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Social Comparison and Optimal Contracts in Competition for Managerial Talent*

Anna Ulrichshofer† and Markus Walzl‡

July 13, 2020

Abstract

We analyze the impact of social comparison on optimal contract design under imperfect labor market competition for managerial talent. Adding a disutility of social comparison as induced by a ranking of verifiable efforts to the multi-task model by Bénabou and Tirole (2016), we demonstrate that rankings can reduce welfare distortions of optimal screening contracts if the degree of competition for talent is sufficiently low. In contrast, a ranking unambiguously reduces welfare if the competition intensity is high and agents suffer from lagging behind while it can enhance welfare (depending on the fraction of high and low productivity types) if agents suffer from leading in a ranking (e.g., because the ranked activity is perceived as a substitute for other potentially pro-social activities).

JEL Classification: D02, D21, D43, D86, D91, G35, G41, J33, J41.
Keywords: Incentive compensation, screening, imperfect labor market competition, social comparison, rankings.

1 Introduction

Numerous incidences of moral hazard, outright fraud, and bad performance (ranging from Enron, VW, or Wirecard to persistent misconduct in financial advice as discussed, e.g., by Egan et al. (2019)) indicate that ill-designed incentive structures can lead to antisocial behavior and negative externalities for the company and society as a whole. A unifying feature of most of the incentive schemes in the background of the above-mentioned scandals and incidences of misconduct is that these schemes are (explicitly or implicitly) a mixture of monetary incentives (e.g., bonuses, prizes, or career concerns) and non-monetary motives mostly addressing social comparison (e.g., rankings). With this paper, we investigate in how far social comparison is a substitute or complement to monetary incentives. To this end, we add psychological costs or benefits of social comparison in a ranking (see, e.g., Maccheroni et al. (2008)) to the model of imperfect labor market competition for managerial talent by Bénabou and Tirole (2016) and demonstrate that rankings complement monetary incentives if monetary incentives are inefficiently low as, e.g., for low degrees of labor market competition but may off-set inefficiently high monetary incentives and associated welfare distortions if, e.g., labor market competition is intense.

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The mixture of monetary incentives and social comparison is prevalent in a variety of industries and sectors. To name just s few examples: First, there has been a widespread debate on the "Wall Street culture" of incentive schemes in the finance industry. While being pecuniary by nature, these incentive schemes are also the basis for social comparison of salaries among peers. In some sub-sectors like the (hedge) fund industry, professionals' salaries are a convex function of past performance relative to other fund managers, resulting in a tournament incentive structure (Brown et al., 1996; Sirri and Tufano, 1998; Kaniel and Parham, 2017) that affects income and status-concerns of managers at the same time. These incentive schemes, however, have been identified as a culprit of excessive risk taking in the finance industry in general (Rajan, 2006; Diamond and Rajan, 2009; Kirchler et al., 2018) and one of the driving forces behind the financial crisis in particular. Second, publication based rankings have an ever increasing impact on career perspectives and funding in academia. As in the finance industry, it has been argued that status seeking may crowd-out intrinsic research motivation (Osterloh and Frey, 2015) and may be one of the reasons for misconduct and sabotage among researchers (Anderson et al., 2007; Fanelli, 2010). Third, if expert providers of a credence goods (e.g., car mechanic, doctor, or financial adviser) are intrinsically motivated to provide appropriate treatment or advice rather than exploiting the informational advantage relative to customers (see, e.g., Dulleck and Kerschbamer (2006) for a parsimonious model of credence goods, and Kerschbamer and Sutter (2017) and Balafoutas and Kerschbamer (forthcoming) for overviews of the recent mainly experimental evidence), performance rankings or customer ratings can have a significant influence on provision behavior ranging from undertreatment (i.e., a choice of a cheap but not necessarily appropriate treatment) in case of a ranking that is based on sales figures to overtreatment (i.e., providing more than the necessary treatment) in case of customer rankings. All these examples demonstrate the motivation behind our study: First, monetary incentives and social comparison in real life are highly relevant and can become the main driving force for employees. Second, social comparison in a multi-tasking environment can complement or substitute monetary incentives and motivations for prosocial activity depending on how the ranking is interpreted. As a consequence, rankings may undermine or promote prosocial behavior and thereby create positive and negative externalities to institutions and the society (Bénabou and Tirole, 2006). In any of these cases, optimal contracts should therefore respond to the presence of a ranking. How optimal responses look like will be analyzed in the following model.

Two agents (each being of high productivity with probability q_H and low productivity with probability $q_L = 1 - q_H$) can decide how much costly effort to invest into a verifiable task and a non-verifiable task. The verifiable task can be thought of as a classical measurable activity such as the number of clients, sold items or assets, while examples for the non-verifiable task include appropriate advice or the generation of a positive externality. Firms offer linear contracts (i.e., a piece-rate for the verifiable task and a fixed payment) and publish performance in the verifiable task in a ranking. In-line with recent evidence on multi-task decision making in a similar setting (see Stefan et al. (2020)), we introduce a psychological cost associated with the relative position in the ranking. To be specific, agents can suffer from lagging behind because being on top of the ranking is considered as desirable or can suffer from leading because a top rank is indicative for a comparably low performance in the non-verifiable task that may have a pro-social dimension. Moreover, we assume that costs are increasing in the difference between efforts. In Stefan et al. (2020) individuals indeed increased effort in the verifiable task (and decreased the corresponding effort in the non-verifiable task) in response to a low rank and decreased effort in the verifiable task (and increased the correspondig effort in the non-verifiable task) in response to a high rank in-line with these assumptions. We analyze this setting for three different models of competition between firms (i.e., employers): (i) a monopsonist employer, (ii) perfectly competitive employers, and (iii) two employers that compete a la Hotelling, i.e., are situated at the ends of a continuum of agents where the agents' position indicates (linear) traveling costs.

Optimal screening contracts and the impact of a ranking depend on the model of competition or its intensity (as captured by traveling costs): If the firm is a monopsonist or traveling costs are sufficiently large, the piece-rate for low productivity agent is inefficiently low to prevent the high productivity agent (who is offered an efficient piece-rate) from signing the low type's contract. If lagging behind in the ranking (e.g., after choosing the low piece-rate designed for the low productivity agent) is undesirable, the low type's contract is less attractive for the high type agent and the corresponding distortion can be reduced. Similarly, if leading the ranking is costly and costs are increasing in the difference between efforts, the high type's contract becomes more attractive as the low type's piece-rate and effort increases. As a result, distortions for low type contracts are reduced in this case as well. In other words, the ranking complements screening contracts in this case and can increase welfare. In contrast, if firms are in perfect competition or traveling costs are sufficiently low, the piece-rate for high productivity agent is inefficiently high to prevent the low productivity agent (who is offered an efficient piece-rate) from signing the high type's contract. If lagging behind in the ranking (e.g., after choosing the low piece-rate designed for the low productivity agent) is undesirable, the high type's contract is more attractive for the low type agent and the corresponding distortion has to be amplified. If leading in the ranking (e.g., after choosing the high piece-rate designed for the high productivity agent) is undesirable, the high type's contract is less attractive for the low type agent and the corresponding distortion will be attenuated. Hence, it depends on the degree of labor market competition *and* the nature of psychological costs of social comparison whether distortions of optimal screening contracts are amplified or attenuated. As a bottom line, the introduction of a ranking has the potential to enhance welfare if agents are under-incentivized by optimal contracts (as is the case if labor market competition is low) or if a ranking attenuates the distortions generated by excessively high boni (as is the case if labor market competition is intense but leading the ranking is considered as undesirable). If the ranking only off-sets inefficiently high incentives (as is the case if labor market competition is intense and lagging behind is perceived as costly), it further reduces welfare compared to an optimal incentive scheme in the absence of a ranking.[1]

Our paper connects two branches of the literature on optimal incentive schemes. The first branch considers optimal bonus payments – in particular in a multi-task environment and under imperfect (labor) market competition (Bénabou and Tirole, 2016; Bannier et al., 2020; Villas-Boas and Schmidt-Mohr, 1999). And the second branch addresses social comparison (Festinger, 1954; Bandiera et al., 2010; Cohn et al., 2015) and status (Moldovanu et al., 2007). Various studies disentangling rank incentives (social comparison) from monetary incentives show the effect of non-incentivized rankings on performance (Tran and Zeckhauser, 2012; Barankay, 2015), portfolio choice (Dijk et al., 2014), risk taking (Kuziemko et al., 2014; Kirchler et al., 2018), and market prices (Ball et al., 2001). With respect to effort provision, the literature reports varying effects of rankings, ranging from an overall increase in effort (Azmat and Iriberri, 2010; Blanes-i-Vidal and Nossol, 2011; Tran and Zeckhauser, 2012) to diverse effects depending on, for instance, expectations, current rank, and principal agent relationships (Al-Ubaydli and List, 2015; Kuhnen and Tymula, 2012; Gill et al., 2019).[2] Most closely related to our study is the paper by Stefan

[1]These findings translate to psychological benefits from social comparison in a subtle way. E.g., if leading the ranking is desirable and generates an additional benefit of social comparison that is increasing in the difference between efforts, efficiency distortions are amplified for intense competition because high type contracts become more attractive. But efficiency distortions are also amplified for low competition intensity as the high type's contract is the more attractive the lower the low type's piece-rate and effort.

[2]Moreover, peer effects need not necessarily be part of an explicit incentive scheme or an explicitly designed ranking, but can also emerge rather naturally (Mas and Moretti, 2009).

et al. (2020) who conduct an experiment with financial professionals demonstrating the impact of a ranking in previous rounds on effort choices in multi-task decisions. Their findings are in-line with the assumption on psychological costs of social comparison made in our contribution. Our inquiry is also related to studies on self-image concerns (Bénabou and Tirole, 2006; Ariely et al., 2009; Falk and Szech, 2019). The trade-off choice in our model can also be interpreted as a balancing of the desire for a positive self-image due to observed rank and their desire for a positive self-image that stems from contributing to a prosocial activity. This also relates to the literature discussing how social comparison and monetary incentives can lower prosocial behavior or even promote misconduct (Shleifer, 2004; Charness et al., 2014).

The remainder of the paper is organized as follows: Section 2 sets-up the model and discusses welfare benchmarks, Section 3 and 4 analyze the limit cases of monopsony and perfect competition while Section 5 discusses the model of imperfect competition. Section 6 concludes with remarks on the robustness of our findings and the discussion of potential implications for optimal contract design.

2 Model

2.1 Benchmark: No Social Comparison

As a starting point consider the model of Bénabou and Tirole (2016) of a unit continuum of agents, who engage in two activities A and B, exerting efforts $(a, b) \in \mathbb{R}_+^2$ respectively. The productivity of a worker is given by $\theta + b$, where θ is a talent parameter (or type), which is private information. Firms offer linear contracts (with fixed payment z and piece-rate y) to workers.[3] The preferences of the agents are then given by[4]

$$U_{BT}(\theta; y, z) = u_{BT}(y) + \theta y + z, \tag{2.1}$$

where u_{BT} denotes the part of the agent's utility that is affected by bonus y via efforts a and b:

$$u_{BT}(y) = v \cdot a(y) + y\, b(y) - C(a(y), b(y)), \tag{2.2}$$

with v parameterizing the strength of the agent's intrinsic motivation to spend effort for activity A and $C(a(y), b(y))$ being the total effort cost, which is strictly increasing and strictly convex in (a, b). Observe that the agent's effort choice in response to bonus y (i.e., $a(y)$ and $b(y)$) maximizes u_{BT} and therefore does not depend on the agent's talent θ. An agent has talent $\theta \in \{\theta_L, \theta_H\}$ with probabilities $\{q_L, q_H = 1 - q_L\}$, respectively. For further reference, we denote $\Delta\theta \equiv \theta_H - \theta_L > 0$. Employers must respect the participation constraint $U_{BT}(y, \theta, z) \geq \bar{U}$, as any agent has an outside option generating utility $\bar{U}$.
A firm has a net profit of

$$\Pi_{BT}(\theta, y, z) = \pi_{BT}(y) + (B - y)\theta - z, \tag{2.3}$$

with π_{BT} denoting the part of the firm's profit that is affected by the agents' effort choice:

$$\pi_{BT}(y) \equiv Aa(y) + (B - y)b(y). \tag{2.4}$$

[3]For the optimality of linear contracts in this setting see Appendix C in Bénabou and Tirole (2016).

[4]Henceforth, we will refer to expressions that are identical to the model in Bénabou and Tirole (2016) with a subscript BT.

Lastly, if a firm hires an agent, social welfare in Bénabou and Tirole (2016) is given by

$$W_{BT}(\theta, y) \equiv U_{BT}(y, \theta, z) + \Pi_{BT}(\theta, y, z) = w_{BT}(y) + B\theta, \tag{2.5}$$

with w_{BT} denoting the part of welfare that is affected by the firm's choice of bonus y

$$w_{BT}(y) \equiv u_{BT}(y) + \pi_{BT}(y) = (A + v)a(y) + Bb(y) - C(a(y), b(y)) \tag{2.6}$$

and $W_{BT,tot}$ denotes total (ex-ante) welfare

$$W_{BT,tot} = q_L W_{BT}(\theta_L, y_L) + q_H W_{BT}(\theta_H, y_H). \tag{2.7}$$

As efforts $a(y)$ and $b(y)$ do not depend on the agent's talent θ (see above), neither does u_{BT} nor π_{BT}. As a consequence also w_{BT} is independent of θ and welfare from hiring the agent is maximized by a piece-rate y^*_{BT} that maximizes $w_{BT}(y)$ and does not differ between agents with talent θ_H and θ_L. Bénabou and Tirole (2016) analyze variations of the following game:

1. Firms simultaneously offer linear contracts, i.e., firm j offers (possibly multiple) contracts (y_j, z_j).
2. Nature draws an agent from the unit continuum, i.e., agent i is located at $x_i \in [0, 1]$ and has (talent) type θ_H with probability q_H and talent θ_L with probability $q_L = 1 - q_H$. Location and talent are private information.
3. The agent chooses at most one contract and decides on efforts a and b.
4. The firm that contracted the agent observes b_i and payments (contingent on b_i) are arranged.

Specifically, Bénabou and Tirole (2016) consider (i) a monopsony firm, (ii) perfect competition between firms, and (iii) a Hotelling-type model of imperfect competition where the location of an agent induces traveling costs to each of two firms located at the two ends of the continuum. We will closely follow the analysis in Bénabou and Tirole (2016) but enrich their set-up with the opportunity (and potential costs) of social comparison.

2.2 Social Comparison

We extend the model introduced in Section 2.1 by an opportunity of social comparison between agents. To be specific, consider two rather than one agent and suppose that their performance in activity B (i.e., effort b) is published in a ranking. For simplicity, we assume that the ranking is not published within a firm but rather captures the performance of all employers in the industry. Observing her own effort b_i next to the other agent's effort b_{-i} in the ranking, generates psychological costs or benefits

$$I_i(b_i, b_{-i}) = \xi_i \delta(b_i, b_{-i}) f(\|b_i - b_{-i}\|)$$

with $\xi_i \geq 0$ parameterizing the relative strength of psychological concerns for agent i, and $\delta(b_i, b_{-i})$ being an indicator function that allows to distinguish psychological costs from being exposed to a ranking. $\delta(b_i, b_{-i}) < 0$ for $b_i < b_{-i}$ captures a psychological cost from lagging behind in a ranking. $\delta(b_i, b_{-i}) < 0$ for $b_i > b_{-i}$ captures a psychological cost from leading in a ranking (e.g., because a good rank is considered as an indicator for low efforts in a pro-social activity A). Finally, $f(.)$ is a convex and monotone increasing function of the absolute difference between b_i and b_{-i} with

$f(0) = 0$ and $f'(0) = 0$. We focus on costs rather than benefits and the specific form of f to guarantee a simple comparative statics for all variants of the model. Many findings regarding equilibrium existence, uniqueness, and the dependence on ξ_i translate to more general settings. We will discuss the corresponding robustness of our findings in Section 6.

Agent i's preferences are then represented by her utility from signing contract (y_i, z_i) vis-a-vis agent $-i$ signing contract (y_{-i}, z_{-i})

$$U(\theta_i, y_i, z_i; y_{-i}, z_{-i}) = U_{BT}(\theta_i, y, z) + I(b(y_i), b(y_{-i})) \tag{2.8}$$

Often, we will refer to $u(y_i, y_{-i}) = u_{BT}(y_i) + I(b(y_i), b(y_{-i}))$ as the part of agent i's utility that is affected by piece-rates y_i and y_{-i} only. Note that (as without social comparison) $u(y_i, y_{-i})$ neither depends on z_i (or z_{-i}) nor θ_i (or θ_{-i}). Observe that for $\xi_i = 0$ for all i our model coincides with the model in Bénabou and Tirole (2016) (with firms hiring two instead of one ex-ante identical agents).

2.3 Welfare and Second-Best Contracts

As firm profits are only affected by social comparison via the efforts chosen by the agents in response to a piece-rate, welfare generated by hiring an agent i of talent θ_i with piece-rate y_i vis-a-vis another agent $-i$ who signed a contract with bonus y_{-i} is $W(\theta_i, y_i, y_{-i}) = w(y_i, y_{-i}) + B\theta_i$ with

$$w(y_i, y_{-i}) = w_{BT}(y_i) + I(b(y_i), b(y_{-i})). \tag{2.9}$$

Recall from the introduction of the benchmark model by Bénabou and Tirole (2016) that in the absence of any costs from social comparison (i.e., for $\xi_i = 0$), welfare generated by hiring an agent with talent θ_i is maximized by a piece-rate y^*_{BT} that maximizes $w_{BT}(y)$ and is independent of the agent's talent. The reason was that the impact of a piece-rate y on efforts $a(y)$ and $b(y)$ is independent of θ because the utility generated by piece-rate y and effort b for talent θ is $(b + \theta)y$.

Throughout this paper, we will distinguish the implications of social comparison on *total welfare* W and *allocative welfare* W_{BT} (the difference between the two concepts being the psychological costs and benefits of social comparison). By definition of y^*_{BT}, allocative welfare from hiring both agents is maximized if $y = y^* = y^*_{BT}$. But also total welfare from hiring both types of agents is maximized for $y = y^* = y^*_{BT}$ as this results in identical efforts chosen by both agents (independent of their location and talent) which implies no costs from social comparison. Hiring agents with talent θ_L and θ_H is then optimal if and only if $W_{BT}(\theta_L, y^*_{BT}) = w_{BT}(y^*_{BT}) + B\theta_L > \overline{U}$ (which is the same condition as in Bénabou and Tirole (2016)). Henceforth, we shall assume that this condition is met i.e., that θ_L is sufficiently large.

To simplify the exposition, we will use the following abbreviations when discussing utility, profits, and welfare for a type i agent when agents of type $-i$ signed contract (y_{-i}, z_{-i}). The utility of type i signing contract (y_i, z_i) is denoted $U_i = U(\theta_i, y_i, z_i; y_{-i}, z_{-i})$ and $u_i = u(y_i; y_{-i})$, the psychological costs are denoted $I_i = I_i(b(y_i), b_{-i})$, and profit and welfare generated by type i is denoted by $\Pi_i = \Pi(\theta_i, y_i, z_i; y_{-i}, z_{-i})$, $\pi_i = \pi(y_i)$, $W_i = W(\theta_i, y_i; y_{-i})$, and $w_i = w(y_i; y_{-i})$, respectively.

To derive second-best contracts, suppose the firm can observe the agent's type θ_i. In the absence of social comparison (i.e., for $\xi_i = 0$), the firm maximizes profits with $y_L = y_H = y^*_{BT}$ (and a choice of z_L and z_H which leaves the agent with her reservation utility $\bar{U}$). This menu of second best contracts remains optimal for the firm in the case of social comparison (i.e., $\xi_i > 0$)

- just observe that $y_L \neq y_H$ (and thereby $b(y_L) \neq b(y_H)$) only reduces welfare (and thereby firm profits).[5]

Lemma 1 *Suppose, for all $i = H, L$, θ_i is observable. Then, $y_L^* = y_H^* = y_{BT}^*$.*

3 Monopsony

Throughout this section, a monopsony employer offers a menu of contracts (y_i, z_i) for $i \in \{L, H\}$. Hence, the probability that i faces an agent of type H (L) is q_H (q_L). So when computing her expected utility U_i from signing contract (y_i, z_i), agent i expects the other agent to sign contract $(y_{-i}, z_{-i}) = (y_H, z_H)$ with probability q_H and contract $(y_{-i}, z_{-i}) = (y_L, z_L)$ with probability q_L. The menu (y_i, z_i) with $i = H, L$ is incentive compatible if $U_H \geq U_L + y_L \Delta\theta$ and $U_L \geq U_H - y_H \Delta\theta$. An agent of type i participates if $U_i \geq \bar{U}$.[6] So if the monopsony employer wants to attract both types, it maximizes its expected profit

$$\max_{(y_i, z_i)_{i=L,H}} \left\{ \sum_{i=L,H} q_i \left[\pi_i + (B - y_i)\theta_i - z_i \right] \right\}, \tag{3.1}$$

subject to the incentive compatibility constraints and participation constraints. By standard arguments, the low type's participation constraint is binding (if it was not, the firm could reduce z_L until it is), i.e.,

$$U_L = \bar{U}, \tag{3.2}$$

and so is the high type's incentive compatibility constraint which fixes the rent for type H agents in excess of $\bar{U}$ to $y_L \Delta\theta$.
We solve

$$\max_{(y_i, z_i)_{i=L,H}} \left\{ \sum_{i=L,H} q_i \left[\pi_i + (B - y_i)\theta_i - z_i \right] \right\}, \tag{3.3}$$

subject to

$$U_L = \bar{U}, \tag{3.4}$$
$$U_H = U_L + \Delta\theta y_L, \tag{3.5}$$

where z_L and z_H are determined by the two binding constraints and y_H and y_L solve the two first-order conditions (for details see appendix A.1):

$$\frac{\partial w}{\partial y_L} = q_H \Delta\theta, \tag{3.6}$$

and

$$\frac{\partial w}{\partial y_H} = 0. \tag{3.7}$$

[5] We will refer to second-best contracts with superscript $(*)$.
[6] By standard arguments, $U_L \geq \bar{U}$ is necessary and sufficient for both participation constraints to hold.

where $w = q_H w_H + q_L w_L$ denotes ex-ante welfare (net of $\theta_i B$) generated by the menu (y_i, z_i) for $i = H, L$ by hiring a random agent.[7] I.e, y_H^m is ex-ante efficient (for a given bonus y_L) while y_L^m below the efficient level (for a given bonus y_H).

Would the monopsonist only hire high talented agents, he would save the high talented agent's rent, but loses the low talented agent's surplus which is suboptimal iff

$$q_L \left[w_L + B\theta_L - \overline{U}\right] \geq q_H y_L^m \Delta\theta. \tag{3.8}$$

Note that this condition is the easier to meet, the smaller the distortion for low types and the smaller the rent for the high type, i.e., iff $q_L > q_L^m$.

If hiring both types of agents is optimal, the optimal menu exhibits two deviations from second-best contracts (provided that ξ is not too large).

Proposition 1 *Suppose $q_L > q_L^m$. There is $\bar{\xi} > 0$ such that for all $\xi_i < \bar{\xi}$: A monopsonist firm offers a menu $((y_H^m, z_H^m), (y_L^m, z_L^m))$ with (i) $y_L^m < y_L^*$ and $y_H^m = y_H^*$, (ii) $\frac{dy_L^m}{d\xi_i} > 0$, (iii) $\frac{dy_H^m}{d\xi_i} = \frac{dy_H^*}{d\xi_i} < 0$.*

According to Proposition 1 and as illustrated in the left graph of Fig. 4.1, the optimal contract menu offers inefficiently low piece-rates for both types of agents whenever $\xi_i > 0$. The reason is that - as usual for monopsonistic screening - y_L^m is downwards distorted to reduce the rent for type H. As a result, y_H^m (which is efficient for a given y_L^m) is also lower than y^*. But as ξ_i increases, y_L^m increases and y_H^m decreases. Hence, the introduction of social comparison is socially beneficial if q_L is sufficiently large as in this case the smaller welfare distortion for low types (as it results from social comparison) outweighs the corresponding distortion for high types. Hence we identify social benefits from social comparison if there is no competition for talent between more than one employer.

4 Perfect Competition

Now consider two firms competing for the two agents, each offering an identical incentive-compatible menu of contracts. Suppose both firms offer identical menus that satisfy the following conditions:

(i) break-even: zero profits for both firms;

(ii) no cross-subsidy: zero profits with both types of agents.

After deriving optimal contracts that satisfy conditions (i) and (ii), we will discuss conditions under which such menus of contract resemble a (unique) equilibrium allocation. For identical menus, agents randomize between firms such that the probability to face a high talented agent in the same firm is $Q_H = \frac{1}{2} q_H$. From condition (ii), we directly find the fixed payments of the two types

$$\Pi_H = 0 \Longleftrightarrow z_H = \pi_H + (B - y_H)\theta_H, \tag{4.1}$$

$$\Pi_L = 0 \Longleftrightarrow z_L = \pi_L + (B - y_L)\theta_L. \tag{4.2}$$

By standard Bertrand-like arguments, the low type must get the maximal surplus, i.e., the surplus W_L generated by y_L^* for any given piece-rate y_H. Would she receive more, the firm made - absent

[7] We will refer to solutions with superscript m.

subsidies - a loss with low talented agents. Would she receive less, a competing firm could profitably deviate by attracting the low talented worker with a positive profit. Hence,

$$y_L^c = y_L^*, \qquad z_L^C = \pi(y_L^*) + (B - y_L^*)\theta_L. \tag{4.3}$$

Additionally, the high and the low type should not benefit from mimicking the other type,

$$U_L \geq U_H - y_H \Delta\theta, \tag{4.4}$$

$$U_H \geq U_L + y_L^* \Delta\theta. \tag{4.5}$$

which implies $y_H \geq y_L^*$. The optimal y_H and U_H maximizes W_H subject to the constraint that $\Delta\theta y_H \geq U_H - U_L$ and $y_H \geq y_L^*$, i.e., the optimal (y_H, U_H) is the contract with smallest distortions which is the contract with $\Delta\theta y_H = U_H - U_L$. With conditions (i) and (ii) this implies that y_H is determined by the binding incentive compatibility constraint of the low type

$$y_H \Delta\theta = W_H - W_L^*. \tag{4.6}$$

The allocation satisfying conditions (i) and (ii), $y_L = y_L^*$, and y_H maximizing W_H subject to incentive compatibility is typically referred to as the least-cost separating (LCS) allocation.

Definition 1 *An incentive-compatible allocation $\{(U_i^{**}, y_i^{**})\}_{i=H,L}$ is interim efficient if there exists no other incentive-compatible $\{(U_i, y_i)\}_{i=H,L}$ that*
(i) Pareto dominates it, i.e. $U_H \geq U_H^{**}, U_L \geq U_L^{**}$, with at least one strict inequality,
(ii) Makes the employer(s) at least break even on average, i.e. $\sum_i q_i[w(y_i) + \theta_i B - U_i] \geq 0$.

As discussed in detail in Bénabou and Tirole (2016), the interim efficiency of the LCS allocation is necessary and sufficient for the LCS allocation to be the unique equilibrium allocation.[8] Hence, the following result establishes the existence of an equilibrium satisfying conditions (i) and (ii).

Lemma 2 *The least-cost separating allocation is interim-efficient iff*

$$q_H \frac{\partial W_H}{\partial y_H} + q_L \Delta\theta \geq 0, \tag{4.7}$$

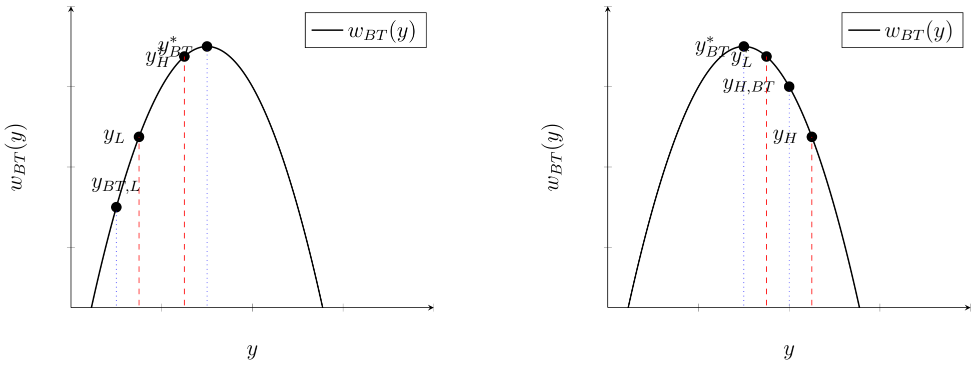
i.e., iff $q_L \geq \tilde{q}_L < 1$.

If the LCS is an equilibrium allocation, the impact of social comparison can be summarized as follows:

Proposition 2 *Suppose $q_L > \tilde{q}_L$. There is $\bar{\xi} > 0$ such that for all $\xi_i < \bar{\xi}$: A firm in perfect competition offers a menu $((y_H^c, z_H^c), (y_L^c, z_L^c))$ with (i) $y_L^c = y_L^*$ and $y_H^c > y_H^*$ with (ii) $\frac{dy_L^c}{d\xi_i} > 0$, and (iii) $\frac{dy_H^c}{d\xi_i} < 0$ for $\delta_H < 0$ and $\frac{dy_H^c}{d\xi_i} > 0$ for $\delta_L < 0$.*

The bonus for the low type (which is efficient for a given bonus of the high type) is enhanced relative to the optimal bonus without social comparison as a higher bonus reduces the costs from social comparison (for an illustration see the right graph in Fig. 4.1). The bonus for the high type is again upwards distorted, and the distortion is amplified if lagging behind induces a cost and attenuated if leading is costly. I.e., social comparison always leads to an up-wards distorted piece-rate for low productivity agents. If lagging behind is considered psychologically costly, this is offset by a further distortion of incentives for high types who receive even larger piece-rates than without social comparison because signing the low type's contract becomes less attractive. If leading is considered costly, the distortion for high types is moderated (relative to the situation without social comparison) because signing the low type's contract becomes more attractive.

[8] It is straightforward to show that this also holds on an open set of $\xi_i \geq 0$ for all i. Material available on request.



(a) Monopsony (b) Perfect Competition

Figure 4.1: Impact of a ranking on welfare (without psychological costs and benefits) if lagging behind is costly.

5 Imperfect Competition

Following Bénabou and Tirole (2016) we model imperfect competition with the full-spectrum Hotelling model. We have a unit continuum of agents, which is uniformly distributed along the unit interval, $x \in [0, 1]$. Two firms $k = 0, 1$, which are located at the extremities, recruit workers (see Fig. 5.1).

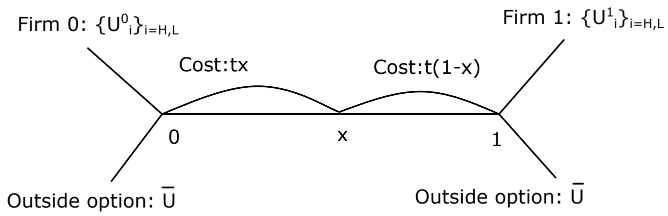


Figure 5.1: Hotelling model

As can also be seen from the figure a worker at position x, who chooses to work for firm 0, incurs a cost of tx equal to the distance he must travel. θ and t are assumed to be independent and that firms cannot observe workers' positions. Moreover, workers also have an outside option $\overline{U}$ and they also have to pay for the distance traveled to get it.[9] Type i located at x will choose firm $k = 0$ iff

$$U_i^k - tx \geq \max_l \{\overline{U} - tx, \overline{U} - t(1-x), U_i^l - t(1-x)\}. \tag{5.1}$$

We focus on the symmetric equilibrium, where each firm attracts half of the labor force. Both firms want to employ positive measures of each type of workers, and they do not want to corner the market, moving x all the way to 0 or 1. As before, we will first derive optimal contracts under these constraints and then discuss the conditions for this to resemble a (unique) equilibrium. Hence, firm k's share of workers of type i is assumed to be

$$x_i^k(U_i^k, U_i^l) = \frac{1}{2} + \frac{U_i^k - U_i^l}{2t}. \tag{5.2}$$

Firm k then chooses (U_L, U_H, y_L, y_H) to solve

$$\max \left\{ q_H \left(U_H - U_H^l + t \right) [w_H + \theta_H B - U_H] + q_L \left(U_L - U_L^l + t \right) [w_L + \theta_L B - U_L] \right\} \tag{5.3}$$

[9]We denote contracts, utilities, and profits (offered) by firm k with a superscript k.

subject to

$$\begin{aligned} &U_H \geq U_L + y_L \Delta\theta && (\mu_H) && (5.4) \\ &U_L \geq U_H - y_H \Delta\theta && (\mu_L) && (5.5) \\ &U_L \geq \overline{U}, && (\nu) && (5.6) \end{aligned}$$

where the Lagrange multipliers are given in parentheses. For $U_i = U_i^l$, the first order conditions are

$$q_H [\pi_H - t] + \mu_H - \mu_L = 0 \tag{5.7}$$
$$q_L [\pi_L - t] - \mu_H + \mu_L + \nu = 0 \tag{5.8}$$
$$q_H t \frac{\partial w_H}{\partial y_H} + q_L t \frac{\partial w_L}{\partial y_H} + \mu_L \Delta\theta = 0 \tag{5.9}$$
$$q_H t \frac{\partial w_H}{\partial y_L} + q_L t \frac{\partial w_L}{\partial y_L} - \mu_H \Delta\theta = 0. \tag{5.10}$$

Were μ_H and μ_L strictly positive, both incentive constraints would bind, and it followed that $y_L = y_H$ and, hence, $I(.) = 0$, which makes (5.9) and (5.10) mutually incompatible. Thus, μ_L and μ_H cannot both be strictly positive and exactly one of the two incentive compatibility constraints binds. [10] As in Bénabou and Tirole (2016) this leads to three different regions in which we can characterize an equilibrium.[11]

Proposition 3 *Let $q_L \geq \bar{q}_L$. There is $\bar{\xi} > 0$ such that for all $\xi_i < \bar{\xi}$: For given ξ_i there exist unique thresholds $t_1 > 0$ and $t_2 > t_1$ such that, in the unique symmetric market equilibrium:*

1. *Region I (strong competition): for all $t < t_1$, bonuses are $\hat{y}_L = y_L^*$ and $\hat{y}_H > y_H^*$. The low type's participation constraint is not binding, $U_L > \bar{U}$, while her incentive constraint is, $U_H - U_L = \hat{y}_H^I(t)\Delta\theta$.*

2. *Region II (medium competition): for all $t \in [t_1, t_2)$, bonuses are $\hat{y}_L = y_L^*$ and $\hat{y}_H > y_H^*$. The low type's participation constraint is binding, $U_L = \bar{U}$, and so is his incentive constraint, $U_H - U_L = \hat{y}_H^{II}(t)\Delta\theta$.*

3. *Region III (weak competition): for all $t \geq t_2$, bonuses are $\hat{y}_L < y_L^*$ and $\hat{y}_H = y_H^*$. The low type's participation and the high type's incentive constraints are binding, $U_L = \bar{U}$ and $U_H - U_L = \hat{y}_L(t)\Delta\theta$.*

In particular, optimal bonuses y_L and y_H are continuous and monotone decreasing in t and so is the difference $U_H - U_L$ offered by the optimal menu (for details see the proof of Proposition 4). For a unique t, $U_H - U_L = y_L^{BT}\Delta\theta$ and the optimal menu is the menu of second-best contracts (see Fig. 4.1). Hence, the main finding by Bénabou and Tirole (2016) that second best contracts are equilibrium contracts for exactly one (intermediate) level of competition translates to our setting with psychological costs from social comparison. For $t = 0$, the optimization problem coincides with Section 4 and for $t \to \infty$ the optimization problem coincides with Section 3. As a consequence, the comparative statics of bonuses in a neighborhood of $t = \infty$ and $t = 0$ is as depicted by Proposition 1 and 2 and for cost of lagging behind illustrated by Fig. 5.2.

[10] We will refer to solutions with $\hat{}$.

[11] Equilibrium existence and uniqueness follows from the interim efficiency of the LCS allocation – for details see Appendix A.6.

Proposition 4 *Suppose $q_L \geq \bar{q}_L$. (i) There is $t^m < \infty$ and $\bar{\xi} > 0$ such that for all $t > t^m$ and $\xi_i < \bar{\xi}$: the optimal menu $((\hat{y}_H, \hat{U}_H), (\hat{y}_L, \hat{U}_L))$ satisfies $\frac{d\hat{y}_L}{d\xi_i} > 0$ and $\frac{d\hat{y}_H}{d\xi_i} < 0$; (ii) There is $t^c > 0$ and $\bar{\xi} > 0$ such that for all $< t^c$ and $\xi_i < \bar{\xi}$: the optimal menu $((\hat{y}_H, \hat{z}_H), (\hat{y}_L, \hat{z}_L))$ satisfies $\frac{d\hat{y}_L}{d\xi_i} > 0$, $\frac{d\hat{y}_H}{d\xi_i} > 0$ for $\delta_H < 0$ and $\frac{d\hat{y}_H}{d\xi_i} > 0$ for $\delta_L < 0$.*

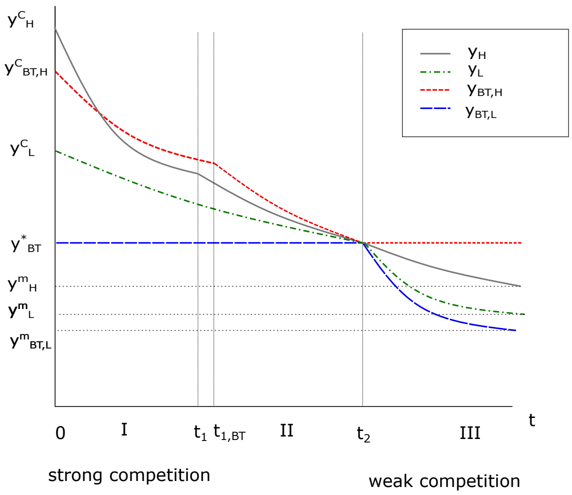


Figure 5.2: Imperfect competition with and without costs of lagging behind, where the subscript BT refers to the results of Bénabou and Tirole (2016). The three regions of imperfect competition as in Bénabou and Tirole (2016) are shown. t measures the level of competition, where high t's are low levels of competition. $y^m_L(y^m_H)$ is the low- (high-) skilled agent's bonus in monopsony, while y^C_L (y^C_H) is the low-(high-) skilled agent's bonus in perfect competition. For $t \to 0$ the bonuses converge to the perfect competition results, while for $t \to \infty$ to the monopsony result. y^*_{BT} is the symmetric efficient allocation of Bénabou and Tirole (2016), when no costs of lagging behind are present. When costs of lagging behind are present, region I shrinks and the efficient bonus with respect to y^*_L (y^*_H) is a function of y_H (y_L), i.e. in region I and II $y_L = y^*_L$ and in region III $y_H = y^*_H$. The intersection in region I of the high-skilled agent's bonuses with and without costs of lagging behind is exactly the point where the dominant effect of the costs of lagging behind switches from the reduction of the imitation incentive through a higher bonus to the importance of decreasing the effect of the costs of lagging behind on the low-skilled worker's utility.

According to Proposition 3 and 4, the impact of psychological costs due to social comparison is twofold. First, if lagging behind or leading in a ranking is perceived as costly, second-best piece-rates for low types are larger and second best piece-rates for high types are smaller than in the absence of social comparison. These modified efficient piece-rates internalize psychological costs and reduce these costs relative to the second-best piece-rates in Bénabou and Tirole (2016). Second, depending on the intensity of competition, piece-rates are distorted for one type of agents and this distortion is either amplified or attenuated by social comparison. If the competition intensity is low, high types have to be prevented from signing low types' contracts and if the competition intensity is high, low types have to be prevented from imitating high types. Introducing psychological costs of social comparison for agents who lag behind therefore reduces the attractiveness to sign low type contracts, which relaxes the distortions if the competition intensity is low and strengthens the distortions if the competition intensity is high. In this sense, social comparison complements screening if agents with inefficiently low incentives are further motivated by the ranking, but it offsets inefficiently high piece-rates if the competition intensity is high. If, in contrast, leading a ranking is considered costly, the problem of overincentivized agents is attenuated with the introduction of social comparison.

6 Discussion and Conclusion

Introducing psychological costs of social comparison, we demonstrated that social comparison can reduce or amplify efficiency distortions depending on the degree of competition and whether lagging behind or leading a ranking is considered as costly. Mirror images of these findings can be derived if one assumes psychological benefits rather than costs. E.g., a psychological benefit from leading renders contracts offered to high type agents more attractive, which is socially beneficial if the competition intensity is low and high types have to be prevented from signing low types' contracts. The introduction of social comparison introduces an additional benefit for highly productive agents that allows to reduce distortions. In contrast, if the competition intensity is high, the additional attractiveness of contracts for high type agents amplifies the problem of preventing low type agents from imitation. For the overall impact of a ranking on welfare, it certainly matters whether social comparison induces costs or benefits. If social comparison generates costs, a ranking ceteris paribus reduces welfare unless reduced distortions overcompensate. This, e.g., is the case if competition intensity is low (and psychological cost reduce distortions for the low type) and the fraction of low productivity types is sufficiently high.

In sum, the welfare conclusions depend on (i) the degree of competition, (ii) the nature of psychological costs and benefits, and (iii) the relative frequency of high and low types. As competition increases, it becomes more and more important to enhance the attractiveness of contracts for low productivity types and lower the attractiveness for high productivity types (which is accomplished by costs of leading or benefits from lagging behind, but hindered by costs from lagging behind and benefits from leading). As we see rankings in particular in industries with a rather strong competition for talent, these results point at a hidden cost of such incentive schemes. If ranked employees perceive lagging behind as costly and/or enjoy being on top of a ranking, inefficiencies as identified by Bénabou and Tirole (2016) are further amplified. If, however, a top position in the ranking is interpreted also on the background of an employees performance in unranked (and unobservable) tasks, the ranking may contribute to an effort to reduce welfare distortions of a bonus culture.

The comparative statics with respect to the costs and benefits of social comparison is robust with respect to details of the functional form. E.g., we assumed psychological costs that are monotone increasing and differentiable in the difference between verifiable efforts by the leader and the loser in the ranking such that costs and the corresponding derivative vanish if the difference is small. We made these assumptions to get a smooth comparative statics with respect to the degree of competition and the intensity of psychological costs. Qualitatively, our results persist if these smoothness assumptions are dropped, e.g., assuming that costs are non-zero and finite if and only if ranks (or efforts) differ. But since this specification introduces a discontinuity, the baseline model by Bénabou and Tirole (2016) would no longer emerge as a limit case for vanishing psychological costs.

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A Appendix

A.1 Proof of Proposition 1

We solve the optimization problem in monopsony, given by

$$\max_{(y_i,z_i)_{i=L,H}} \left\{ \sum_{i=L,H} q_i \left[\pi(y_i) + (B - y_i)\theta_i - z_i \right] \right\}, \tag{A.1}$$

subject to participation of both types of agents and incentive compatibility of the contract menu. To simplify exposition, we restate the monopsonist's problem as choosing $((y_L.U_L), (y_H, U_H))$ to solve

$$\max_{(y_i,U_i)_{i=L,H}} \left\{ \sum_{i=L,H} q_i \left[W_i - U_i \right] \right\}, \tag{A.2}$$

$$U_L = \bar{U}, \qquad (\mu) \tag{A.3}$$

$$U_H = U_L + y_L \Delta\theta, \qquad (\lambda) \tag{A.4}$$

where (A.4) is the high type's binding incentive constraint and the Lagrange multipliers are given in parentheses. Plugging the binding constraints into the objective function yields the problem to choose y_H and y_L to solve

$$\max_{(y_i)_{i=L,H}} \left\{ q_L(W_L - \bar{U}) + q_H(W_H - \bar{U} - y_L\Delta\theta) \right\}. \tag{A.5}$$

For this problem, first order conditions are given by

$$\begin{aligned} \Delta_L = q_L \frac{\partial W_L}{\partial y_L} + q_H \left(\frac{\partial w_H}{\partial y_L} - \Delta\theta \right) = 0 \\ \Delta_H = q_L \frac{\partial W_L}{\partial y_H} + q_H \frac{\partial w_H}{\partial y_H} = 0. \end{aligned} \tag{A.6}$$

Proposition 1 (i) follows directly from the FOCs (whenever ξ_i is sufficiently small to guarantee the concavity of welfare which is satisfied on an open set of ξ_i). (ii) and (iii) follow directly from

$$\frac{dy_i}{d\xi_i} = -\frac{\frac{\partial \Delta_i}{\partial \xi_i}\frac{\partial \Delta_{-i}}{\partial y_{-i}} - \frac{\partial \Delta_{-i}}{\partial \xi_i}\frac{\partial \Delta_i}{\partial y_{-i}}}{\frac{\partial \Delta_i}{\partial y_i}\frac{\partial \Delta_{-i}}{\partial y_{-i}} - \frac{\partial \Delta_{-i}}{\partial y_i}\frac{\partial \Delta_i}{\partial y_{-i}}}. \tag{A.7}$$

for ξ_i sufficiently small. Again, it is straightforward to check that the results of the Proposition hold on an open set of ξ including $\xi_i = 0$.

A.2 Proof of Lemma 2 (interim efficiency)

The proof is analogous to the proof of Lemma 1 in Bénabou and Tirole (2016) with $w(y_H)$ in Bénabou and Tirole (2016) being substituted by W_H.

A.3 Proof of Proposition 2

(i) follows from the concavity of W_H (for ξ_i sufficiently small) and (4.7). (ii)-(iii) follows directly from (A.7) with the implicit functions determining y_L^c and y_H^c as Δ_L and δ_H, respectively. I.e., $\Delta_L = \frac{\partial W}{\partial y_L} = 0$ and $\Delta_H = W_H - W_L^* - y_H\Delta\theta$.

A.4 Proof of Proposition 3

Solving with the different feasible combinations of Lagrange Multipliers yields the following regions.

A.4.1 Region I

Suppose (5.6) is non binding, $U_L > \overline{U}$, such that $\nu = 0$.

Lemma 3 *If $\nu = 0$, then $\mu_H = 0 \leq \mu_L$ and $y_L = y_L^* \leq y_H$.*

Proof

- *If $\mu_L = \mu_H = 0$, then (5.9) and (5.10) imply $y_H = y_L = y^*$, so (5.4) and (5.5) imply that $U_H - U_L = y^*\Delta\theta$. Next from (5.7) and (5.8) we have $\pi_H - \pi_L = t - \pi_L = 0$, whereas $\pi_H - \pi_L = B\Delta\theta - (U_H - U_L) = (B - y^*)\Delta\theta > 0$, a contradiction.*
- *If $\mu_H > 0 = \mu_L$ condition (5.10) implies $q_H t\frac{\partial w_H}{\partial y_L} + q_L t\frac{\partial w_L}{\partial y_L} > 0$, thus $y_L < y_L^*$ and condition (5.9) implies $y_H = y_H^*$. Moreover, (5.7) and (5.8) and $\mu_H > \mu_L$ require $\pi_H < t < \pi_L$. But*

$$\pi_H - \pi_L = w_H - w_L + (B - y_L)\Delta\theta > 0, \tag{A.8}$$

a contradiction.

- *We are thus left with $\mu_H = 0 < \mu_L$, implying $y_L = y_L^* < y_H$ by (5.9) and (5.10).* □

We now want to express y_H as a function of t. We rewrite (5.9) as

$$q_H t\frac{\partial w_H}{\partial y_H} + q_L t\frac{\partial w_L}{\partial y_H} = -\mu_L\Delta\theta = q_L\Delta\theta(\pi_L - t) \tag{A.9}$$

We sum (5.7) and (5.8), and recall the definition of π_i

$$U_L + t = q_H\left[w_H + \theta_H B - y_H\Delta\theta\right] + q_L\left[w_L + \theta_L B\right], \tag{A.10}$$

where $y_H\Delta\theta$ comes from the binding constraint due to $\mu_L > 0$. Hence,

$$\pi_L - t = w_L + \theta_L B - U_L - t \tag{A.11}$$

$$= q_H\left[w_L - w_H - (B - y_H)\Delta\theta\right]. \tag{A.12}$$

Substituting into (A.9) we define

$$\Phi(y_H; t) \equiv w_H - w_L + (B - y_H)\Delta\theta + \frac{t}{q_L\Delta\theta}\frac{\partial w_H}{\partial y_H} + \frac{t}{q_H\Delta\theta}\frac{\partial w_L}{\partial y_H} = 0, \tag{A.13}$$

with $y_L = y_L^*$. We now want to characterize the equilibrium value of y_H over region I, denoted by $\hat{y}_H^I(t)$.

Lemma 4 *For any $t \geq 0$ there exists a unique $\hat{y}_H^I(t) \in [y_L^*, B)$ to (A.13). It is strictly decreasing in t, starting from the perfectly competitive value $\hat{y}_H^I(0) = y_H^C$.*

Proof *$\Phi(y; t)$ is strictly decreasing in y on the interval $[y_L^*, B)$ with $\Phi(B; t) < 0$ and $\Phi(y_L^*; t) > 0$. Hence, we have uniqueness and existence. Strict monotonicity follows from Φ being strictly decreasing in t. Setting $t = 0$ we find $\hat{y}_H^I(0) = y_H^C$ from (A.13). Thus, we only have to verify that the initial assumption $U_L > \overline{U}$, or $\nu = 0$ holds. By (A.10) we have for all y_H and $y_L = y_L^*$*

$$U_L + t = q_H\left(w_H + \theta_H B - y_H\Delta\theta\right) + q_L\left(w_L + \theta_L B\right) \tag{A.14}$$

$$= q_H\left[w_H + \Delta\theta(B - y_H) - w_L\right] + w_L + \theta_L B. \tag{A.15}$$

For $y_H = \hat{y}_H^I(t)$ the corresponding value of U_L is strictly above $\overline{U}$ iff $\Psi(t) > \overline{U} + t$, with

$$\Psi(t) \equiv w_L + \theta_L B + q_H \left[\left(B - \hat{y}_H^I(t) \right) \Delta\theta - w_L + \hat{w}_H \right], \tag{A.16}$$

where $y_L = y_L^*$ and $\hat{w}_H = w(\hat{y}_H^I(t))$.

Lemma 5 *There exists a unique $t_1 > 0$ such that $\Psi(t) \geq \overline{U} + t$ iff $t \leq t_1$. On $[0, t_1]$, the low type's utility U_L is strictly decreasing in t, reaching $\overline{U}$ at t_1.*

Proof *For $t = 0$ the bracket term is equal to 0, so $\Psi(0) = w_L + \theta_L B > \overline{U}$ by (3.8), and $\lim_{t\to+\infty} \left[\Psi(t) - \overline{U} - t \right] = -\infty$, then there exists at least one solution to $\Psi(t) = \overline{U} + t$. We want to establish the monotonicity of U_L and the uniqueness of the solution. Hence, we show $\Psi'(t) < 1$ for all $t > 0$. Therefore, we use the implicit function theorem to derive*

$$\begin{aligned} \Psi'(t) =& \frac{\partial \Psi(t)}{\partial \hat{y}_H^I(t)} (-1) \frac{\partial \Phi(\hat{y}_H^I(t); t)}{\partial t} \left[\frac{\partial \Phi(\hat{y}_H^I(t); t)}{\partial \hat{y}_H^I(t)} \right]^{-1} \\ =& \left[-\frac{\partial w_L}{\partial y_H} + q_H \left(\Delta\theta + \frac{\partial w_L}{\partial y_H} - \frac{\partial w_H}{\partial y_H} \right) \right] \frac{-\frac{1}{q_L \Delta\theta} \frac{\partial w_H}{\partial y_H} - \frac{1}{q_H \Delta\theta} \frac{\partial w_L}{\partial y_H}}{\Delta\theta - \frac{\partial w_H}{\partial y_H} + \frac{\partial w_L}{\partial y_H} - \frac{t}{q_L \Delta\theta} \frac{\partial^2 w_H}{\partial y_H^2} - \frac{t}{q_H \Delta\theta} \frac{\partial^2 w_L}{\partial y_H^2}}. \end{aligned}$$

Suppose now $\Psi'(t) < 1$, we find

$$t \left(\frac{\partial^2 w_H}{\partial y_H^2} + \frac{q_L}{q_H} \frac{\partial^2 w_L}{\partial y_H^2} \right) - q_L q_H \Delta\theta \frac{\partial w_L}{\partial y_H} < \left(\Delta\theta - \frac{\partial w_H}{\partial y_H} - \frac{q_L}{q_H} \frac{\partial w_L}{\partial y_H} \right) \left(q_L \Delta\theta + q_L \frac{\partial w_L}{\partial y_H} + q_H \frac{\partial w_H}{\partial y_H} \right),$$

where the right hand side is positive (due to (4.7)) and the left hand side is negative.[12] □

Overall, Region I consists of the interval $[0, t_1]$, where t_1 is uniquely defined by $\Psi(t_1) = t_1 + \overline{U}$. In this interval $y_L = y_L^*$ and $y_H = \hat{y}_H^I(t)$ strictly decreasing in t. [13]
For $t \geq t_1$ the constraint $U_L \geq \overline{U}$ is binding. We recall $\mu_L \mu_H = 0$, we distinguish two subregions, Region II ($\mu_H = 0$) and Region III ($\mu_L = 0$) and show that these are two intervals $[t_1, t_2]$ and $[t_2, +\infty)$ with $t_1 < t_2$. Inside Region II the low type's incentive constraint is binding, but not the high type's, inside Region III it is the reverse.

A.4.2 Region II

Here, we consider first the values of t, where $\mu_H = 0 < \mu_L$, which implies $y_L = y_L^* < y_H$. Moreover, $U_H - U_L = y_H \Delta\theta$ or $U_H = \overline{U} + y_H \Delta\theta$ since $U_L = \overline{U}$. Therefore,

$$\mu_L = q_H(\pi_H - t) = q_H \left[w_H + \theta_H B - U_H - t \right] \tag{A.17}$$

$$= q_H \left[w_H + \theta_H B - \overline{U} - y_H \Delta\theta - t \right]. \tag{A.18}$$

Substituting into (5.9)

$$q_H t \frac{\partial w_H}{\partial y_H} + q_L t \frac{\partial w_L}{\partial y_H} + q_H \Delta\theta \left[w_H + \theta_H B - \bar{U} - t - y_H \Delta\theta \right] = 0. \tag{A.19}$$

[12] For costs of being behind, one needs to restrict ξ_i for this to hold

[13] In the case of costs for being behind, one can show that t_1 is smaller than in Bénabou and Tirole (2016). Thus, Region I is in this case smaller than in Bénabou and Tirole (2016).

We then define

$$\Gamma(y_H;t) \equiv w_H + \theta_H B - y_H\Delta\theta - \bar{U} - t + \frac{t}{\Delta\theta}\frac{\partial w_H}{\partial y_H} + \frac{q_L t}{q_H\Delta\theta}\frac{\partial w_L}{\partial y_H} = 0. \tag{A.20}$$

On the interval $[y_L^*, B)$ the function $\Gamma(y;t)$ is strictly decreasing in y and t. From (A.13) it follows

$$w_H = w_L - (B - y_H)\Delta\theta - \frac{t}{q_L\Delta\theta}\frac{\partial w_H}{\partial y_H} - \frac{t}{q_H\Delta\theta}\frac{\partial w_L}{\partial y_H}, \tag{A.21}$$

then

$$\Gamma(\hat{y}_H^I;t) = w_L + B\theta_L - \bar{U} - t\left[1 + \frac{q_H}{q_L\Delta\theta}\frac{\partial w_H}{\partial y_H} + \frac{1}{\Delta\theta}\frac{\partial w_L}{\partial y_H}\right]. \tag{A.22}$$

At $t = t_1$, $\Psi(t_1) - \overline{U} - t_1 = \Gamma(\hat{y}_H^I(t_1);t_1) = 0$. Furthermore, when t is larger than t_1, $\hat{y}_H^I(t)$ decreases, so $\frac{\partial \hat{w}_H}{\partial y_H}$ increases. Since

$$\begin{aligned} q_L\Delta\theta + q_H\frac{\partial w_H}{\partial y_H} + q_L\frac{\partial w_L}{\partial y_H} &> q_L\Delta\theta + q_H\frac{\partial w_H(t=0)}{\partial y_H} + q_L\frac{\partial w_L(t=0)}{\partial y_H} \\ &= q_L\Delta\theta + q_H\frac{\partial w_H^C}{\partial y_H^C} + q_L\frac{\partial w_L}{\partial y_H^C} > 0, \end{aligned}$$

by (4.7), where $w_H^C = w(y_H^C)$, therefore $t[q_L\Delta\theta + q_H\frac{\partial w_H}{\partial y_H} + q_L\frac{\partial w_L}{\partial y_H}]$ is also increasing in t, such that $\Gamma(\hat{y}_H^I(t);t)$ is decreasing in t and negative over $(t_1, +\infty)$. We observe,

$$\Gamma(y_L^*;t) = w_L + \theta_H(B - y_L^*) + y_L^*\theta_L - \bar{U} - t, \tag{A.23}$$

therefore we define

$$t_2 \equiv w_L + \theta_H(B - y_L^*) + y_L^*\theta_L - \bar{U}, \tag{A.24}$$

so we observe

$$\begin{aligned} t_1 &= q_L w_L + \theta_L B + q_H\left[\left(B - \hat{y}_H^I(t_1)\right)\Delta\theta + \hat{w}_H\right] - \bar{U} \\ &< w_L + \theta_L B + q_H\left(B - \hat{y}_H^I(t_1)\right)\Delta\theta - \bar{U} \\ &< w_L + \theta_L B + (B - y_L^*)\Delta\theta - \bar{U} = t_2, \end{aligned}$$

with $\hat{w}_H = w(\hat{y}_H^I(t_1))$ and $y_L = y_L^*$.

Lemma 6 *For all $t \in [t_1, t_2]$, there exists a unique $\hat{y}_H^{II}(t) \in [y_L^*, \hat{y}_H^I(t_1)]$ such that $\Gamma(\hat{y}_H^{II}(t);t) = 0$. Furthermore, $\hat{y}_H^{II}(t)$ is strictly decreasing in t, starting at $\hat{y}_H^{II}(t_1) = \hat{y}_H^I(t_1)$ and reaching y_L^* at $t = t_2$. For all $t > t_2$, $\Gamma(y_H;t) < 0$ over all $y_H \geq y_L^*$.*

Proof *For $t \in [t_1, t_2]$ we have shown $\Gamma(\hat{y}_H^I(t_1);t) \leq 0 \leq \Gamma(y_L^*;t)$, with strict inequalities except at points t_1 and t_2 respectively. Since $\Gamma(y;t)$ is strictly decreasing in y and t, the result follows. Since $\hat{y}_H^{II}(t) < \hat{y}_H^I(t)$ on $(t_1, t_2]$, a possible kink between the two curves at t_1 must be a convex one. So by differentiating* (A.13) *and* (A.20)*, we find $-(\hat{y}_H^I)'(t_1) < -(\hat{y}_H^{II})'(t_1)$ iff*

$$\frac{\frac{\partial w_H}{\partial y_H} + \frac{q_L}{q_H}\frac{\partial w_L}{\partial y_H}}{q_L\Delta\theta\left(\frac{\partial w_H}{\partial y_H} - \frac{\partial w_L}{\partial y_H} - \Delta\theta\right) + t\left(\frac{\partial^2 w_H}{\partial y_H^2} + \frac{q_L}{q_H}\frac{\partial^2 w_L}{\partial y_H^2}\right)} < \frac{-\Delta\theta + \frac{\partial w_H}{\partial y_H} + \frac{q_L}{q_H}\frac{\partial w_L}{\partial y_H}}{\Delta\theta\left(\frac{\partial w_H}{\partial y_H} - \Delta\theta\right) + t\left(\frac{\partial^2 w_H}{\partial y_H^2} + \frac{q_L}{q_H}\frac{\partial^2 w_L}{\partial y_H^2}\right)}$$

equivalent to

$$q_H \frac{\partial w_H}{\partial y_H} + q_L \frac{\partial w_L}{\partial y_H} + q_L \Delta\theta > \frac{-t\left(\frac{\partial^2 w_H}{\partial y_H^2} + \frac{q_L}{q_H}\frac{\partial^2 w_L}{\partial y_H^2}\right) + \frac{q_L}{q_H}\Delta\theta \frac{\partial w_L}{\partial y_H}}{\frac{\partial w_H}{\partial y_H} - \Delta\theta + \frac{q_L}{q_H}\frac{\partial w_L}{\partial y_H}},$$

where the left hand side is positive due to (4.7) *and the right hand side negative.*[14] *Note, all derivatives are evaluated at* $\hat{y}_H^I(t_1) = \hat{y}_H^{II}(t_1)$. □

t_2 is the only point where $\mu_L = 0 = \mu_H$ (the only intersection of Region II and Region III). Then it follows $y_H = y_H^* = y^* = y_L^* = y_L$ by (5.9) and (5.10), and condition (5.7) with $U_L = \bar{U}$ implies $t = \pi_H = w(y^*) + (B - y^*)\theta_H + y^*\theta_L - \bar{U} = t_2$. In Region II $[t_1, t_2]$, $y_L = y_L^*$ and $y_H = \hat{y}_H^{II}(t)$ is strictly decreasing in t, like the high types utility $U_H = \bar{U} + \hat{y}_H^{II}(t)\Delta\theta$, while $U_L = \bar{U}$.

A.4.3 Region III

In this region, we have $U_L = \bar{U}$ but now $\mu_H > \mu_L = 0$. This implies $y_H^m = y_H^* > y_L$ by (5.9) and (5.10), and $U_H = \bar{U} + y_L\Delta\theta$ by (5.4). Moreover we find

$$\mu_H = q_H[t + \bar{U} + y_L\Delta\theta - w_H - \theta_H B], \tag{A.25}$$

with $w_H = w(y_H^*)$. Substituting into (5.10), we define

$$\Lambda(y_L; t) \equiv q_H[w_H + \theta_H B - t - \bar{U} - y_L\Delta\theta] + \frac{q_L t}{\Delta\theta}\frac{\partial w_L}{\partial y_L} + \frac{q_H t}{\Delta\theta}\frac{\partial w_H}{\partial y_L} = 0. \tag{A.26}$$

On the interval $[0, y_H^m]$, $\Lambda(y; t)$ is strictly decreasing in y_L, with

$$\Lambda(y^*; t) = q_H[w_H + \theta_H B - y^*\Delta\theta - \bar{U} - t] = q_H(t_2 - t) < 0. \tag{A.27}$$

We recall y_L^m is defined by $\frac{\partial w}{\partial y_L} = q_H\Delta\theta$. Hence,

$$\Lambda(y_L^m; t) = q_H[w_H + \theta_L B - \bar{U} + (B - y_L^m)\Delta\theta] > 0. \tag{A.28}$$

Lemma 7 *For all* $t \geq t_2$ *there exist a unique* $\hat{y}_L(t)$ *such that* $\Lambda(\hat{y}_L(t); t) = 0$*, and* $y_L^m < \hat{y}_L(t) \leq y_H^m$*, with equality at* $t = t_2$*. Furthermore,* $\hat{y}_L(t)$ *is strictly decreasing in* t *and* $\lim_{t\to+\infty} \hat{y}_L(t) = y_L^m$*.*

Proof *Existence and uniqueness have been established. Next,* $\frac{\partial\Lambda(y;t)}{\partial t} = -q_H + \frac{q_L}{\Delta\theta}\frac{\partial w_L}{\partial y_L} + \frac{q_H}{\Delta\theta}\frac{\partial w_H}{\partial y_L}$*. At* $y = \hat{y}_L(t)$ *this yields*

$$-\frac{q_H}{t}[w_H + \theta_H B - \bar{U} - y_L\Delta\theta] < 0, \tag{A.29}$$

so the function is strictly decreasing in t *and* $\lim_{t\to+\infty} \hat{y}_L(t) = y_L^m$. □

A.5 Proof of Proposition 4

Comparative Statics with respect to t The monotonicity of optimal y_H and y_L with respect to t directly follows from

$$\frac{dy_i}{dt} = -\frac{\frac{\partial\Delta_i}{\partial t}\frac{\partial\Delta_{-i}}{\partial y_{-i}} - \frac{\partial\Delta_{-i}}{\partial t}\frac{\partial\Delta_i}{\partial y_{-i}}}{\frac{\partial\Delta_i}{\partial y_i}\frac{\partial\Delta_{-i}}{\partial y_{-i}} - \frac{\partial\Delta_{-i}}{\partial y_i}\frac{\partial\Delta_i}{\partial y_{-i}}}. \tag{A.30}$$

with implicit functions Δ_H and Δ_L defined by FOCs (5.9) and (5.10), respectively.

[14] For costs of being behind, one needs to impose restrictions on ξ for this to hold.

Comparative Statics with respect to ξ_i The comparative statics with respect to ξ_i follows directly from (A.7) with implicit functions Δ_H and Δ_L defined by FOCs (5.9) and (5.10), respectively.

A.6 General optimization program under imperfect competition

In this proof we follow Appendix D of Bénabou and Tirole (2016). We use the notation as above and focus on costs of either being ahead $\delta_H < 0$ or behind $\delta_L < 0$.
Denote by $\hat{C} \equiv (\hat{U}_H, \hat{U}_L, \hat{y}_H, \hat{y}_L)$ the symmetric equilibrium strategies and payoffs given in Proposition 3, and played by the other firm. Let $X(u) = \min\{\max\{u, 0\}, 2t\}$ $\forall u \in \mathbb{R}$. The firm then solves:

$$\begin{aligned}\max \{&q_H \; X(U_H + t - \hat{U}_H)[w_H + \theta_H B - U_H] \\ &+ q_L \; X(U_L + t - \hat{U}_L)\mathbb{1}_{\{U_L \geq \bar{U}\}}[w_L + \theta_L B - U_L]\Big\}\end{aligned} \tag{A.31}$$

subject to

$$U_H \geq U_L + y_L \Delta\theta \tag{A.32}$$
$$U_L \geq U_H - y_H \Delta\theta \tag{A.33}$$
$$y_L \geq 0. \tag{A.34}$$

(A.31) is not everywhere differentiable, nor globally concave. If either $U_L \leq \hat{U}_L - t$ or $U_L < \bar{U}$, the firm employs zero low type workers, and must sell to a positive measure of H type agents, requiring $U_H > \max\{\hat{U}_H - t, \bar{U}\}$. Moreover the measure of H-types is given by $\frac{X(U_H + t - \hat{U}_H)}{2t} q_H$ and the measure of L-types by $\frac{X(U_L + t - \hat{U}_L)}{2t} q_L$. Therefore, the expected costs of being ahead/behind are

$$\frac{X(U_i + t - \hat{U}_i)}{2t} q_i I_i(b_i, b_{-i}), \tag{A.35}$$

with $I_i(b_i, b_{-i}) = \xi_i \delta(b_i, b_{-i}) f(\|b_i - b_{-i}\|)$ as described in section 2.2.
First, we show that there is no exclusion of low/high type workers, and then that it is not optimal to corner the market on either type.

A.6.1 No exclusion

Lemma 8 *There exists $\bar{q}_L \in [q_L, 1)$, independent of t, such that $\forall q_L \geq \bar{q}_L$, it is strictly suboptimal not to employ a positive measure of L-type agents. In particular, $U_L \geq \bar{U}$.*

Proof *Selling only to H-type agents under some contract (y_H, U_H) is less profitable than sticking to the symmetric strategy $(\hat{y}_H, \hat{U}_H)$ if*

$$\begin{aligned}q_H \bar{\omega}_H \equiv& q_H X(U_H - \hat{U}_H + t)[w_H + B\theta_H - U_H] \\ \leq& q_H t[\hat{w}_H + B\theta_H - \hat{U}_H] + q_L t[\hat{w}_L + B\theta_L - \hat{U}_L] \\ \equiv& q_H \hat{\hat{\omega}}_H + q_L \hat{\hat{\omega}}_L \equiv \hat{\hat{\omega}},\end{aligned} \tag{A.36}$$

with $w_i = w(y_i, y_{-i})$ and $\hat{w}_i = w(\hat{y}_i, \hat{y}_{-i})$. Since for any $t > 0$, $\hat{\hat{\omega}}_L$ is greater than 0, (A.36) is satisfied for q_H low enough, or $\frac{q_L}{q_H}$ large enough. The ratio $(\bar{\omega}_H - \hat{\hat{\omega}}_H)/\hat{\hat{\omega}}_L$ must remain bounded above as t tends to 0, even though $\lim_{t\to 0} \hat{\hat{\omega}}_L = 0$. For t small enough we can even show $\bar{\omega}_H(t) <$

$\hat{\omega}_H(t)$.
Firstly, to exclude L types, $U_L \le \max\{\bar{U}, \hat{U}_L - t\}$. *We have* $\hat{U}_L > \bar{U}\ \forall\, t < t_1$, *so for small* t *the relevant constraint is* $U_L \le \hat{U}_L - t$. *The firm then solves:*

$$\max\{X(U_H - \hat{U}_H + t)[w_H + B\theta_H - U_H]\} \tag{A.37}$$

subject to

$$U_H \ge U_L + y_L\Delta\theta \qquad (\mu_H) \tag{A.38}$$
$$U_L \ge U_H - y_H\Delta\theta \qquad (\mu_L) \tag{A.39}$$
$$U_L \le \hat{U}_L - t \qquad (\phi) \tag{A.40}$$
$$y_L \ge 0. \qquad (\psi) \tag{A.41}$$

For a positive share of H types, $U_H - \hat{U}_H > -t > U_L - \hat{U}_L$, *implying* $y_H > \hat{y}_H$. *The first order conditions are*

$$\begin{aligned} &-2t \le \mu_L - \mu_H \le w_H + B\theta_H - 2U_H + \hat{U}_H - t \\ &\textit{with equality for } U_H - \hat{U}_H > t \textit{ and } U_H - \hat{U}_H < t \textit{ respectively;} \end{aligned} \tag{A.42}$$

$$X(U_H - \hat{U}_H + t)\frac{\partial w_H}{\partial U_L} - \mu_H + \mu_L - \phi = 0 \tag{A.43}$$

$$X(U_H - \hat{U}_H + t)\frac{\partial w_H}{\partial y_H} + \mu_L\Delta\theta = 0 \tag{A.44}$$

$$X(U_H - \hat{U}_H + t)\frac{\partial w_H}{\partial y_L} - \mu_H\Delta\theta + \psi = 0 \tag{A.45}$$

If $\mu_L = 0$ *then* (A.44) *and* $X(U_H - \hat{U}_H + t) > 0$, *which implies* $y_H = y_H^* \le \hat{y}_H$, *a contradiction. Therefore* $\mu_L > 0$, *such that* $U_H - U_L = y_H\Delta\theta$ *with* $\hat{y}_H < y_H$. *If* $\psi > 0$, *then* $y_L = 0$ *and* $\mu_H > 0$, *which implies* $U_H - U_L = y_L\Delta\theta$, *such that* $y_H = y_L = 0$, *a contradiction. It follows that* $\mu_H = 0$, *and for small* ξ $\phi > 0$, *and* $U_L = \hat{U}_L - t$. $\hat{U}_H - \hat{U}_L = \hat{y}_H\Delta\theta$ *for* $t \le t_2$ *implies* $U_H - \hat{U}_H + t = (y_H - \hat{y}_H)\Delta\theta$, *which cannot be larger than* $2t$, *since* $-2t < \mu_L - \mu_H$. *Hence,* $X(U_H - \hat{U}_H + t) = U_H - \hat{U}_H + t$. *Eliminating* μ_L,

$$0 \le w_H + B\theta_H - 2U_H + \hat{U}_H - t + (U_H - \hat{U}_H + t)\frac{1}{\Delta\theta}\frac{\partial w_H}{\partial y_H} \tag{A.46}$$

with equality for $U_H - \hat{U}_H < t$.
From (5.7), (5.9) *and* (5.10) *with* $\hat{y}_L = y_L^*$, *we have for* $\hat{y}_H$

$$\hat{w}_H + \theta_H B - \hat{U}_H - t + t\frac{1}{\Delta\theta}\frac{\partial \hat{w}_H}{\partial \hat{y}_H} + \frac{t}{\Delta\theta}\frac{q_L}{q_H}\frac{\partial \hat{w}_L}{\partial \hat{y}_H} = 0. \tag{A.47}$$

Subtracting (A.47) *from* (A.46) *and using* $U_H - \hat{U}_H + t = (y_H - \hat{y}_H)\Delta\theta$, *we find*

$$\begin{aligned} \Upsilon(y_H; \hat{y}_H, t) \equiv\, & w_H - \hat{w}_H - 2[(y_H - \hat{y}_H)\Delta\theta - t] + (y_H - \hat{y}_H)\frac{\partial w_H}{\partial y_H} \\ & - t\frac{1}{\Delta\theta}\frac{\partial \hat{w}_H}{\partial \hat{y}_H} - t\frac{1}{\Delta\theta}\frac{q_L}{q_H}\frac{\partial \hat{w}_L}{\partial \hat{y}_H} \ge 0 \end{aligned} \tag{A.48}$$

with equality for $U_H - \hat{U}_H < t$. *For* $U_H - \hat{U}_H = t$, *we got* $(y_H - \hat{y}_H)\Delta\theta = 2t$, *and for small* ξ $\Upsilon < 0$[15], *a contradiction. Hence* (A.48) *is an equality, and since* $\frac{\partial \Upsilon}{\partial y_H} = 2\frac{\partial w_H}{\partial y_H} - 2\Delta\theta + (y_H - \hat{y}_H)\frac{\partial^2 w_H}{\partial y_H^2} < 0$,

[15] Note, for costs of being ahead one does not need restrictions on ξ.

it uniquely defines y_H as a function $y_H = Y(\hat{y}_H, t)$. If we take it as a function of t, $y_H(t) = Y(\hat{y}_H(t), t)$ tends to $Y(\hat{y}_H(0), 0) = \hat{y}_H(0) = y_H^C$, as can be seen from taking limits in (A.48) *as an equality. Further we use the notation $w_H^C = w(y_H^C, y_{-i})$. Expanding $\Upsilon(y_H(t); \hat{y}_H(t), t) = 0$ yields*

$$2\left[\Delta\theta - \frac{\partial w_H^C}{\partial y_H^C}\right](y_H(t) - \hat{y}_H(t)) = t\left[2 - \frac{1}{\Delta\theta}\left(\frac{w_H^C}{\partial y_H^C} + \frac{q_L}{q_H}\frac{\partial w_L}{\partial y_H^C}\right)\right] + \mathcal{O}(t^2) \tag{A.49}$$

equivalent to

$$y_H(t) - \hat{y}_H(t) = \omega t + \mathcal{O}(t^2) \tag{A.50}$$

with

$$\omega \equiv \frac{2 - \frac{1}{\Delta\theta}\left(\frac{w_H^C}{\partial y_H^C} + \frac{q_L}{q_H}\frac{\partial w_L}{\partial y_H^C}\right)}{2\Delta\theta - 2\frac{\partial w_H^C}{\partial y_H^C}}, \tag{A.51}$$

and $\omega\Delta\theta \in (0, 1)$ for small ξ. From (A.47) *and* (A.46)*, we have the associated profit margins*

$$\hat{w}_H + B\theta_H - \hat{U}_H = t\left[1 - \frac{1}{\Delta\theta}\frac{\partial \hat{w}_H}{\partial \hat{y}_H} - \frac{1}{\Delta\theta}\frac{q_L}{q_H}\frac{\partial \hat{w}_L}{\partial \hat{y}_H}\right] \tag{A.52}$$

$$w_H + B\theta_H - U_H = \left(U_H - \hat{U}_H + t\right)\left[1 - \frac{1}{\Delta\theta}\frac{\partial w_H}{\partial y_H}\right]. \tag{A.53}$$

Therefore, as $t \to 0$

$$\frac{\bar{\omega}_H(t)}{\hat{\bar{\omega}}_H(t)} = \frac{(U_H - \hat{U}_H + t)^2}{t^2}\frac{1 - \frac{1}{\Delta\theta}\frac{\partial w_H}{\partial y_H}}{1 - \frac{1}{\Delta\theta}\frac{\partial \hat{w}_H}{\partial \hat{y}_H} - \frac{1}{\Delta\theta}\frac{q_L}{q_H}\frac{\partial \hat{w}_L}{\partial \hat{y}_H}} \tag{A.54}$$

$$\to (\omega\Delta\theta)^2 \frac{1 - \frac{1}{\Delta\theta}\frac{\partial w_H^C}{\partial y_H^C}}{1 - \frac{1}{\Delta\theta}\frac{\partial w_H^C}{\partial y_H^C} - \frac{1}{\Delta\theta}\frac{q_L}{q_H}\frac{\partial w_L}{\partial y_H^C}} < 1, \tag{A.55}$$

which is exactly what we wanted to show. □

Now we want to show that excluding high types is also not optimal. Therefore, we prove the following Lemma.

Lemma 9 *It is always strictly suboptimal not to employ a positive measure of H-type agents.*

Proof [16] *For e.g. Firm 0 not selling to H types, implies selling to a positive measure of L types and getting strictly positive profits from their contracts (y_L, U_L). Moreover, $y_L = y_L^*$ to be optimal. Hence, $\bar{U} \leq U_L$ and $\hat{U}_L - t < U_L < w(y_L^*, y_{-i}) + B\theta_L$.*
In Region III, let the firm deviate offering the single contract (y_L, U_L). If a H-type takes it, he gets $\tilde{U}_H = U_L + y_L^*\Delta\theta > \hat{U}_L - t + y_L^*\Delta\theta \geq \hat{U}_H - t$, so it is preferred by a positive measure of them to working for Firm 1 or taking the outside option ($\tilde{U}_H > \bar{U}$). Their profits are then $w(y_L^*, y_{-i}) + B\theta_H - \tilde{U}_H = w(y_L^*, y_{-i}) + B\theta_L - U_L + (B - y_L^*)\Delta\theta > 0$. Hence, the contract excluding H-types could not have been optimal.
In Region I and II, there always exists a contract $(\tilde{y}_H, \tilde{U}_H)$, which can be offered along with (y_L, U_L) such that it attracts H types, and is not strictly preferred by L types generating positive profit. We

[16] This proof is not altered by adding a social value function. See also Bénabou and Tirole (2016).

will show this in the following. First, if $U_L \geq \hat{U}_L$, one can choose $(\tilde{y}_H, \tilde{U}_H) = (\hat{y}_H, \hat{U}_H)$, the same contract offered by Firm 1. The L types working at Firm 0 (weakly) prefer their original contract (y_L, U_L), since $U_L \geq \hat{U}_L \geq \hat{U}_H - \hat{y}_H\Delta\theta = \tilde{U}_H - \tilde{y}_H\Delta\theta$. For H-types $\tilde{U}_H = \hat{U}_H > \bar{U}$, where getting it from Firm 0 is preferred to getting it from Firm 1 for all agents located at $x < \frac{1}{2}$. Hence, this deviation is strictly profitable.
Suppose $U_L < \hat{U}_L$ and consider $(\tilde{y}_H, \tilde{U}_H) \equiv (\hat{y}_H, U_L + \hat{y}_H\Delta\theta)$. There is no reason for the L-types to switch, whereas a positive measure of H-types prefers this offer to Firm 1's, because $\tilde{U}_H = U_L + \hat{y}_H\Delta\theta > \hat{U}_L + \hat{y}_H\Delta\theta - t = \hat{U}_H - t$. Moreover at Firm 0 they also prefer it to the L-types' contract, since $\tilde{U}_H \leq U_L + y_L^*\Delta\theta$. The firm can offer the incentive compatible menu $\{(y_L, U_L), (\tilde{y}_H, \tilde{U}_H)\}$ and attract a positive measure of H-types with unit profit

$$\hat{w}_H + B\theta_H - \tilde{U}_H = \hat{w}_H + B\theta_H - \hat{y}_H\Delta\theta - U_L \tag{A.56}$$

$$> \hat{w}_H + B\theta_H - \hat{y}_H\Delta\theta - \hat{U}_L \tag{A.57}$$

$$= \hat{w}_H + B\theta_H - \hat{U}_H > 0. \tag{A.58}$$

Hence, the deviation is profitable. □

A.6.2 A property at an optimum

From Lemma 8 and 9, $X_H \equiv X(U_H + t - \hat{U}_H) > 0$ and $X_L \equiv X(U_L + t - \hat{U}_L)\mathbb{1}_{\{U_L \geq \bar{U}\}} > 0$ at an optimum. Therefore,

Lemma 10 *At any optimum, it must be that either:*

1. $y_L^* = y_L \leq y_H$ *and* $U_H - U_L = y_H\Delta\theta$*, with multiplier* $\mu_H = 0$ *on* (A.32)*, or*
2. $y_L \leq y_H = y_H^*$ *and* $U_H - U_L = y_L\Delta\theta$*, with multiplier* $\mu_L = 0$ *on* (A.33).

Proof *We look at the sub-problem of maximizing* (A.31) *over* (y_H, y_L) *while keeping* (U_H, U_L) *and therefore* $(X_H > 0, X_L > 0)$ *fixed, which is then a differentiable and concave problem. We denote by* μ_H *and* μ_L *the multipliers on H- and L- type's incentive constraints and find the first order conditions*

$$q_H X_H \frac{\partial w_H}{\partial y_H} + q_L X_L \frac{\partial w_L}{\partial y_H} + \mu_L \Delta\theta = 0 \tag{A.59}$$

$$q_H X_H \frac{\partial w_H}{\partial y_L} + q_L X_L \frac{\partial w_L}{\partial y_L} - \mu_H \Delta\theta + \psi = 0. \tag{A.60}$$

It cannot be that $\mu_H > 0$ *and* $\mu_L > 0$*, otherwise* (A.32) *and* (A.33)*, together with* (A.59) *imply* $y_H = y_L > y_H^*$*, and* $\psi = 0$*, a contradiction in* (A.60).
Suppose $\mu_H = 0$*, implying* $\psi = 0$ *and* $y_L = y_L^*$*. If* (A.33) *were not binding, we would have* $\mu_L = 0$*, hence* $y_L = y_H$*, and* $U_L > U_H - y_H\Delta\theta = U_H - y_L\Delta\theta \geq U_L$*, a contradiction. Thus,* $y_H\Delta\theta = U_H - U_L \geq y_L\Delta\theta = y_L^*\Delta\theta$*, which is case 1. If* $\mu_H > 0$*, then* (A.32) *is binding and* $\mu_L = 0$*, hence* $y_H = y_H^*$*. Moreover,* $y_L\Delta\theta = U_H - U_L \leq y_H\Delta\theta$*, which is case 2.* □

A.6.3 No cornering

Lemma 11 *At an optimum,* $X_H \equiv U_H + t - \hat{U}_H$ *and* $X_L \equiv U_L + t - \hat{U}_L$ *must both lie in* $(0, 2t]$.

Proof *From before we know* $X_H > 0$ *and* $X_L > 0$*. First suppose* $\min\{U_H + t - \hat{U}_H, U_L + t - \hat{U}_L\} > 2t$*, which implies* $U_L > \hat{U}_L + t > \bar{U}$*. By reducing both* U_H *and* U_L *slightly while keeping the full*

market of both types, $X_H = X_L = 1$ and not violating any constraint, the firm can increase profits, which is a contradiction.
Suppose now $U_H + t - \hat{U}_H \leq 2t < U_L + t - \hat{U}_L$ implying $U_L > \bar{U}$ and $U_H - U_L \leq \hat{U}_H - \hat{U}_L$. Hence, the following equation must be satisfied

$$\max \Big\{ q_H X(U_H + t - \hat{U}_H)[w_H + \theta_H B - U_H] + q_L 2t[w_L + \theta_L B - U_L] \Big\} \tag{A.61}$$

subject to

$$U_H \geq U_L + y_L \Delta\theta \qquad (\mu_H) \tag{A.62}$$
$$U_L \geq U_H - y_H \Delta\theta \qquad (\mu_L) \tag{A.63}$$
$$U_L \geq \bar{U} \qquad (\nu), \tag{A.64}$$

where the participation constraint is not binding. Therefore, when maximizing over U_L, we find the first order conditions

$$-q_L 2t - \mu_H + \mu_L = 0, \tag{A.65}$$

which must hold in addition to (A.59) and (A.60) with $X_L = 1$. It cannot be that $\mu_L = 0$. Therefore $\mu_H = 0 < \mu_L = 2tq_L$, implying that (A.59) becomes

$$\frac{q_H X_H}{2t} \frac{\partial w_H}{\partial y_H} + q_L \frac{\partial w_L}{\partial y_H} + q_L \Delta\theta = 0. \tag{A.66}$$

Moreover, $y_H \Delta\theta = U_H - U_L \leq \hat{U}_H - \hat{U}_L \leq y_H^C \Delta\theta$, so $y_H \leq y_H^C$. But then interim-efficiency condition (4.7) implies that $q_H \frac{\partial w_H}{\partial y_H} + q_L \frac{\partial w_L}{\partial y_H} + q_L \Delta\theta > 0$, which leads to a contradiction because $X_H \leq 2t$.
Lastly, suppose $U_L + t - \hat{U}_L \leq 2t < U_H + t - \hat{U}_H$. The allocation must thus satisfy

$$\max \Big\{ q_H 2t[w_H + \theta_H B - U_H] + q_L X(U_L + t - \hat{U}_L)[w_L + \theta_L B - U_L] \Big\} \tag{A.67}$$

subject to

$$U_H \geq U_L + y_L \Delta\theta \qquad (\mu_H) \tag{A.68}$$
$$U_L \geq U_H - y_H \Delta\theta \qquad (\mu_L) \tag{A.69}$$
$$U_L \geq \bar{U} \qquad (\nu), \tag{A.70}$$

maximizing over U_H yields the first order condition

$$-2tq_H + \mu_H - \mu_L = 0. \tag{A.71}$$

Therefore, $\mu_H \neq 0$, so $\mu_L = 0 < \mu_H = 2tq_H$, $y_H = y_H^*$ and $q_L X_L \frac{\partial w_L}{\partial y_L} + q_H 2t \frac{\partial w_H}{\partial y_L} = 2tq_H \Delta\theta - \psi \equiv 2t \frac{\partial w^m}{\partial y_L^m} - \psi$, with w^m denoting the welfare function at the monopsony level. If $\psi > 0$ then $y_L = 0 < y_L^m$, and if $\psi = 0$ then $\frac{q_L X_L}{2t} \frac{\partial w_L}{\partial y_L} + q_H \frac{\partial w_H}{\partial y_L} = \frac{\partial w^m}{\partial y_L^m}$, so $y_L \leq y_L^m$ because $X_L \leq 2t$. But we also have $y_L \Delta\theta = U_H - U_L > \hat{U}_H - \hat{U}_L > y_L^m \Delta\theta$. which yields a contradiction. □

A.6.4 Global optimality

The objective function

$$\max\left\{q_H(U_H+t-\hat{U}_H)[w_H+\theta_H B-U_H]+q_L(U_L+t-\hat{U}_L)[w_L+\theta_L B-U_L]\right\} \tag{A.72}$$

is not globally concave, which can be seen by computing the Hessian. To proof global optimality, we will first show that for any $C=(U_H,U_L,y_H,y_L)$ to be an optimum, it must lie in either of the following subspaces:

$$S_H\equiv\left\{(U_H,U_L,y_H,y_L)|y_L^*=y_L\le y_H\le y_H^C \text{ and } U_H-U_L=y_H\Delta\theta\right\} \tag{A.73}$$
$$S_L\equiv\left\{(U_H,U_L,y_H,y_L)|y_H^*=y_H\ge y_L\ge y_L^m \text{ and } U_H-U_L=y_L\Delta\theta\right\}. \tag{A.74}$$

Next, we will show that concavity is given on S_H and S_L separately, implying that $\hat{C}=(\hat{U}_H,\hat{U}_L,\hat{y}_H,\hat{y}_L)$ is a maximum over all feasible allocations in the subspace it belongs (S_H for $t\le t_2$ (Region I and II), or S_L for $t\ge t_2$ (Region III)). Lastly, the global optimum can never lie in the other subspace than the one to which $\hat{C}$ belongs to.

Lemma 12 *A global optimum $C=(U_H,U_L,y_H,y_L)$ must lie in S_H or S_L.*

Proof *Let $S'_H(S'_L)$ be the superset of $S_H(S_L)$ obtained by omitting the inequality $y_H\le y_H^C$ ($y_L\ge y_L^m$) from* (A.73)*(*(A.74)*). By Lemma 10, an optimum must belong to S'_L or S'_H. Moreover, due to Lemma 8, 9, 11, solving* (A.31), (A.32) *and* (A.33) *is equivalent to solving the program*

$$\max\left\{q_H(U_H+t-\hat{U}_H)[w_H+\theta_H B-U_H]+q_L(U_L+t-\hat{U}_L)[w_L+\theta_L B-U_L]\right\} \tag{A.75}$$

subject to

$$X_H\equiv U_H+t-\hat{U}_H\le 2t \qquad (\tau_H) \tag{A.76}$$
$$X_L\equiv U_L+t-\hat{U}_L\le 2t \qquad (\tau_L) \tag{A.77}$$
$$U_H\ge U_L+y_L\Delta\theta \qquad (\mu_H) \tag{A.78}$$
$$U_L\ge U_H-y_H\Delta\theta \qquad (\mu_L) \tag{A.79}$$
$$U_L\ge\bar{U} \qquad (\nu) \tag{A.80}$$
$$y_L\ge 0 \qquad (\psi). \tag{A.81}$$

The first order conditions are

$$q_H[w_H+\theta_H B-2U_H-t+\hat{U}_H]+q_L(U_L+t-\hat{U}_L)\frac{\partial w_L}{\partial U_H}-\tau_H+\mu_H-\mu_L=0 \tag{A.82}$$
$$q_H(U_H+t-\hat{U}_H)\frac{\partial w_H}{\partial U_L}+q_L[w_L+\theta_L B-2U_L-t+\hat{U}_L]-\tau_L-\mu_H+\mu_L+\nu=0 \tag{A.83}$$
$$q_H(U_H+t-\hat{U}_H)\frac{\partial w_H}{y_H}+q_L(U_L+t-\hat{U}_L)\frac{w_L}{\partial y_H}+\mu_L\Delta\theta=0 \tag{A.84}$$
$$q_H(U_H+t-\hat{U}_H)\frac{\partial w_H}{\partial y_L}+q_L(U_L+t-\hat{U}_L)\frac{\partial w_L}{\partial y_L}-\mu_H\Delta\theta+\psi=0, \tag{A.85}$$

with $X_H>0$ and $X_L>0$ at an optimum.

(a) *Consider $C \in S'_H$. Then we have $y_L = y_L^*$, implying $\psi = 0$ and $\mu_H = 0$, so eliminating μ_L:*

$$\begin{aligned} &w_H + \theta_H B - 2U_H - t + \hat{U}_H + \frac{q_L}{q_H}(U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial U_H} - \frac{\tau_H}{q_H} \\ &+ \frac{1}{\Delta\theta}\left[(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial y_H} + \frac{q_L}{q_H}(U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial y_H}\right] = 0 \end{aligned} \tag{A.86}$$

$$\begin{aligned} &\frac{q_H}{q_L}(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial U_L} + w_L + \theta_L B - 2U_L + \hat{U}_L - t - \frac{\tau_L}{q_L} \\ &- \frac{1}{\Delta\theta}\left[\frac{q_H}{q_L}(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial y_H} + (U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial y_H}\right] + \frac{\nu}{q_L} = 0 \end{aligned} \tag{A.87}$$

Using $U_H - U_L = y_H\Delta\theta$ and $\hat{U}_H - \hat{U}_L = \hat{y}\Delta\theta$ (with $\hat{y} = \hat{y}_H$ in Region I and II, $\hat{y} = \hat{y}_L$ in Region III) and subtracting (A.86)-(A.87) *gives*

$$\begin{aligned} w_H - w_L + (B + \hat{y} - 2y_H)\Delta\theta =& \frac{\tau_H}{q_H} - \frac{\tau_L}{q_L} + \frac{\nu}{q_L} \\ &- \frac{1}{\Delta\theta}\left(1 + \frac{q_H}{q_L}\right)\left[(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial y_H} + (U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial y_H}\right] \\ &+ \frac{q_H}{q_L}(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial U_L} - \frac{q_L}{q_H}(U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial U_H} \end{aligned} \tag{A.88}$$

Next subtracting $w_H^C - w(y_L^*, y_{-i}) + (B - y_H^C)\Delta\theta = 0$, we find

$$\begin{aligned} w_H - w_H^C + (\hat{y} - 2y_H + y_H^C)\Delta\theta =& \frac{\tau_H}{q_H} - \frac{\tau_L}{q_L} + \frac{\nu}{q_L} \\ &- \frac{1}{\Delta\theta}\left(1 + \frac{q_H}{q_L}\right)\left[(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial y_H} + (U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial y_H}\right] \\ &+ \frac{q_H}{q_L}(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial U_L} - \frac{q_L}{q_H}(U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial U_H} \end{aligned} \tag{A.89}$$

If $y_H > y_H^C \geq \hat{y}$ then the LHS is negative, and since $U_H - U_L > \hat{U}_H - \hat{U}_L$ then implies $U_L - \hat{U}_L < U_H - \hat{U}_H \leq t$, such that $\tau_L = 0$, the RHS is positive for small ξ, a contradiction. Hence, $y_H \leq y_H^C$, so that $C \in S_H$.

(b) *Consider now $C \in S'_L$. We have $y_H = y_H^*$ and $\mu_L = 0$, so eliminating μ_H*

$$\begin{aligned} &w_H + \theta_H B - 2U_H - t + \hat{U}_H + \frac{q_L}{q_H}(U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial U_H} - \frac{\tau_H}{q_H} + \frac{\psi}{q_H\Delta\theta} \\ &+ \frac{1}{q_H\Delta\theta}\left[q_H(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial y_L} + q_L(U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial y_L}\right] = 0 \end{aligned} \tag{A.90}$$

$$\begin{aligned} &w_L + \theta_L B - 2U_L - t + \hat{U}_L + \frac{q_H}{q_L}(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial U_L} - \frac{\tau_L}{q_L} + \frac{\nu}{q_L} - \frac{\psi}{q_L\Delta\theta} \\ &- \frac{1}{q_L\Delta\theta}\left[q_H(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial y_L} + q_L(U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial y_L}\right] = 0 \end{aligned} \tag{A.91}$$

If $y_L < y_L^m$ then $U_H - U_L = y_L\Delta\theta < \hat{y}\Delta\theta = \hat{U}_H - \hat{U}_L$ so $U_H - \hat{U}_H < U_L - \hat{U}_L \leq t$, hence $\tau_H = 0$.
Suppose first $U_L > \bar{U}$, then $\nu = 0$ and from the last two equations we get

$$w_H + \theta_H B - 2U_H + \hat{U}_H - t < 0 < w_L + \theta_L B - 2U_L - t + \hat{U}_L$$

equivalent to

$$w_H - w_L + (B - y_L)\Delta\theta + (\hat{y} - y_L)\Delta\theta < 0,$$

which is a contradiction. Therefore, $U_L = \bar{U}$. Next, for $y_L < y_L^m$, we have $\frac{\partial w}{\partial y_L} > \frac{\partial w^m}{\partial y_L^m} = q_H\Delta\theta$, hence by (A.90)

$$\begin{aligned}
-\frac{\psi}{q_H\Delta\theta} >& w_H + \theta_H B - 2U_H - t + \hat{U}_H + \frac{q_L}{q_H}(U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial U_H} \\
&+ \frac{1}{q_H\Delta\theta}\left[q_H\Delta\theta(U_L + t - \hat{U}_L) + q_H(U_H - \hat{U}_H - U_L + \hat{U}_L)\frac{\partial w_H}{\partial y_L}\right]
\end{aligned} \quad (A.92)$$

$$\begin{aligned}
=& w_H + \theta_L B - \bar{U} + (B - y_L)\Delta\theta + (\hat{y} - y_L)\Delta\theta \\
&+ \frac{q_L}{q_H}(U_L + t - \hat{U}_L)\frac{\partial w_H}{\partial U_H} + \frac{1}{\Delta\theta}\frac{\partial w_H}{\partial y_L}(y_L - \hat{y})
\end{aligned} \quad (A.93)$$

which is strictly positive for small ξ. Hence, another contradiction. So we find $y_L \geq y_L^m$, such that $C \in S_L$. □

Lemma 13 *The objective function in* (A.72) *is strictly concave over S_H and S_L.*

Note, Lemma 13 implies that the symmetric solution $\hat{C} \equiv (\hat{U}_H, \hat{U}_L, \hat{y}_H, \hat{y}_L)$ always strictly satisfies the local second-order conditions for a maximum of the program (A.72).

Proof *First, over $S_H = \{(U_H, U_L, y_H, y_L) | y_L = y_L^* \leq y_H \leq y_H^C \text{ and } U_H - U_L = y_H\Delta\theta\}$ the objective function becomes*

$$\begin{aligned}
\Phi(U_H, y_H) \equiv& q_H(U_H + t - \hat{U}_H)\left[w_H + \theta_H B - U_H\right] \\
&+ (U_H - y_H\Delta\theta) - \hat{U}_L + t)\left[w_L + \theta_L B - U_H + y_H\Delta\theta\right]
\end{aligned} \quad (A.94)$$

and the determinant of its Hessian

$$\begin{aligned}
det(H(\Phi)) = &\left[-2 + 2q_L\frac{\partial w_L}{\partial U_H}\right]\left[q_H(U_H + t - \hat{U}_H)\frac{\partial^2 w_H}{\partial y_H^2} + q_L(U_H - y_H\Delta\theta - \hat{U}_L + t)\frac{\partial^2 w_L}{\partial y_H^2}\right. \\
&\left. -2q_L\Delta\theta^2 - 2q_L\Delta\theta\frac{\partial w_L}{\partial y_H}\right] \\
&- \left[q_H\frac{\partial w_H}{\partial y_H} + 2q_L\Delta\theta + q_L\frac{\partial w_L}{\partial y_H} - q_L\Delta\theta\frac{\partial w_L}{\partial U_H}\right. \\
&\left. + q_L(U_H - y_H\Delta\theta - \hat{U}_L + t)\frac{\partial^2 w_L}{\partial U_H\partial y_H}\right]^2,
\end{aligned} \quad (A.95)$$

which by using (4.7) can be shown to be positive for small ξ.
Next, over S_L the objective function reduces to

$$\begin{aligned}\Phi(U_L, y_L) \equiv & q_H(U_L + y_L\Delta\theta - \hat{U}_H + t)\,[w_H + \theta_H B - U_L - y_L\Delta\theta] \\ & + q_L(U_L + t - \hat{U}_L)\,[w_L + \theta_L B - U_L]\,.\end{aligned} \tag{A.96}$$

and the determinant of its Hessian

$$\begin{aligned}det(H(\Phi)) = & \left[-2 + 2q_H\frac{\partial w_H}{\partial U_L}\right]\left[q_L(U_L + t - \hat{U}_L)\frac{\partial^2 w_L}{\partial y_L^2} + q_H(U_L + y_L\Delta\theta - \hat{U}_H + t)\frac{\partial^2 w_H}{\partial y_L^2}\right. \\ & \left. -2q_H\Delta\theta^2 + 2q_H\Delta\theta\frac{\partial w_H}{\partial y_L}\right] \\ & - \left[q_L\frac{\partial w_L}{\partial y_L} - 2q_H\Delta\theta + q_H\frac{\partial w_H}{\partial y_L} + q_H\Delta\theta\frac{\partial w_H}{\partial U_L}\right. \\ & \left. + q_H(U_L + y_L\Delta\theta - \hat{U}_H + t)\frac{\partial^2 w_H}{\partial U_L \partial y_L}\right]^2\end{aligned} \tag{A.97}$$

which can be shown to be positive for small ξ. □

Proposition 5 *The unique global optimum to* (A.31), (A.32), *and* (A.33) *is the allocation $\hat{C} \equiv (\hat{U}_H, \hat{U}_L, \hat{y}_H, \hat{y}_L)$ characterized in Proposition 3, which is therefore an equilibrium (the unique symmetric one) of the game between the two firms.*

Proof *By Lemma 8 and 9 the global solution to $C = (U_H, U_L, y_H, y_L)$ to* (A.31), (A.32) *and* (A.33) *is also the global solution to* (A.72) *and satisfies the associated first order conditions* (A.82)-(A.85)*, with $X_H \equiv U_H - \hat{U}_H + t$ and $X_L \equiv U_L - \hat{U}_L + t$ both in $(0, 2t]$. By Proposition 3, $\hat{C} \equiv (\hat{U}_H, \hat{U}_L, \hat{y}_H, \hat{y}_L)$ solves these conditions (with $\hat{X}_H = \hat{X}_L = t$), is the unique candidate for a symmetric equilibrium, and is such that $\hat{C} \in S_H$ when t is in Region I and II, while $\hat{C} \in S_L$ when t is in Region III. Furthermore, by Lemma 13, the objective function is strictly concave over each of these subspaces, so in each case $\hat{C}$ maximizes the program over the one to which it belongs to. By Lemma 13, moreover, the global optimum C must also belong to S_H or S_L. Two cases therefore remain to consider.*

(a) *t lies in Region I or II, so that $\hat{C} \in S_H$. If $C \in S_H$ as well they must coincide. If $C \in S_L$ then $y_H = y_H^*$, and $U_H - U_L = y_L\Delta\theta \le \hat{y}_H\Delta\theta = \hat{U}_H - \hat{U}_L$.*

  (a.1) *If the inequality is strict*

$$U_H - \hat{U}_H < U_L - \hat{U}_L. \tag{A.98}$$

  Note, this requires $\tau_H = 0$, otherwise $t = U_H - \hat{U}_H < U_L - \hat{U}_L \le t$, a contradiction. Next, subtracting from (A.82) *its counterpart for $\hat{C}$, and likewise for* (A.83)*, we have*

$$q_H[w_H - \hat{w}_H - 2(U_H - \hat{U}_H)] = -\hat{\mu}_L - \mu_H - q_L(U_L + t - \hat{U}_L)\frac{\partial w_L}{\partial U_H} \tag{A.99}$$

$$q_L[w_L - \hat{w}_L - 2(U_L - \hat{U}_L)] = \tau_L + \mu_H + \hat{\mu}_L + \hat{\nu} - \nu - q_H(U_H + t - \hat{U}_H)\frac{\partial w_H}{\partial U_L} \tag{A.100}$$

  The first equation implies for small ξ that $w_H - \hat{w}_H \le 2(U_H - \hat{U}_H)$. Since $w_H - \hat{w}_H \ge 0$ it follows $U_L - \hat{U}_L > 0$ by (A.98)*. Thus $U_L > \bar{U}$, implying $\nu = 0$. From* (A.100) *it follows that $2(U_L - \hat{U}_L) \le w_L - \hat{w}_L \le 0$, which contradicts $U_L > \hat{U}_L$.*

(a.2) *Therefore,* (A.98) *is an equality, which implies* $y_L = \hat{y}_H = y^* = y_H$ *and* $\psi = 0$*, thus* $U_H - U_L = y_H \Delta\theta$ *and* $y_L = y^*$*, implying that* $C \in S_H$*, so it must coincide with* $\hat{C}$*. Note, that* $\hat{C} \in S_H \cap S_L$ *can only occur at* $t = t_2$*.*

(b) t *lies in Region III, so that* $\hat{C} \in S_L$*. If* $C \in S_L$ *they must again coincide. If* $C \in S_H$ *then* $y_L = y_L^*$*,* $\mu_H = 0$ *and* $U_H - U_L = y_H \Delta\theta \geq \hat{y}_L \Delta\theta = \hat{U}_H - \hat{U}_L$*. Therefore,*

$$U_H - \hat{U}_H \geq U_L - \hat{U}_L = U_L - \bar{U} \geq 0 \tag{A.101}$$

and from (A.82)

$$q_H \left[w_H - \hat{w}_H - 2(U_H - \hat{U}_H) \right] = \tau_H + \mu_L + \hat{\mu}_H - q_L(U_L + t - \hat{U}_L) \frac{\partial w_L}{\partial U_H} \tag{A.102}$$

therefore

$$w_H - \hat{w}_H \geq 2(U_H - \hat{U}_H) \tag{A.103}$$

which together with (A.101) *and* $w_H - \hat{w}_H \leq 0$ *for small* ξ *requires that* $U_H = \hat{U}_H$*,* $U_L = \hat{U}_L$ *and* $y_H = y^* = \hat{y}_L$*, so that* $C = \hat{C}$*. Here again it must be that* $t = t_2$*, which corresponds to the only intersection of* S_H *and* S_L*.*

□

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Anna Ulrichshofer, Markus Walzl

Social Comparison and Optimal Contracts in Competition for Managerial Talent

Abstract
We analyze the impact of social comparison on optimal contract design under imperfect labor market competition for managerial talent. Adding a disutility of social comparison as induced by a ranking of verifiable efforts to the multi-task model by Bénabou and Tirole (4238), we demonstrate that rankings can reduce welfare distortions of optimal screening contracts if the degree of competition for talent is sufficiently low. In contrast, a ranking unambiguously reduces welfare if the competition intensity is high and agents suffer from lagging behind while it can enhance welfare (depending on the fraction of high and low productivity types) if agents suffer from leading in a ranking (e.g., because the ranked activity is perceived as a substitute for other potentially pro-social activities).

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