explicit expressions for the equilibrium expected social welfare in the market. This is easily obtained: since all prices paid are pure transfers, social welfare is given by the expected net surplus realized through consumption minus the total search costs incurred.

In the “Bertrand region” where Proposition 1 applies, each consumer searches only one random firm, obtains a match value of v_H or v_L with probability θ and $1-\theta$, respectively, and buys there deterministically. Hence, total social welfare in the Bertrand region equals $W_B = \theta v_H + (1-\theta)v_L - s$. In all other regions, we have established that each consumer keeps searching until a full match is found (if at any firm). In these regions, the aggregate search friction incurred is thus given by[19]

$$S = \left(\sum_{k=1}^{N-1} \theta(1-\theta)^{k-1}ks\right) + (1-\theta)^{N-1}Ns = s\left[\frac{1-(1-\theta)^N}{\theta}\right]. \tag{16}$$

At the same time, the net consumption surplus depends on the equilibrium which is played. In the high-price equilibrium, a fraction $(1-\theta)^N$ of consumers does not find a full match at any firm and therefore drops out of the market, such that the net consumption surplus is given by $(v_H - c)\left[1-(1-\theta)^N\right]$. In the low-price equilibrium, once again a fraction $(1-\theta)^N$ of consumers does not find a full match at any firm, but now these consumers will also buy with their partial match (at the lowest-priced firm). Hence, the net consumption surplus is given by $(v_H - c)\left[1-(1-\theta)^N\right] + (v_L - c)(1-\theta)^N$. Finally, in the gap equilibrium, the fraction $(1-\theta)^N$ of consumers who do not have a full match at any firm will only buy with their partial match if the lowest-priced firm prices below v_L, which happens with probability $1-(1-\kappa)^N$. Hence, the *expected* consumption surplus in this case is given by $(v_H - c)\left[1-(1-\theta)^N\right] + (v_L - c)(1-\theta)^N\left[1-(1-\kappa)^N\right]$. Subtracting the aggregate search friction S from these aggregate match values, the subsequent lemma is immediate.

[19] Note that for $k = 1, \ldots, N-1$, a fraction $(1-\theta)^{k-1}\theta$ of consumers has no full match at the first $k-1$ sampled firms and a full match at the k'th sampled firm, with a per-consumer search cost of ks (first term). A fraction $(1-\theta)^{N-1}$ of consumers has no full match at the first $N-1$ firms and therefore searches all firms, with a per-consumer search cost of Ns (second term). The second equality can then easily be established via induction starting from $N = 2$.

Lemma 3. *Total social welfare in the market is given by*

$$W = \begin{cases} \left(v_H - \frac{s}{\theta} - c\right)\left[1-(1-\theta)^N\right] & \text{if } \frac{v_L}{v_H} \leq \underline{\gamma} \\ \left(v_H - \frac{s}{\theta} - c\right)\left[1-(1-\theta)^N\right] + (v_L - c)(1-\theta)^N\left[1-(1-\kappa)^N\right] & \text{if } \frac{v_L}{v_H} \in (\underline{\gamma}, \tilde{\gamma}) \\ \left(v_H - \frac{s}{\theta} - c\right)\left[1-(1-\theta)^N\right] + (v_L - c)(1-\theta)^N & \text{if } \frac{v_L}{v_H} \in [\tilde{\gamma}, \overline{\gamma}) \\ \theta v_H + (1-\theta)v_L - s - c & \text{if } \frac{v_L}{v_H} \geq \overline{\gamma}. \end{cases} \tag{17}$$

As discussed in detail above, welfare losses occur in the high-price and gap equilibrium regions: if all firms were forced to, for example, set some common price between c and v_L, the mass $(1-\theta)^N$ of consumers without a full match at any firm would purchase (deterministically instead of probabilistically in the gap-equilibrium region), creating an additional surplus of $v_L - c$ for each additional consumer served. Moreover, the aggregate search friction would not be affected, since all consumers would still find it optimal to search until they find a full match. We may hence state the following:

Proposition 5. *In the high-price equilibrium ($v_L/v_H \leq \underline{\gamma}$), a deterministic welfare loss of $(v_L - c)(1-\theta)^N$ occurs. In the gap equilibrium ($v_L/v_H \in (\underline{\gamma}, \tilde{\gamma})$), an expected welfare loss of $(v_L - c)(1-\theta)^N(1-\kappa)^N$ occurs.*

5 The Effects of Lower Search Costs

The widespread adoption of the Internet, the emergence of a wide array of price-comparison websites and product search engines, as well as the ongoing improvement of smartphones and mobile applications has arguably led to a steady decline in consumers' costs of searching and comparing products. In this section, we therefore study the comparative effects of a reduction of search costs within our model framework.

We will subsequently define "sales" as price draws that do not exceed v_L, such that firms have a chance to sell also to partially matched consumers when pricing accordingly. It is then first easy to establish the following.

Proposition 6. *Suppose that $s < \theta(v_H - v_L)$ (equivalently, $v_L/v_H < \overline{\gamma}$) such that the Bertrand equilibrium is not played. Then a decrease in search costs leads to strictly higher equilib-*

rium prices (in the sense of first order stochastic dominance and therefore also in expectation) and equilibrium expected profits and a weakly lower probability that firms engage in sales (strictly so in the gap equilibrium).

Proof. See Appendix. □

The intuition is that a lower search cost makes consumers more willing to search on after having obtained only partial matches so far, enabling firms to attract these consumers (and sell to them when a full match is realized) with higher prices and thereby relaxing competition. As a direct consequence, firms' expected prices and profits increase and they may reduce their propensity to engage in sales. The finding that lower search costs unambiguously increase prices and profits is also featured in the models of price-directed search by Armstrong and Zhou (2011), Choi et al. (2018) and Haan et al. (2018), and it is in stark contrast to the result in standard models of random search with unobservable firm pricing (such as Wolinsky (1986), Stahl (1989) and Anderson and Renault (1999)).

Notably, different from the closely related model of price-directed search in Ding and Zhang (2018), the price-increasing effect of a lower search cost also prevails in our gap equilibrium. The reason is that in this equilibrium, firms mix between choosing a sale price below v_L, which gives them a chance to sell to returning consumers who are everywhere only partially matched, and pricing in a high range (up to some endogenous maximal price strictly below $v_H - \frac{s}{\theta}$) catering to fully matched consumers only. When now s decreases, then for fixed rival pricing, higher prices in the high range could be chosen without losing traffic from so-far only partially matched consumers, making pricing in the high range more attractive. The equilibrium adjusts such that the highest price in firms' pricing support increases and firms sample prices from the high range more often, reducing their frequency of sales.

In contrast, in Ding and Zhang's gap equilibrium, firms mix between pricing below $v_H - \frac{s}{\theta}$, which allows them to sell to uninformed consumers who need to engage in costly search – these authors' interpretation of "sales" – and pricing in a high range up to v_H, which only attracts demand from informed consumers. In this case, a decrease in s increases the highest price $v_H - \frac{s}{\theta}$ uninformed consumers are willing to search, rendering pricing in the low range more attractive. This leads firms to increase their frequency of sales and choose lower prices on average.

We finally turn to the impact of lower search costs on total welfare and consumer surplus.

Proposition 7. *A decrease in search costs (i) strictly increases the expected total social welfare whenever $N \geq 3$ and (ii) may increase or decease the expected consumer surplus.*

Proof. See Appendix. □

A decrease in search costs s has two effects on welfare. On the one hand, it directly lowers the aggregate search friction. On the other hand, as documented above, it makes pricing less competitive, shifting the equilibrium price distribution to the right. However, since the prices paid are mere redistributions, we only have to examine the effect of lower search costs on the expected consumption surplus generated in order to evaluate their impact on welfare. In the Bertrand, high-price and low-price equilibrium, this is deterministic and independent of s, thus a decrease of search costs unambiguously improves welfare. In the gap equilibrium, a decrease in search costs actually decreases the expected consumption surplus, since the probability of at least one firm engaging in a sale decreases (compare with Proposition 6 above). Still, we can show that the reduction of the aggregate search friction outweighs the expected loss of net consumption utility for almost all parameters combinations where the gap equilibrium is played, the only exception being when $N = 2$ and both v_L/v_H and θ are small.

Interestingly, the expected consumer surplus often decreases after a reduction of search costs. The reason is that consumers have to pay higher prices on average due to the strategic effect on firms' pricing, which may dominate their gains stemming from less costly search. In particular, we show in the proof of Proposition 7 that this happens when the gap equilibrium is played and both v_L/v_H and θ are relatively small.[20] A partial intuition is that for small v_L, sale prices below v_L create a large surplus for fully matched consumers at the respective firms; moreover, they allow the segment $(1-\theta)^N$ of consumers who do not find a full match at any firm to recover some of their losses from search. A reduction of search costs now makes firms less likely to price below v_L, causing a large expected harm for consumers. Figure 3 illustrates how the expected consumer surplus depends on s for the case of

[20]When the Bertrand equilibrium is played, consumers clearly always benefit from lower search costs, and we show in the proof of Proposition 7 that the same holds for the high-price equilibrium. In contrast, the expected consumer surplus may also fall as s decreases in the low-price equilibrium, however we can prove that this only occurs when $N = 2$ and $\theta \leq 0.5$. Details to the latter are available from the authors upon request.

two, three and four firms. It can clearly be seen that a reduction of s may indeed decrease the expected consumer surplus in the market over a wide range of search costs.

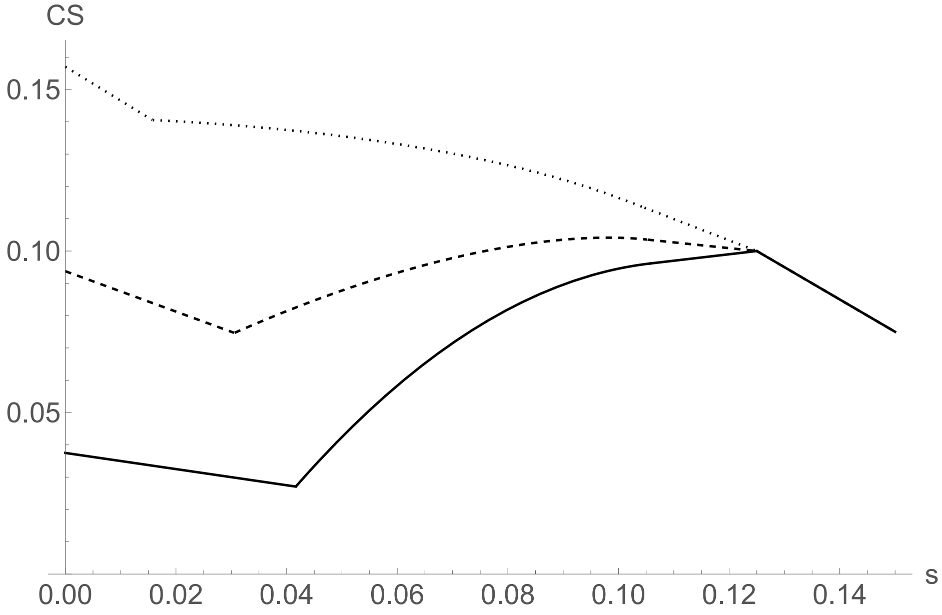

Figure 3: Expected consumer surplus as a function of s for $N = 2$ (solid line), $N = 3$ (dashed line) and $N = 4$ (dotted line). The parameters used are $v_H = 1$, $v_L = 0.5$, $c = 0.4$, $\theta = 0.25$.

6 Random Search

So far, we have limited the analysis to the case where prices are freely observable to consumers, so that their optimal search behavior is directly guided by them. Our main motivation in doing so was to develop a tractable model of price-directed search that allows for a flexible degree of product differentiation and leads to more realistic consumer search patterns, compared to previous work involving mixed-strategy pricing equilibria. However, it is also interesting to explore what our model framework predicts – and how this differs from the baseline model's results – when product prices can only be discovered upon search, such that consumers' search is essentially random.

To this end, we modify the baseline model by assuming that consumers cannot observe any firm's price before searching it: searching a firm's product now reveals *both* its price and match value. In line with standard models of random sequential search (see e.g. Anderson and Renault (1999), Stahl (1989, 1996)), we suppose that the number of remaining options is still known to consumers (e.g., they can access a list of all relevant firms, but their product prices are not directly observable), but because of the indistinguishability of these yet unexplored options, a new search selects one of them at random. As is also common in the literature, and in order to avoid situations of market breakdown, we further assume in what follows that consumers' first search is costless.[21] All other model features remain unchanged.

We look for symmetric Perfect Bayesian Equilibria (PBEs) wherein (i) consumers search and purchase optimally, given their available offers and their beliefs about unexplored firms' pricing strategies, (ii) firms price optimally, given consumers' search behavior and their rivals' pricing strategies, and (iii) consumers' beliefs are consistent with firms' pricing strategies. We also adopt the standard assumption of "passive" out-of-equilibrium beliefs. This means that if a consumer observes a price that is not supposed to be charged in equilibrium, she does not alter her beliefs regarding the pricing of yet unexplored options.

In the following, we start with an equilibrium characterization under random search before turning to a welfare analysis and comparison with the baseline model.

6.1 Equilibrium Analysis with Random Search

A common feature and major tractability advantage of models of random sequential search for differentiated products in the spirit of Wolinsky (1986) and Anderson and Renault (1999) is that these normally give rise to (symmetric) pure-strategy pricing equilibria. In our setting with binary product match values, simple symmetric pure-strategy equilibria are however sometimes jeopardized, as utility ties would occur which give firms scope for profitable undercutting.[22] The following proposition characterizes the set of symmetric pure-strategy PBEs in our model variant with random search (compare also with Figure 1 above).

[21] We will outline below when this assumption can be dispensed with.

[22] It may be noted that utility ties are however not a prerequisite for the failure of existence of a pure-strategy equilibrium, so that our setup is not a knife-edge case. Indeed, in the parameter region of our model where no pure-strategy equilibrium exists, it can be shown that adding a sufficiently small amount of additional differentiation drawn from a continuous distribution would not restore its existence.

Proposition 8. *Suppose that consumers cannot observe firms' prices before searching them, such that their search order is random. Suppose moreover that their first search is costless. Then, if $v_L/v_H < \check{\gamma}$, where*

$$\check{\gamma} \equiv \frac{c}{v_H} + \left(1 - \frac{c}{v_H}\right)\theta \in (\underline{\gamma}, 1), \tag{18}$$

in the unique symmetric pure-strategy PBE firms set $p^* = v_H$. Consumers search a single firm and buy there if they find a full match, otherwise they leave the market. If instead $v_L/v_H > \max\{\overline{\gamma}, \check{\gamma}\}$, in the unique symmetric pure-strategy PBE firms set $p^* = v_L$. Consumers search a single firm and buy there no matter whether they find a full or partial match. Finally, if $v_L/v_H \in (\check{\gamma}, \overline{\gamma})$, no symmetric pure-strategy PBE exists.[23]

Similar to the setting with price-directed search, we find that also under random search, the degree of product differentiation crucially affects the equilibrium outcome. When product differentiation is large, $v_L/v_H < \check{\gamma}$, firms choose the highest possible price $p^* = v_H$ and only serve fully matched consumers in the unique symmetric PBE; in contrast, when product differentiation is small and search costs are large, $v_L/v_H > \max\{\check{\gamma}, \overline{\gamma}\}$,[24] firms settle for the intermediate price $p^* = v_L$ and also serve partially matched consumers in the unique symmetric PBE. However, since firms cannot induce consumers to seek them out early by choosing low prices under random search, firms have no reason to randomize prices in these two equilibria, and active consumer search does not occur. Moreover, it is easy to see that in the case of a costly first search, these two equilibria would break down, leading to market collapse as in homogeneous-goods (Diamond 1971-type) models with costly first search.

Interestingly, when product differentiation is small and search costs are not too large, $v_L/v_H \in (\check{\gamma}, \overline{\gamma})$, despite consumer's random search, no symmetric pure-strategy PBE can be sustained. The reason is that, in contrast to standard sequential search models for differentiated goods such as Wolinsky (1986) and Anderson and Renault (1999), in our model match values are drawn from a discrete rather than continuous distribution. This implies that sym-

[23]In the knife-edge case where $v_L/v_H = \check{\gamma}$, the described PBE for $v_L/v_H < \check{\gamma}$ with $p^* = v_H$ is always an equilibrium, whereas the described PBE for $v_L/v_H > \max\{\overline{\gamma}, \check{\gamma}\}$ with $p^* = v_L$ is additionally an equilibrium if and only if $s \geq \theta(v_H - v_L)$. In the other knife-edge case where $v_L/v_H = \overline{\gamma}$ and $s \leq \theta(1-\theta)(v_H - c)$ such that $\max\{\overline{\gamma}, \check{\gamma}\} = \overline{\gamma}$, there is actually a continuum of symmetric pure-strategy PBEs where all firms choose the same $p^* \in [c + \frac{\theta(v_H - v_L)}{1-\theta}, v_L]$, and consumers immediately purchase when discovering a partial match at p^*. Further information about these non-generic cases is available from the authors upon request.

[24]Recall that $v_L/v_H > \overline{\gamma}$ is equivalent to $s > \theta(v_H - v_L)$.

metric pure-strategy equilibria involving active search (for full matches) do not exist, as in any such candidate equilibrium firms have an incentive to marginally undercut the candidate equilibrium price to resolve utility ties in their favor.

Naturally, it would be desirable to characterize the set of symmetric mixed-strategy PBEs that arise in the above parameter region. Unfortunately, it turns out that even for the most straightforward case of duopoly, the analysis becomes nearly intractable, and a closed-form equilibrium characterization does not appear to exist.[25] In order to still gain some insight into the equilibrium outcome in this scenario, we therefore consider the limit case of monopolistic competition as the number of firms becomes infinite (cf. Wolinsky (1986)). The major tractability advantage of this model variation is that for any anticipated equilibrium price distribution, consumers' optimal search behavior is stationary (i.e., independent of the price/match-value combinations discovered so far), and in particular returning demand can be ruled out. The following proposition characterizes the unique symmetric PBE when there is a continuum of firms and $v_L/v_H \in (\check{\gamma}, \overline{\gamma})$, such that neither $p^* = v_H$ nor $p^* = v_L$ can be supported in equilibrium under oligopolistic competition.

Proposition 9. *Suppose that consumers cannot observe firms' prices before searching them, there is a continuum of firms with measure* 1*, and* $v_L/v_H \in (\check{\gamma}, \overline{\gamma})$*. Then no symmetric pure-strategy equilibrium exists. There is a unique symmetric mixed-strategy equilibrium in which each firm randomizes whether to set price* p_L^* *with probability* α *or to set price* $p_H^* > p_L^*$ *with probability* $1-\alpha$*, where*

$$p_L^* = c + \frac{\theta(v_H - v_L)}{1-\theta} < v_L,$$
$$p_H^* = c + \frac{v_H - v_L}{1-\theta} < v_H,$$
$$\alpha = \frac{s}{\theta(v_H - v_L)} \in (0,1).$$

[25] For $s \approx 0$, we conjecture that the symmetric mixed-strategy PBE will be "close" to the concurrent mixed-strategy PBE under price-directed search, i.e., close to the low-price or gap equilibrium, depending on v_L/v_H. For example, for v_L/v_H large such that the low-price equilibrium arises under price-directed search and s close to zero, we conjecture that there is a unique symmetric mixed-strategy PBE with the following structure: Firms draw prices either from a low range, $[\underline{p}, \rho_H]$, or from a high range, $[\underline{p}', \overline{p}]$, where ρ_H denotes the "reservation price" for fully matched consumers (i.e., the critical price threshold above which fully matched consumers optimally search on, given firms' anticipated pricing strategies) and where $c < \underline{p} < \rho_H < \underline{p}' < \overline{p} = \underline{p} + v_H - v_L \leq v_L$. As s tends to zero, ρ_H and $\underline{p}'$ would move closer to $\underline{p}$ and eventually coincide with it for $s = 0$, giving rise to the same equilibrium CDF as under price-directed search and $s = 0$.

When a consumer discovers a partial (full) match at some firm i, she buys immediately when $p_i \leq p_L^*$ ($p_i \leq p_H^*$), and otherwise searches on (and never returns).

The "monopolistic competition" limit equilibrium under random search and $v_L/v_H \in (\check{\gamma}, \overline{\gamma})$ shares several properties with the concurrent equilibrium under oligopolistic competition and price-directed search. A lower degree of product differentiation again drives down prices, both by directly reducing p_L^* and p_H^* and by increasing the frequency with which firms sample the low price. Moreover, and in contrast to the usual result that higher search costs drive up prices under random search (see e.g. Stahl (1989); Wolinsky (1986); Anderson and Renault (1999)), our finding from the price-directed-search baseline model that a higher search cost induces firms to price more aggressively carries over also to the case of random search. Indeed, while the equilibrium prices p_L^* and p_H^* are independent of s, higher search costs lead firms to set the low price more often.[26]

It may also be noted that the limit equilibrium would not break down even with a costly first search. To see this, note that this is the case if and only if the expected consumer payoff from searching a single firm following the equilibrium search strategy,

$$\theta[v_H - \alpha p_L^* - (1-\alpha)p_H^*] + (1-\theta)\alpha(v_L - p_L^*) - s, \tag{19}$$

is non-negative across the relevant parameter range.[27] As (19) clearly strictly increases in v_L (since both prices decrease in v_L while α – the probability that firms choose the low price – increases in v_L), it thus suffices to check that (19) is non-negative for the lowest possible $v_L = v_H\check{\gamma}$, which is true as then (19) is exactly zero. Although we have not been able to characterize the pricing equilibrium under random search with a finite number of firms, this suggests that for any $v_L/v_H \in (\check{\gamma}, \overline{\gamma})$, the market would not break down under random search and a costly first search when the number of firms is sufficiently large.

[26]The reason for this is rather subtle. In the proof of the proposition, we show that the only possible symmetric mixed-strategy PBE is a two-price equilibrium in which partially (fully) matched consumers are indifferent between purchasing immediately and searching on when facing the low (high) price. But if this is the case in equilibrium, low- and high-price firms' demands and profits are independent of s. When now s increases, to keep consumers that are partially matched at low-price firms indifferent between buying and searching on, firms need to increases their frequency of choosing the low price.

[27]While it is possible that $v_L - p_H^* > 0$ so that a single search gives a strictly higher expected payoff, it is easy to check that then (19) is automatically positive.

6.2 Welfare Comparison

We conclude our discussion of the model variant with random search with a brief comparison of its welfare results with those of the baseline model with price-directed search. For this, note first that for $v_L/v_H < \check{\gamma}$ and $v_L/v_H > \max\{\check{\gamma},\bar{\gamma}\}$, the pure-strategy equilibria under random search characterized in Proposition 8 do not exist if consumers' first search is costly; rather, the market would collapse. Since, given our parameter assumptions, market breakdown is never an issue under price-directed search, welfare is clearly strictly higher in that case. If we assume instead that consumers' first search is costless both under random and price-directed search, it is immediate that welfare is identical across the two search regimes for $v_L/v_H > \max\{\check{\gamma},\bar{\gamma}\}$, while it is still strictly lower under random search for $v_L/v_H < \check{\gamma}$.

Most interesting is the welfare comparison of the search regimes when $v_L/v_H \in (\check{\gamma},\bar{\gamma})$, where mixed-strategy equilibria ensue both under random and price-directed search. Moreover, at least for the characterized limit equilibrium under random search, market breakdown does not occur even with a costly first search, hence for the welfare comparison it does not matter whether we take the first search as costly or not.[28] Now, in order to be able to compare the search regimes on an equal footing, we need to consider also a limit version of the baseline model with price-directed search as the number of firms tends to infinity. Since by assumption $v_L/v_H < \bar{\gamma}$, it is then easy to see from the welfare expressions under price-directed search in Lemma 3 that as N tends to infinity, total social welfare gets arbitrarily close to $W_{lim}^{dir} = v_H - \frac{s}{\theta} - c$.[29] In contrast, using the characterization of the limit equilibrium under random search provided in Proposition 9, it may be observed that welfare in this case can be defined recursively via

$$W_{lim}^{rand} = -s + \theta(v_H - c) + (1-\theta)\alpha(v_L - c) + (1-\theta)(1-\alpha)W_{lim}^{rand},$$

which gives

$$W_{lim}^{rand} = \frac{-s + \theta(v_H - c) + (1-\theta)\alpha(v_L - c)}{1-(1-\theta)(1-\alpha)}.$$

[28] For the welfare expressions to be derived below, we will treat the first search as costly, same as in the baseline model.

[29] The reason is simple: In equilibrium, consumers keep searching until they find a full match, which eventually creates a surplus of $v_H - c$. The expected search cost to find a full match is $s + (1-\theta)s + (1-\theta)^2 s + \ldots = \frac{s}{\theta}$.

It is straightforward to check that W_{lim}^{rand} strictly *decreases* in α, the probability that firms choose the low price. This is because the higher α, the higher the probability that consumers stop after finding a partial match at some firm; however, since by assumption $s < \theta(v_H - v_L)$, from society's point of view consumers *should* keep searching until they find a full match. As the limit of W_{lim}^{rand} as $\alpha \to 0$ is W_{lim}^{dir}, but $\alpha > 0$ in the considered parameter range, this shows that at least for the limit equilibria, price-directed search gives rise to a strictly higher total social welfare than random search. Hence, observability of prices unambiguously improves the market outcome. Unfortunately, since we have not been able to pin down the equilibria under random search for a finite number of firms, we cannot conduct the same welfare comparison under oligopolistic competition. However, it seems natural to expect that price-directed search will also lead to a higher market performance for every finite number of firms N. One indication for this is that even when there is only duopoly competition under price-directed search, for a large set of parameters, still a higher social welfare is obtained than in the limit equilibrium under random search.[30]

7 Conclusion

We have set up a tractable model of price-directed search in which consumers observe prices, but need to engage in costly sequential search to find out whether products fully or only partially match their needs. We have characterized consumers' optimal search behavior and the set of symmetric pricing equilibria caused by different degrees of product differentiation. While it turns out that consumers' equilibrium search behavior is always efficient from a social point of view, welfare losses still occur, as all firms may (deterministically or stochastically) price above consumers' valuation for partial matches. If this happens, part of the consumers inefficiently drop out of the market. Investigating the impact of lower search costs on market outcomes, we establish that these lead to stochastically higher prices and firm profits, but typically also to higher total social welfare. In contrast, consumers' expected surplus may well fall when search costs decrease. We have also provided evidence that unobservability of prices, leading to random rather than price-directed search, should tend to decrease market performance.

[30] A numerical analysis, also for larger N, is available from the authors upon request.

For future work, it might be interesting to incorporate a share of fully informed consumers without search costs into the model, essentially blending our framework for the analysis of price-directed search with the one in Ding and Zhang (2018). While for a sufficiently low share of such informed consumers, we can show that our pricing equilibria prevail, we expect that for larger shares, quite intricate pricing patterns will emerge. For example, we conjecture that a price equilibrium with two gaps may exist in which the firms randomize between targeting partially matched consumers, fully matched uninformed consumers, and fully matched informed consumers. Another promising route may be to introduce various forms of observable or unobservable firm heterogeneity into tractable models of price-directed search, examining the effects on equilibrium pricing and market outcomes. In particular, the impact of unobservable quality differences on the interaction between firms' pricing and consumers' search behavior does not seem to be well understood.

Ultimately, we hope that our model will both serve as useful building block for applied researchers studying markets with price-directed search – for which it may be seen as complementary to the contribution of Ding and Zhang (2018) by allowing for a variable degree of product differentiation – and as a starting point for further modeling developments.

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8 Appendix: Technical Proofs

Proof of Proposition 2. We first give a detailed existence proof. We then provide a sketch how uniqueness can be established.

Existence. When setting some price p anywhere in the candidate equilibrium's support, firm i's expected demand is $(1-\theta F_H(p))^{N-1}\theta$. This is because any given rival firm will only stop some consumer from visiting firm i if it provides a full match to the consumer and has a lower price, which happens with probability $\theta F_H(p)$. With complementary probability $1-\theta F_H(p)$, this is not the case for any given rival firm, such that with probability $(1-\theta F_H(p))^{N-1}$, not a single rival firm blocks the consumer from visiting firm i. In this case, the consumer purchases if and only if firm i provides a full match, which happens with probability θ.

Given that all other firms sample prices from the CDF $F_H(p)$ as defined in equation (4), it is then easy to see that for any price in the candidate equilibrium's support $[\underline{p}_H, \overline{p}_H]$, it indeed holds that $\pi_i(p) = (p-c)(1-\theta F_H(p))^{N-1}\theta = \pi_H^*$, with π_H^* as defined in equation (7). It is moreover straightforward to check that given the imposed parameter restriction $v_L/v_H \leq \underline{\gamma}$, $F_H(p)$ is strictly increasing over its support, and that $\underline{p}_H > v_L$. Hence, the candidate equilibrium is well-behaved.

We now rule out profitable deviations outside the candidate equilibrium's pricing support. For this, note first that it clearly cannot be optimal to deviate to any price in the range $(v_L, \underline{p}_H)$, as the same demand would already be achieved when pricing at $\underline{p}_H$. Note next that when deviating to v_L, a firm would make an expected profit of $\pi_i(v_L) = (v_L - c)\left[\theta + (1-\theta)^N\right]$, as it would become the lowest-priced firm that is sampled first with certainty, attracting all of its fully matched consumers as well as all consumers with no full match at any firm (who would eventually return to the deviating firm after having searched all firms). Note moreover that those consumers who are only partially matched at the deviating firm would always continue to search, since even if all rival firms priced at $\overline{p}_H$, the expected gains from search would be non-negative. It is then easy to see that $\pi_i(v_L) \leq \pi_H^*$ if and only if $v_L/v_H \leq \underline{\gamma}$.

We next establish that under the relevant parameter restrictions, it is never profitable to price below v_L, as the expected profits for any deviation price $p \in (c, v_L)$ are lower than when deviating to v_L. To see this, note that since the deviating firm is guaranteed to be searched first, the fraction θ of consumers who find a full match at this firm will immediately buy there. Furthermore, consumers who only find a partial match will only search those rival firms j (and buy there in case they find a full match) whose price difference is not too large relative to the deviation price, that is, for which $p_j \leq p + v_H - v_L - \frac{s}{\theta}$ (compare with Lemma 1). The probability that one rival sets $p_j \leq p + v_H - v_L - \frac{s}{\theta}$ (such that it will be searched) and provides a full match (such that it will attract the deviating firm's partially matched consumers) is given by $F_H\left(p + v_H - v_L - \frac{s}{\theta}\right)\theta$. Hence, the probability that *not a single* rival firm does so is given by $\left[1 - F_H\left(p + v_H - v_L - \frac{s}{\theta}\right)\theta\right]^{N-1}$. In turn, the expected deviation profits for $p \leq v_L$ can be written as

$$\begin{aligned}\pi_i(p) &= (p-c)\left[\theta + (1-\theta)\left[1 - F_H\left(p + v_H - v_L - \frac{s}{\theta}\right)\theta\right]^{N-1}\right] \\ &= (p-c)\left[\theta + (1-\theta)^N\left(\frac{v_H - \frac{s}{\theta} - c}{p - c + v_H - v_L - \frac{s}{\theta}}\right)\right].\end{aligned}$$

Since $\frac{p-c}{p-c+v_H-v_L-\frac{s}{\theta}}$ is strictly increasing in p when $v_H - v_L - \frac{s}{\theta} > 0$ (as holds in the considered parameter region), it is easy to see that the last expression is strictly increasing in p. It is thus indeed maximized for $p = v_L$, such that deviations below v_L cannot be optimal.

It remains to show that there is no profitable deviation above $\overline{p}_H = v_H - \frac{s}{\theta}$. But since no firm would ever be searched for $p > \overline{p}_H$ (compare once again with Lemma 1), this is immediately evident. This completes the proof of existence.

Uniqueness. For brevity, we only provide a sketch how uniqueness can be established in the class of symmetric equilibria. This sketch also applies for the subsequent Propositions 3 and 4.

Note first that the parameter requirement for Proposition 2 (as well as Propositions 3 and 4) is that $v_L/v_H < \overline{\gamma}$, which is equivalent to $\theta(v_H - v_L) > s$. Only in this case, consumers may have an incentive to search on after discovering only a partial match at the lowest-priced firm, avoiding the Bertrand outcome as unique symmetric equilibrium.

Second, since the parameter requirement $\theta(v_H - v_L) > s$ is equivalent to $v_H - \frac{s}{\theta} > v_L$, it follows immediately from consumers' optimal search rule in Lemma 1 that no firm can make a positive profit when pricing strictly above $p_{max} \equiv v_H - \frac{s}{\theta}$, as it would never be searched. But clearly, each firm can guarantee a positive profit by pricing at $c + v_H - v_L - \frac{s}{\theta} > c$, since it would be searched by only partially matched consumers even for $p_1 = c$. Hence, no firm may ever price above p_{max} in equilibrium.

Third, given that $\theta(v_H - v_L) > s$, clearly no symmetric pure-strategy equilibrium can exist, as marginally undercutting any symmetric candidate equilibrium price $p^* \in (c, v_H - \frac{s}{\theta}]$ would give a firm a discretely higher profit (by being searched first by all consumers). By a similar logic, there can be no mass points in any symmetric equilibrium.

Denoting $\overline{p}$ and $\underline{p}$ as the upper and lower support bound of any symmetric candidate equilibrium, with $\overline{p} \leq p_{max}$, the crucial steps are now to establish that either (i) $\overline{p} - \underline{p} < \Delta \equiv v_H - v_L - \frac{s}{\theta}$ and $\overline{p} = p_{max}$ or (ii) $\overline{p} - \underline{p} = \Delta$. The former is trivial to see by contradiction: in any candidate equilibrium where $\overline{p} - \underline{p} < \Delta$ and $\overline{p} < p_{max}$, a firm choosing $p_i = \overline{p}$ could increase its profit by choosing $p_i = \min\{\underline{p} + \Delta, p_{max}\}$ instead. This gives the firm an identical demand of $(1-\theta)^{N-1}\theta$ at a higher price (in particular, since there can be no mass point at $\overline{p}$).

It is somewhat more demanding to show that $\overline{p} - \underline{p} > \Delta$ cannot hold. This can be proven by contradiction via the following steps: (1) For $\overline{p} - \underline{p} > \Delta$, it must hold that the density $f(\overline{p}) \overset{!}{=} 0$ by comparing $\lim_{p \uparrow \overline{p}} \pi_i'(p)$ with $\lim_{p \downarrow \overline{p}} \pi_i'(p)$, (2) from this, it follows that the density $f(\overline{p} - \Delta) > 0$, such that $\overline{p} - \Delta$ must lie in the equilibrium support, (3) $\lim_{p \uparrow (\overline{p}-\Delta)} \pi_i'(p) = \lim_{p \downarrow (\overline{p}-\Delta)} \pi_i'(p)$ as a consequence of $f(\overline{p}) = 0$, (4) combining the conditions $\pi_i'(\overline{p}) \stackrel{!}{=} 0$ and $\pi_i'(\overline{p} - \Delta) \stackrel{!}{=} 0$[31] and finally observing that this leads to a contradiction.

Using the result that either (i) $\overline{p} - \underline{p} < \Delta$ and $\overline{p} = p_{max}$ or (ii) $\overline{p} - \underline{p} = \Delta$, the required profit indifference at $\overline{p}$ and $\underline{p}$ gives rise to a respectively unique solution for $\overline{p}$, $\underline{p}$ and the candidate equilibrium profit π^*, both for (i) (as provided in equations (5), 6) and (7)) and (ii) (as provided in equations (10), (11) and (14)). The corresponding $\overline{p}$ for (ii) is however not compatible with $\overline{p} \leq p_{max}$ if $v_L/v_H \leq \underline{\gamma}$,[32] as assumed for Proposition 2 (while it is compatible with it for $\frac{v_L}{v_H} \in (\underline{\gamma}, \overline{\gamma})$, in which case the candidate equilibrium following (i) does not exist). Noting finally that with $\overline{p} - \underline{p} \leq \Delta$ there can be no holes in the equilibrium support apart possibly from some range immediately above v_L, in each case the respective equilibrium follows uniquely from construction. □

Proof of Proposition 3 It is convenient to first provide the slightly simpler proof of Proposition 4, which we do below. The actual proof of Proposition 3 follows immediately afterwards.

Proof of Proposition 4. In what follows, we prove existence. For uniqueness, the argument at the end of the proof of Proposition 2 applies.

Existence. It is first easy to check that $\overline{p}_M - \underline{p}_M = v_H - v_L - \frac{s}{\theta}$, $\overline{p}_M \leq v_L$ due to $v_L/v_H \geq \tilde{\gamma}$, and $\underline{p}_M > c$ due to $v_L/v_H < \overline{\gamma}$. Moreover, it holds that $\pi_M^* = (\overline{p}_M - c)(1-\theta)^{N-1}\theta = (\underline{p}_M - c)[\theta + (1-\theta)^N]$. Consumers' optimal search rule (see Lemma 1) implies that firm *i*'s expected demand when setting some price p anywhere in the candidate equilibrium support is given by $\theta(1 - \theta F_{M_1}(p))^{N-1} + (1 - F_{M_1}(p))^{N-1}(1-\theta)^N$. The first term follows from the same logic as in the proof of Proposition 2, whereas the second term stems from firm *i*'s returning demand: with probability $(1 - F_{M_1}(p))^{N-1}$, all rival firms choose a higher price, such that firm *i* attracts the mass $(1-\theta)^N$ of consumers who don't find a full match anywhere and therefore return to firm *i*.

By construction, the implicit definition of $F_{M_1}(p)$ in equation (9) now ensures that all prices in the candidate equilibrium's support yield the same expected profit. One may also note from equation (9) that $F_{M_1}(p)$ is strictly increasing in its support. Hence, all equilibrium objects are well-behaved.

We now rule out profitable deviations outside the candidate equilibrium's pricing support. First, we show that there is no profitable deviation above $\overline{p}_M$. A deviating firm pricing at some $p > \overline{p}_M$

[31] The latter must be true since $\overline{p} - \Delta$ lies in the equilibrium support, such that there must also be probability mass immediately below or above $\overline{p} - \Delta$ (or both).

[32] In the borderline case where $v_L/v_H = \underline{\gamma}$, it actually holds that $\overline{p} = p_{max}$ and $\underline{p} = v_L$. In particular, this would mean that the firms choose prices weakly lower than v_L with zero probability, yet $p = v_L$ lies in the support, with discretely higher demand than when setting $p + \varepsilon$ for any $\varepsilon > 0$ (due to returning demand). Hence, there would need to be a gap in the equilibrium distribution for prices slightly above v_L, which is however incompatible with $F(v_L) = 0$ and v_L being part of the support.

will only be searched if its price is not too high relative to the lowest-priced firm, which holds if $p_1 \geq p - \left(v_H - v_L - \frac{s}{\theta}\right)$ (compare with Lemma 1). Hence, in order for the deviating firm to be searched at all, all rival firms' prices must lie above $p - \left(v_H - v_L - \frac{s}{\theta}\right)$. Then, the deviating firm will cater to the mass $(1-\theta)^{N-1}\theta$ consumers who don't have a full match at any rival firm, but a full match at this firm. Thus, the expected profit at any such price $p > \overline{p}_M$ can be written as

$$\pi_i(p) = (p-c)\left[1 - F_{M_1}\left(p - \left(v_H - v_L - \frac{s}{\theta}\right)\right)\right]^{N-1}(1-\theta)^{N-1}\theta. \tag{20}$$

For prices which lie in the support of the candidate equilibrium, i.e. $p \in [\underline{p}_M, \overline{p}_M]$, the expected profit is by construction equal to π_M^*, where we replicate here the implicit definition of $F_{M_1}(p)$, equation (9), for convenience:

$$\pi_i(p) = (p-c)\left[\left(1 - \theta F_{M_1}(p)\right)^{N-1}\theta + \left(1 - F_{M_1}(p)\right)^{N-1}(1-\theta)^N\right] = \pi_M^*. \tag{21}$$

Since $F_{M_1}(p)$ cannot be obtained in closed form for an arbitrary number of firms N, we will use an estimation. Rewriting (21), it holds for $p \in [\underline{p}_M, \overline{p}_M]$ that

$$(1 - F_{M_1}(p))^{N-1} = \frac{\frac{\pi_M^*}{p-c} - (1 - \theta F_{M_1}(p))^{N-1}\theta}{(1-\theta)^N} \leq \frac{\frac{\pi_M^*}{p-c} - (1 - F_{M_1}(p))^{N-1}\theta}{(1-\theta)^N},$$

such that by isolating $(1 - F_{M_1}(p))^{N-1}$ we obtain

$$(1 - F_{M_1}(p))^{N-1} \leq \frac{\pi_M^*}{(p-c)\left[\theta + (1-\theta)^N\right]}. \tag{22}$$

For $p \in [\overline{p}_M, \overline{p}_M + (v_H - v_L - \frac{s}{\theta})]$, it holds that $p - (v_H - v_L - \frac{s}{\theta}) \in [\underline{p}_M, \overline{p}_M]$. Hence, by inequality (22), we have that for $p \in [\overline{p}_M, \overline{p}_M + (v_H - v_L - \frac{s}{\theta})]$,

$$\left[1 - F_{M_1}\left(p - \left(v_H - v_L - \frac{s}{\theta}\right)\right)\right]^{N-1} \leq \frac{\pi_M^*}{\left[p - c - \left(v_H - v_L - \frac{s}{\theta}\right)\right]\left[\theta + (1-\theta)^N\right]}.$$

In turn, this implies that the following estimation can be given for equation (20) and $p \in [\overline{p}_M, \overline{p}_M + (v_H - v_L - \frac{s}{\theta})]$:

$$\begin{aligned}\pi_i(p) &= (p-c)\left[1 - F_{M_1}\left(p - \left(v_H - v_L - \frac{s}{\theta}\right)\right)\right]^{N-1}(1-\theta)^{N-1}\theta \\ &\leq (p-c)\left[\frac{\pi_M^*}{\left[p - c - \left(v_H - v_L - \frac{s}{\theta}\right)\right]\left[\theta + (1-\theta)^N\right]}\right](1-\theta)^{N-1}\theta.\end{aligned}$$

Since $\frac{p-c}{p-c-(v_H-v_L-\frac{s}{\theta})}$ is strictly decreasing in p for $v_H - v_L - \frac{s}{\theta} > 0$, as assumed for the proposition, the last expression is thereby maximized for $p = \overline{p}_M$. This implies that for $p \in [\overline{p}_M, \overline{p}_M + (v_H - v_L - \frac{s}{\theta})]$,[33]

$$\pi_i(p) \leq (\overline{p}_M - c)\left[\frac{\pi_M^*}{\left[\overline{p}_M - c - \left(v_H - v_L - \frac{s}{\theta}\right)\right]\left[\theta + (1-\theta)^N\right]}\right](1-\theta)^{N-1}\theta = \pi_M^*.$$

Hence, deviations above $\overline{p}_M$ are indeed not profitable.

Next, we show that there is no profitable deviation below $\underline{p}_M$. For such low prices, there is now a positive probability that some or all rival firms draw high enough prices such that consumers who are only partially matched at the deviating firm do not search them. Precisely, for deviation prices $p < \underline{p}_M$, consumers that are only partially matched at the deviating firm will only search rival firms j for which $p_j \leq p + v_H - v_L - \frac{s}{\theta}$ (compare with Lemma 1). Moreover, consumers will only buy at such firms if they are fully matched at them. The probability to lose the mass $1-\theta$ of partially matched consumers towards a single rival is therefore given by $F_{M_1}\left(p + v_H - v_L - \frac{s}{\theta}\right)\theta$. Consequently, the probability *not* to lose these consumers against *any* rival firm is given by $\left[1 - F_{M_1}\left(p + v_H - v_L - \frac{s}{\theta}\right)\theta\right]^{N-1}$. Hence, we can write a deviating firm's expected profit for $p < \underline{p}_M$ as

$$\pi_i(p) = (p-c)\left[\theta + (1-\theta)\left[1 - F_{M_1}\left(p + v_H - v_L - \frac{s}{\theta}\right)\theta\right]^{N-1}\right]. \tag{23}$$

Again, our strategy will be to use an estimation for the additional expected demand, which will be derived from the only implicitly defined CDF F_{M_1}. Using once more equation (21), we find that for $p \in [\underline{p}_M, \overline{p}_M]$ it holds that

$$(1 - \theta F_{M_1}(p))^{N-1} = \frac{\frac{\pi_M^*}{p-c} - (1 - F_{M_1}(p))^{N-1}(1-\theta)^N}{\theta} \leq \frac{\pi_M^*}{\theta(p-c)}. \tag{24}$$

For $p \in [\underline{p}_M - (v_H - v_L - \frac{s}{\theta}), \overline{p}_M - (v_H - v_L - \frac{s}{\theta})] = [\underline{p}_M - (v_H - v_L - \frac{s}{\theta}), \underline{p}_M]$, it holds that $p + (v_H - v_L - \frac{s}{\theta}) \in [\underline{p}_M, \overline{p}_M]$. Hence, by inequality (24), we have that for $p \in [\underline{p}_M - (v_H - v_L - \frac{s}{\theta}), \underline{p}_M]$,

$$\left[1 - \theta F_{M_1}\left(p + \left(v_H - v_L - \frac{s}{\theta}\right)\right)\right]^{N-1} \leq \frac{\pi_M^*}{\theta(p - c + v_H - v_L - \frac{s}{\theta})}.$$

[33] For $p > \overline{p}_M + (v_H - v_L - \frac{s}{\theta})$, $\pi_i(p) = 0$, since no consumer would ever search the deviating firm.

In turn, this implies that the following approximation can be given for equation (23) and $p \in [\underline{p}_M - (v_H - v_L - \frac{s}{\theta}), \underline{p}_M]$:

$$\pi_i(p) = (p-c)\left[\theta + (1-\theta)\left[1 - F_{M_1}\left(p + v_H - v_L - \frac{s}{\theta}\right)\theta\right]^{N-1}\right]$$
$$\leq (p-c)\left[\theta + (1-\theta)\left[\frac{\pi_M^*}{\theta(p - c + v_H - v_L - \frac{s}{\theta})}\right]\right].$$

Since $\frac{p-c}{p-c+v_H-v_L-\frac{s}{\theta}}$ is strictly increasing in p for $v_H - v_L - \frac{s}{\theta} > 0$, as assumed for the proposition, the last expression is thereby maximized for $p = \underline{p}_M$. This implies that for $p \in [\underline{p}_M - (v_H - v_L - \frac{s}{\theta}), \underline{p}_M]$,[34]

$$\pi_i(p) \leq (\underline{p}_M - c)\left[\theta + (1-\theta)\left[\frac{\pi_M^*}{\theta(\underline{p}_M - c + v_H - v_L - \frac{s}{\theta})}\right]\right] = \pi_M^*.$$

Hence, deviations below $\underline{p}_M$ are indeed not profitable. This completes the proof. □

Proof of Proposition 3. In what follows, we prove existence. For uniqueness, the argument at the end of the proof of Proposition 2 applies.

Existence. It is first straightforward to check that $\overline{p}_M \in (v_L, v_H - \frac{s}{\theta})$ due to $v_L/v_H \in (\underline{\gamma}, \tilde{\gamma})$ and that $\underline{p}_M \in (c, v_L)$ due to $v_L/v_H \in (\underline{\gamma}, \overline{\gamma})$. To see that $\underline{p}'_M > v_L$, note the following. First, since $(v_L - c)\left[\theta(1-\theta\kappa)^{N-1} + (1-\kappa)^{N-1}(1-\theta)^N\right]$ is strictly increasing in v_L for $\kappa \in [0,1]$ while π_M^* is strictly decreasing in v_L, one can clearly see via the implicit definition of $\kappa = F_{M_1}(v_L)$ in equation (15) that κ must be strictly increasing in v_L whenever $\kappa \in [0,1)$. Moreover, for $v_L/v_H = \underline{\gamma}$ it holds that $\kappa = 0$, while for $v_L/v_H = \tilde{\gamma}$, it holds that $\kappa = 1$. Hence, $\kappa \in (0,1)$ in the considered parameter region. Substituting π_M^* from equation (15) into equation (13) now yields

$$\underline{p}'_M = c + (v_L - c)\left[1 + \frac{(1-\theta)^N}{\theta}\left(\frac{1-\kappa}{1-\theta\kappa}\right)^{N-1}\right],$$

which indeed strictly exceeds v_L for all $\kappa \in [0,1)$.

A firm's expected profit when choosing a price in the range $[\underline{p}_M, v_L]$ is given by

$$\pi_i(p) = (p-c)\left[\theta(1-\theta F_{M_1}(p))^{N-1} + (1-F_{M_1}(p))^{N-1}(1-\theta)^N\right],$$

[34] For $p < \underline{p}_M - (v_H - v_L - \frac{s}{\theta})$, $\pi_i(p) < \pi_i(\underline{p}_M - (v_H - v_L - \frac{s}{\theta}))$, since all consumers already purchase deterministically at the deviating firm for $p = \underline{p}_M - (v_H - v_L - \frac{s}{\theta})$.

such that $\pi_i(p) = \pi_M^*$ for all prices in that interval via the implicit definition of $F_{M_1}(p)$ in equation (9). A firm's expected profit when choosing a price in the range$[\underline{p}_M', \overline{p}_M]$ is given by $\pi_i(p) = (p - c)\left[1 - F_{M_2}(p)\theta\right]^{N-1}\theta$, such that for

$$F_{M_2}(p) = \frac{1}{\theta}\left[1 - \left(\frac{\pi_M^*}{\theta(p-c)}\right)^{\frac{1}{N-1}}\right],$$

it also holds that $\pi_i(p) = \pi_M^*$ for all prices in that interval. It is moreover easy to see that both $F_{M_1}(p)$ and $F_{M_2}(p)$ are strictly increasing in p. Hence, all equilibrium objects are well-behaved.

We now rule out profitable deviations outside the candidate equilibrium's pricing support. First, it clearly cannot be optimal to deviate to a price $p \in (v_L, \underline{p}_M')$, as the deviating firm would not achieve a higher expected demand than when pricing at $\underline{p}_M' > p$. When deviating to a price $p > \overline{p}_M$, the deviating firm will only be searched if all rival firms price above $p - \left(v_H - v_L - \frac{s}{\theta}\right)$ (compare with Lemma 1). Then, the deviating firm will cater to the mass $(1-\theta)^{N-1}\theta$ consumers who don't have a full match at any rival firm, but a full match at this firm. Thus, the expected profit at any such price $p > \overline{p}_M$ can be written as

$$\pi_i(p) = (p-c)\left[1 - F_{M_1}\left(p - \left(v_H - v_L - \frac{s}{\theta}\right)\right)\right]^{N-1}(1-\theta)^{N-1}\theta, \tag{25}$$

where $F_{M_1}(p - (v_H - v_L - \frac{s}{\theta}))$ (rather than $F_{M_2}(p - (v_H - v_L - \frac{s}{\theta}))$) is the relevant probability that a rival firm prices below $p - (v_H - v_L - \frac{s}{\theta})$.[35] The same estimation as in the proof of Proposition 4 can now be used to show that $\pi_i(p) \leq \pi_M^*$ for all $p > \overline{p}_M$. Hence, deviations above $\overline{p}_M$ are not profitable.

We finally show that there are no profitable deviations to prices $p \in (c, \underline{p}_M)$. Following the argument in the proof of Proposition 4, a firm deviating to such a price makes an expected profit of

$$\pi_i(p) = (p-c)\left[\theta + (1-\theta)\left[1 - F_{M_r}\left(p + v_H - v_L - \frac{s}{\theta}\right)\theta\right]^{N-1}\right], \tag{26}$$

where $r = 1$ if $p + v_H - v_L - \frac{s}{\theta} \leq v_L$ and $r = 2$ otherwise. Since $F_{M_1}(p)$ is implicitly defined by

$$(p-c)\left[\theta(1-\theta F_{M_1}(p))^{N-1} + (1-F_{M_1}(p))^{N-1}(1-\theta)^N\right] - \pi_M^* = 0,$$

while $F_{M_2}(p)$ is implicitly defined by

$$(p-c)\left[\theta(1-\theta F_{M_2}(p))^{N-1}\right] - \pi_M^* = 0,$$

[35] Otherwise $p > v_H - \frac{s}{\theta}$, implying zero demand for the deviating firm.

it is straightforward to see that $F_{M_1}(p) > F_{M_2}(p)$ when applied to the same price. Comparing with (26), a sufficient condition to have no profitable deviations below $\underline{p}_M$ is then that for all $p \in (c, \underline{p}_M)$,

$$\pi_i(p) \leq (p-c)\left[\theta + (1-\theta)\left[1 - F_{M_2}\left(p + v_H - v_L - \frac{s}{\theta}\right)\theta\right]^{N-1}\right] \leq \pi_M^*.$$

Inserting $F_{M_2}(\cdot)$ from equation (12), the above condition is equivalent to

$$(p-c)\left[\theta + (1-\theta)\left[\frac{\pi_M^*}{\theta(p-c+v_H-v_L-\frac{s}{\theta})}\right]\right] \leq \pi_M^* \qquad \forall p \in (0, \underline{p}_M).$$

Since $\frac{p-c}{p-c+v_H-v_L-\frac{s}{\theta}}$ is strictly increasing in p for $v_H - v_L - \frac{s}{\theta} > 0$, as assumed for the proposition, the LHS in the last expression is maximized for $p = \underline{p}_M$. Hence, for $p \in (c, \underline{p}_M]$,

$$\pi_i(p) \leq (\underline{p}_M - c)\left[\theta + (1-\theta)\left[\frac{\pi_M^*}{\theta(\underline{p}_M - c + v_H - v_L - \frac{s}{\theta})}\right]\right] = \pi_M^*,$$

such that deviations below $\underline{p}_M$ are indeed not profitable. This completes the proof. □

Proof of Proposition 6. One may first check that the gap equilibrium smoothly transitions to the high-price equilibrium as $v_L/v_H \downarrow \underline{\gamma}$ (equivalently, $s \downarrow \theta(v_H - c) - (v_L - c)\frac{\theta+(1-\theta)^N}{(1-\theta)^{N-1}}$) and that it smoothly transitions to the low-price equilibrium as $v_L/v_H \uparrow \tilde{\gamma}$ (equivalently, $s \uparrow \theta(v_H - c) - \theta(v_L - c)\frac{2[\theta+(1-\theta)^N]-\theta(1-\theta)^{N-1}}{\theta+(1-\theta)^N}$). Moreover, the low-price equilibrium smoothly transitions to the Bertrand equilibrium as $v_L/v_H \uparrow \bar{\gamma}$ (equivalently, $s \uparrow \theta(v_H - v_L)$).

Inspection of $F_H(p)$ in Proposition 2 then immediately reveals that $F_H(p)$ increases in s – which means that as s decreases, prices increase in the sense of first order stochastic dominance (FOSD in what follows), as claimed. Next, inspection of the implicit definition of $F_{M_1}(p)$ together with π_M^* in Proposition 3 shows that $F_{M_1}(p)$ also increases in s (since π_M^* decreases in s). Thereby, also $\kappa = F_{M_1}(v_L)$ increases in s. It is moreover easy to see that $F_{M_2}(p)$ increases in s and that $\underline{p}_M$ and $\overline{p}_M$ decrease in s. $\underline{p}'_M$ decreases in s as it can be rewritten as $\underline{p}'_M = c + (v_L - c)\left[1 + \frac{(1-\theta)^N}{\theta}\left(\frac{1-\kappa}{1-\theta\kappa}\right)^{N-1}\right]$ (see the proof of Proposition 3), which decreases in s because κ increases in s. Together, this again implies that prices increase in the sense of FOSD as s decreases in the gap equilibrium. Since the same expressions for $F_{M_1}(p)$, $\underline{p}_M$ and $\overline{p}_M$ are also relevant for the low-price equilibrium, the same conclusion can be reached there. Overall, prices thus increase in the sense of FOSD as s decreases.

That a decrease in s strictly increases equilibrium profits is trivial to see from the respective expressions. The final claim that a decrease in s weakly decreases the probability that firms price below v_L follows from the fact that this probability is constant in the high-price equilibrium and low-price equilibrium (zero and one, respectively) and strictly increasing in s in the gap equilibrium since $\kappa = F_{M_1}(v_L)$ strictly increases in s. □

Proof of Proposition 7. For (i), note first that total social welfare W strictly decreases in s whenever the gap equilibrium is *not* played, as follows immediately from the expressions in Lemma 3. It thus remains to show that W strictly decreases in s also in the gap equilibrium whenever $N \geq 3$. Now, in the gap equilibrium, it holds that

$$\begin{aligned}\frac{dW}{ds} &= -\frac{1}{\theta}\left[1-(1-\theta)^N\right] + (v_L - c)(1-\theta)^N N(1-\kappa)^{N-1}\frac{d\kappa}{ds} \\ &= -\frac{1}{\theta}\left[1-(1-\theta)^N\right] + \frac{(1-\theta)^N N(1-\kappa)^{N-1}\frac{\left[\theta+(1-\theta)^N\right](1-\theta)^{N-1}}{\theta+(1-\theta)^N-\theta(1-\theta)^{N-1}}}{(N-1)\left[\theta^2(1-\theta\kappa)^{N-2}+(1-\theta)^N(1-\kappa)^{N-2}\right]} \\ &= -\frac{1}{\theta}\left[1-(1-\theta)^N\right] + \frac{(1-\theta)^{2N-1}N(1-\kappa)\frac{\theta+(1-\theta)^N}{\theta+(1-\theta)^N-\theta(1-\theta)^{N-1}}}{(N-1)\left[\theta^2\left(\frac{1-\theta\kappa}{1-\kappa}\right)^{N-2}+(1-\theta)^N\right]},\end{aligned}$$

where the second equality follows from computing $\frac{d\kappa}{ds}$ using the implicit definition of κ given in (15). The first term in the last expression for dW/ds above is independent of s, while the second term indirectly depends on s via its effect on κ. Indeed, it is straightforward to see that the second term strictly decreases in s since (i) it is strictly positive for $\kappa < 1$, (ii) it strictly decreases in κ since its nominator strictly decreases in κ while its denominator weakly increases in it (the latter because $\frac{1-\theta\kappa}{1-\kappa}$ strictly increases in κ and its exponent $N-2$ is nonnegative), and (iii) κ strictly increases in s. Overall, we may thus conclude that $d^2W/ds^2 < 0$ in the gap equilibrium. By the result that the second term of dW/ds strictly decreases in κ, it moreover clearly holds that

$$\frac{dW}{ds} \leq \left.\frac{dW}{ds}\right|_{\kappa=0} = -\frac{1}{\theta}\left[1-(1-\theta)^N\right] + \frac{(1-\theta)^{2N-1}N\frac{\theta+(1-\theta)^N}{\theta+(1-\theta)^N-\theta(1-\theta)^{N-1}}}{(N-1)\left[\theta^2+(1-\theta)^N\right]}.$$

The above upper bound for $\frac{dW}{ds}$ decreases in N, as follows from the fact that all of $-\frac{1}{\theta}\left[1-(1-\theta)^N\right]$, $\frac{N}{N-1}$, $\frac{(1-\theta)^{2N-1}}{\theta^2+(1-\theta)^N}$ and $\frac{\theta+(1-\theta)^N}{\theta+(1-\theta)^N-\theta(1-\theta)^{N-1}}$ decrease in N. To show that $\frac{dW}{ds} < 0$ for $N \geq 3$, it thus suffices to establish that

$$\left.\frac{dW}{ds}\right|_{\kappa=0,N=3} < 0,$$

which is true if and only if

$$-3+3\theta-\theta^2+\frac{3(1-\theta)^5\left[\theta+(1-\theta)^3\right]}{2\left(1-3\theta+4\theta^2-\theta^3\right)\left(1-3\theta+5\theta^2-2\theta^3\right)} < 0 \quad \text{for all } \theta \in (0,1).$$

This inequality indeed holds, as can easily be shown numerically. This proves (i). On the other hand, it may be noted that

$$\left.\frac{dW}{ds}\right|_{\kappa=0,N=2}=\frac{\theta\left(1-6\theta+10\theta^2-8\theta^3+2\theta^4\right)}{\left(1-2\theta+2\theta^2\right)^2}>0 \text{ for } \theta \lessapprox 0.2533.$$

Hence, for $N=2$, welfare locally increases in s in the gap equilibrium when v_L/v_H lies sufficiently close above $\underline{\gamma}$ (where by continuity κ is close to 0) and θ is sufficiently small.

For (ii), note first that in the Bertrand equilibrium, the consumer surplus satisfies $CS_B=W_B=\theta v_H+(1-\theta)v_L-c-s$, thus it clearly increases as s decreases. For the other equilibria, we may simply compute the expected consumer surplus by subtracting the expected industry profit (i.e., $N\pi_H^*$ in the high-price equilibrium and $N\pi_M^*$ in both the gap and low-price equilibrium) from the relevant welfare expression as given in Lemma 3. For the high-price equilibrium, this implies that

$$CS_H=\left(v_H-\frac{s}{\theta}-c\right)\left[1-(1-\theta)^N-N(1-\theta)^{N-1}\theta\right].$$

Clearly, this expression is strictly decreasing in s if

$$\eta(\theta,N)\equiv 1-(1-\theta)^N-N(1-\theta)^{N-1}\theta>0.$$

This is indeed the case, since $\eta(\theta,N)$ is strictly increasing in θ (as can trivially be established), and $\eta(0,N)=0$. Hence, the consumer surplus in the high-price equilibrium strictly increases as s decreases.

Having shown that the the expected consumer surplus always strictly decreases in s in the Bertrand and high-price equilibrium, we will finally prove that the expected consumer surplus strictly *increases* in s in the gap equilibrium when v_L/v_H lies sufficiently close above $\underline{\gamma}$ and θ is sufficiently small. To see this, note that in the gap equilibrium we have that

$$\frac{d^2CS}{ds^2}=\frac{d^2W}{ds^2}-N\frac{d^2\pi_M^*}{ds^2}<0,$$

where the inequality follows from $\frac{d^2W}{ds^2} < 0$ (as shown in part (i) of the proof) and $\frac{d^2\pi_M^*}{ds^2} = 0$. Hence, $\frac{dCS}{ds}$ is clearly largest at the boundary to the high-price equilibrium where $\kappa = 0$. There, we have that

$$\begin{aligned}\left.\frac{dCS}{ds}\right|_{\kappa=0} &= \left.\frac{dW}{ds}\right|_{\kappa=0} - N\frac{d\pi_M^*}{ds} \\ &= -\frac{1}{\theta}\left[1-(1-\theta)^N\right] + \frac{(1-\theta)^{2N-1}N\frac{\theta+(1-\theta)^N}{\theta+(1-\theta)^N-\theta(1-\theta)^{N-1}}}{(N-1)\left[\theta^2+(1-\theta)^N\right]} + N\frac{[\theta+(1-\theta)^N](1-\theta)^{N-1}}{\theta+(1-\theta)^N-\theta(1-\theta)^{N-1}},\end{aligned}$$

where the second equality uses the expression for $\frac{dW}{ds}\big|_{\kappa=0}$ obtained in part (i) of the proof above.

The limit of this as θ tends to zero is $\frac{N}{N-1}$, as is straightforward to check. Hence, we have that $\frac{dCS}{ds}\big|_{\kappa=0,\theta=0} = \frac{N}{N-1} > 0$. By continuity, this implies that for $\kappa \approx 0$ – that is, v_L/v_H sufficiently close above $\underline{\gamma}$ – and θ sufficiently close to zero, it holds that $\frac{dCS}{ds} > 0$ in the gap equilibrium. Thus, in this case, the expected consumer surplus falls as s decreases. This completes the proof. □

Proof of Proposition 8. Consider first candidate symmetric pure-strategy PBEs where $p^* \in (v_L, v_H]$. Clearly, in such equilibria, consumers would immediately buy upon discovering a full match, and never purchase (or return to purchase) with just a partial match. For $s > 0$, it then has to hold that $p^* = v_H$: otherwise, firms could slightly increase their price without losing demand from their fully matched consumers. The best deviation from the candidate equilibrium $p^* = v_H$ is to set $p = v_L$ and also sell to one's partially matched consumers (note that given the expectation of $p^* = v_H$ at every firm, no consumer would search more than one firm). The equilibrium thus exists if and only if $(v_H - c)\theta/N \geq (v_L - c)/N$, which is equivalent to $v_L/v_H \leq \check{\gamma}$. Given the latter condition and $s > 0$, $p^* = v_H$ is the unique symmetric pure-strategy PBE satisfying $p^* > v_L$.

Consider next candidate symmetric pure-strategy PBEs where $p^* \leq v_L$, and suppose first that $s < \theta(v_H - v_L)$. Then, consumers would find it strictly optimal to search on after discovering a partial match at p^*, which however cannot be true in equilibrium for any $p^* > c$. To see why, note that then there would be a mass $(1-\theta)^N$ of consumers that exhaust all their search options, as they do not find a full match anywhere. Hence, due to indifference, these consumers would end up buying from a random firm (e.g., from the last firm that they sampled). But by pricing marginally below p^*, each firm could break this indifference in its favor, attracting the full mass $(1-\theta)^N$ of non-fully matched consumers at a negligible loss of margin – a profitable deviation. Thus, for $s < \theta(v_H - v_L)$, in a symmetric pure-strategy PBE with $p^* \leq v_L$, only $p^* = c$ remains possible. But this cannot be true in equilibrium either, as firms would have an incentive to slightly increase their price and sell to fully matched consumers at a positive margin. Symmetric pure-strategy PBEs where $p^* \leq v_L$ thus require that $s \geq \theta(v_H - v_L)$ – where in the knife-edge case of $s = \theta(v_H - v_L)$, consumers need to buy immediately when they find a partial match at p^*. Observe moreover that for $s > \theta(v_H - v_L)$, the only possible equilibrium with $p^* \le v_L$ has $p^* = v_L$: for any lower p^*, firms could slightly increase their price and still retain their partially matched consumers. The best deviation from the candidate equilibrium $p^* = v_L$ is now to set $p = v_H$: this loses all demand from only partially matched consumers, but still allows to sell to fully matched consumers. Equilibrium existence of $p^* = v_L$ thus requires, next to $s \ge \theta(v_H - v_L)$ – equivalently, $v_L/v_H \ge \bar{\gamma}$ – that $(v_L - c)/N \ge (v_H - c)\theta/N$, which is equivalent to $v_L/v_H \ge \check{\gamma}$. To sum up, $p^* = v_L$ is a PBE if and only if $v_L/v_H \ge \max\{\bar{\gamma}, \check{\gamma}\}$. It is the unique symmetric pure-strategy PBE satisfying $p^* \le v_L$ if $v_L/v_H > \max\{\bar{\gamma}, \check{\gamma}\}$.

Since, as argued above, no pure-strategy PBE with $p_L^* \le v_L$ can exist for $v_L/v_H < \check{\gamma}$, $p^* = v_H$ is the unique symmetric pure-strategy PBE for $s > 0$ and $v_L/v_H < \check{\gamma}$. On the other hand, since for $s > 0$ the only possible symmetric pure-strategy PBE with $p^* \in (v_L, v_H]$ has $p^* = v_H$, but this equilibrium does not exist for $v_L/v_H > \check{\gamma}$, we can also refine the statement on the equilibrium uniqueness of $p^* = v_L$ from the end of the previous paragraph: $p^* = v_L$ is the unique symmetric pure-strategy PBE whenever $v_L/v_H > \max\{\bar{\gamma}, \check{\gamma}\}$.

Finally, what remains is the case $v_L/v_H \in (\check{\gamma}, \bar{\gamma})$, which requires that $s < \theta(1-\theta)(v_H - c)$. Then, neither the symmetric pure-strategy PBE with $p^* = v_L$ exists (as $v_L/v_H < \bar{\gamma}$) nor that with $p^* = v_H$ exists (as $v_L/v_H > \check{\gamma}$), while all other symmetric pure-strategy PBEs have been ruled out anyway. Hence, in this case, any symmetric PBE must be in mixed strategies. □

Proof of Proposition 9. Note first that any symmetric pure-strategy PBE with $p^* < v_H$ can be ruled out due to $s < \theta(v_H - v_L)$ (equivalently, $v_L/v_H < \bar{\gamma}$). The reason is that in such candidate equilibria, firms could only sell to fully matched consumers (for $p^* \le v_L$, this is because $s < \theta(v_H - v_L)$ makes it optimal for partially matched consumers to search another firm), for which however a slightly higher price would also be acceptable due to their strictly positive search cost. But also $p^* = v_H$ cannot be an equilibrium, as $v_L/v_H > \check{\gamma}$ implies that firms could make a strictly higher profit by deviating to $p = v_L$ and also selling to partially matched consumers. Hence, no symmetric pure-strategy equilibrium exists.

Note next that in any candidate symmetric mixed-strategy PBE, firms' price distribution, say $F(p)$ with lower support bound $\underline{p}$, induces exactly two "reservation prices" $\rho_L \in (\underline{p} - (v_H - v_L), v_L]$ and $\rho_H = \rho_L + (v_H - v_L) \in (\underline{p}, v_H]$, which denote the highest acceptable prices to consumers partially matched (ρ_L) or fully matched (ρ_H) at a firm that induce them buy.[36] But given this, it is obvious

[36]Precisely, then $\rho_L \le v_L$ is the unique solution (if any) to

$$\theta \int_{\underline{p}}^{\rho_L + (v_H - v_L)} [(v_H - p) - (v_L - \rho_L)] dF(p) + (1-\theta) \int_{\underline{p}}^{\rho_L} (\rho_L - p) dF(p) = s.$$

that in any symmetric mixed-strategy PBE, firms must put all probability mass on ρ_L and ρ_H: pricing below ρ_L is strictly dominated by pricing at ρ_L, pricing between ρ_L and ρ_H is strictly dominated by pricing at ρ_H, while pricing above ρ_H is strictly dominated by pricing at ρ_H.

We thus look for a symmetric mixed-strategy PBE where firms price at some $p_L^* = \rho_L \leq v_L$ with probability $\alpha \in (0,1)$ and at $p_H^* = \rho_H = \rho_L + (v_H - v_L)$ with probability $1-\alpha$. Given this, indifference of partially matched consumers to buy at $p_L^* = \rho_L$ or to search another firm (in which case a utility gain is only realized when a full match is found at the low price) implies that $\theta\alpha(v_H - v_L) - s = 0$, or $\alpha = \frac{s}{\theta(v_H - v_L)}$, as claimed in the proposition. Given consumers' search strategies, the expected demand of a firm choosing $p_L^* = \rho_L$ is given by $D_L = 1 + (1-\theta)(1-\alpha) + [(1-\theta)(1-\alpha)]^2 + ... = \frac{1}{1-(1-\theta)(1-\alpha)}$. Likewise, the expected demand of a firm choosing $p_H^* = \rho_H$ is given by $D_H = \theta + (1-\alpha)(1-\theta)\theta + [(1-\alpha)(1-\theta)]^2\theta + ... = \frac{\theta}{1-(1-\theta)(1-\alpha)} = \theta D_L$. Indifference which price to choose thus requires that $(p_L^* - c)D_L = (p_H^* - c)D_H$, that is, $p_L^* - c = (p_L^* + v_H - v_L - c)\theta$. From this we finally conclude that $p_L^* = c + \frac{\theta(v_H - v_L)}{1-\theta}$ and $p_H^* = p_L^* + v_H - v_L = c + \frac{v_H - v_L}{1-\theta}$ in the unique symmetric PBE. □

(If no solution $\rho_L \leq v_L$ to this equation exists, this means that even a consumer partially matched at price $p = v_L$ – holding a purchase option of value zero – would not be willing to search another firm as the expected gross utility provided by it would fall short of her search cost. In this case, clearly $\rho_L = v_L$ is the highest acceptable price.)

Similarly, $\rho_H \leq v_H$ is the unique solution (if any) to

$$\theta \int_{\underline{p}}^{\rho_H} (\rho_H - p) dF(p) + (1-\theta) \int_{\underline{p}}^{\rho_H - (v_H - v_L)} [(v_L - p) - (v_H - \rho_H)] dF(p) = s.$$

(By an analogous argument as above, if no solution $\rho_H \leq v_H$ to this equation exists, $\rho_H = v_H$ is the highest acceptable price.)

Comparing the two equations, clearly $\rho_H = \rho_L + (v_H - v_L)$ holds.

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Price-Directed Search, Product Differentiation and Competition

Abstract
Especially in many online markets, consumers can readily observe prices, but may need to further inspect products to assess their suitability. We study the effects of product differentiation and search costs on competition and market outcomes in a tractable model of price-directed consumer search. We find that (i) firms' equilibrium pricing always induces efficient search behavior, (ii) for relatively large product differentiation, welfare distortions still occur because some consumers (may) forgo consumption, and (iii) lower search costs lead to stochastically higher prices, increasing firms' expected profits and decreasing their frequency of sales. Consumer surplus often falls when search costs decrease.

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