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Nonlinear Incentives and Advisor Bias *

Jun Honda† Roman Inderst‡

November 2017

Abstract

We analyze firms' competition to steer an advisor's recommendations through potentially non-linear incentives. Even when firms are symmetric, so that the overall size of compensation would not distort advice when incentives were linear, advice is biased when firms are allowed to make compensation non-linear, which they optimally do. Policies that target an advisor's liability are largely ineffective, as firms react to such increased liability by making incentives even steeper, increasing bonus payments while reducing the linear (commission) part at the same time. This observation may justify policymakers' direct interference with firms' compensation practice, as frequently observed notably in consumer finance.

JEL Classification: L51 (Economics of Regulation), M52 (Compensation and Compensation Methods and Their Effects).

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1 Introduction

Commissions have been a key focus of policy intervention over the last decade.[1] This concerns, in particular, commissions paid to financial intermediaries, such as mortgage brokers or investment advisors.[2] As the financial crisis unfolded, policymakers have been quick to point to distorted incentives and a lack of consumer financial protection as two main culprits.[3] Health care, where medical advice can be compromised by gifts or other inducements that physicians receive from pharmaceutical companies, as well as the residential rental market, are further examples where size, structure, or transparency of commissions is often regulated.

An immediate way to rectify possible problems of biased advice could be to ramp up liability or liability standards, rather than interfering directly with incentives. In fact, one may argue that this should represent the preferred type of intervention, as it does not interfere with parties' contractual freedom, which could risk distorting the efficient design of institutional and contractual solutions. In this paper, we identify however a case where direct interference with advisors' incentive contracts may be warranted. Precisely, we show that the unregulated equilibrium design of non-linear incentives may lead to biased advice. This bias is, at least within bounds, immune to variations in the advisor's liability. In fact, we show that in the unregulated equilibrium, as the advisor's liability increases, the effect of this is fully counteracted by product providers' adjustment of compensation, as they increase the bonus and reduce the linear (commission) part of their compensation.[4]

In our model, two product providers compete for the recommendations offered to con-

[1] This interest even precedes the recent financial crisis, as witnessed, for instance, by the high-profile lawsuit against the largest insurance brokerage firm in the US, Marsh & McLennan Companies, where compensation allegedly steered businesses towards insurers with which the company had lucrative "contingent commission" (or "placement service") agreements. The case ended with a settlement under which Marsh agreed to pay \$850 million. Cummins and Doherty (2006) provide a detailed discussion and empirical analysis of brokerage intermediation in the US insurance market.

[2] In this paper, given its theoretical focus, we do not distinguish between different types of (financial) advisors or brokers, as defined by law in different jurisdictions. Clearly, the fiduciary duties and legal requirements imposed on particular financial intermediaries differ substantially, so that, for instance, broker-dealers may be excluded from regulations when their offer of advice is "solely incidental". See, for instance, Hung et al. (2008) for a discussion of legal definitions, at least in the context of the US, as well as on how consumers perceive the different roles.

[3] Cf. Financial Stability Board (2011). The key legislative measure in Europe was the Markets in Financial Instruments Directive (MiFID), which is currently overhauled, notably to ban various commissons as of January 2018 (MiFID II).

[4] There may of course be other reasons for why an increase in liability alone may not be sufficient or may even have negative consequences by itself. To the extent that this requires a larger capital base (or, say, the posting of a "surety bond" to cover liability), this may, for instance, lead to exit, thereby strenghtening the market power of the remaining firms.

sumers by a single intermediary, the advisor. When giving recommendations, the advisor faces a trade-off between the monetary incentives provided by product providers and his concern for the suitability of advice. This setting follows closely Inderst and Ottaviani (2009, 2012a), albeit there, as otherwise in the literature, attention is restricted to serving a single customer, so that the issue of non-linear incentives across sales to different customers does not arise.

Though the advisor may also be concerned about his reputation or may have intrinsic preferences for providing suitable advice, we take this as given and, for the purpose of our analysis, we focus on his concern for liability, as captured by a single parameter w that represents a penalty imposed on the advisor when the recommended product ultimately turns out to be unsuitable for the particular customer.

The literature has considered the case where a product provider n competes for a single customer by offering the advisor a commission (or fee) f_n that is paid only if the respective customer ends up buying the firm's product. An interesting benchmark case is that where firms are symmetric in all respects, in which case also the equilibrium outcome will be symmetric, with $f_n = f$. While commissions could end up being substantial, in equilibrium the advisor's recommendation will remain unbiased.[5] We show that even with such symmetry, advice is biased when the advisor serves multiple customers, arriving either sequentially or simultaneously, and when firms' incentives (optimally) become non-linear.

To describe our results, when there are two customers who purchase at most one unit each, a firm's compensation is now fully captured by two variables: A commission f_n that is paid for each individual sale and a bonus b_n that is paid, in addition, when both customers purchase product n. When $b_n > 0$, the advisor's recommendations become distorted, even when firms' compensation is perfectly symmetric. Intuitively, taking the case of a sequential arrival of customers, with $b_n > 0$ the recommendation to the second customer will be biased towards the product that the advisor already recommended to the first visiting customer. We show that such a bias exists as well when recommendations are made simultaneously. In both cases, the presence of a bonus makes the advisor more willing to recommend the same product to both customers, even when such a linkage in recommendations is not warranted. Bonuses are always part of an unregulated equilibrium,[6] and

[5] The key insight of Inderst and Ottaviani (2012a) is that when there is asymmetry, a cap on commissions or any other measure that restricts the use of commissions, such as greater transparency with respect to customers, risks having a larger effect on a more efficient firm, as this firm would typically pay a larger commission. Sales then shift inefficiently to the less efficient firm.

[6] As we argue below in more detail, this represents a novel result in the theory of incentives, as it is notably different from the well-known use of bonus contracts with moral hazard (Innes 1990).

we are able to provide a simple characterization of the prevailing pattern of advice. We find that this pattern of advice and the thereby implied inefficiency are independent of the agent's liability. Notably, when w increases, firms optimally adjust their incentives to fully counteract the advisor's increased liability. This implies in particular that when the advisor's liability increases, this has no effect on the suitability of his recommendations: Recommendations remain biased to the same extent as when liability was lower.

When determining the optimal size of their respective commission and bonus, firms face a trade-off between the two instruments, in terms of their cost-effectiveness, which does *not* depend on w. The agent's liability only affects the overall sensitivity to compensation. It is only when w is sufficiently large that liability starts having an impact on recommendations, as then firms no longer pay commissions and the remaining positive bonus $b_n = b > 0$ decreases as liability increases.

From the preceding observation we know that increasing the agent's liability alone, at least when this remains within bounds, does not affect the pattern of advice and thus does not reduce the advisor's bias. Instead, advice becomes unbiased when regulation prohibits the use of bonuses and forces firms to employ only linear compensation schemes. Our theory thus supports direct interference with firms' compensation practice.

In the financial industry, we indeed observe regulators attempting to influence firms' incentive schemes in the way analyzed in this paper. Even when this does not lead to an outright prohibition of non-linear incentives, financial authorities may impose strict standards to curb misselling and unsuitable advice. For instance, regulators conduct on-site assessments of incentive arrangements, focusing in particular on bonuses and how these are earned.[7] Also, bonuses and other compensation arrangements that are made contingent on a particular target or quota should frequently fall foul of increasingly strict standards.[8] Our analysis supports such policies and shows that imposing stricter liability on brokers and advisors is not an adequate substitute. We should note however that in this paper, we do not moel potential drawbacks of the envisaged restriction on incentives. Such potential drawbacks clearly need to be considered when implementing such policies, and we turn to them briefly in our concluding remarks (where we also point to the respective analysis in the Online Appendix).

There exists a small empirical literature that shows, mostly outside the financial in-

[7]See, for instance, the thematic review of the UK's Financial Conduct Authority in 2014 and the guidance given to financial firms with respect to (still) permissible incentive arrangements: https://www.fca.org.uk/publication/thematic-reviews/tr14-04.pdf.

[8]In the US, in September 2010 an amendment of Regulation Z (Loan Originator Compensation and Steering 12 CFR 226) was published that prohibits various compensation practices.

dustry, how bonuses affect agents' behavior, notably by leading to a surge of sales at the end of the respective quarter or year.[9] Theoretically, Steenburgh (2008) and Chung et al. (2013) provide a simulated quantification exercise of the trade-off between such "timing games" of agents and the potential of lump-sum bonuses to motivate harder work so as to attain incremental orders, in the spirit of the well-known result by Innes (1990).[10] These papers do not address an agent's role of advising customers and consequently the potential of biased advice.

In fact, also in the theoretical literature the primary role of bonuses is to motivate effort at lowest cost to the principal in the presence of limited liability. Essentially, the aim is then to reward an agent only for the highest sales performance while punishing him for any other (lower) performance, so as to minimize his agency rent. Economists have appealed to this potentially efficiency enhancing role of "discontinuous" incentive schemes in a variety of contexts, such as in antitrust, and we return to our relationship to this literature below. In this respect, we both add a new policy perspective, focusin on biased advice, and provide a new theoretical motivation for why bonuses are optimal. In our model, a bonus does not act as an optimal instrument to increase effort at lower cost to the principal. In fact, if this was the case, firms would *only* pay a bonus, that is compensation only in the "highest state", which is typically not the case in our model.[11]

What we do not analyze in this paper is the outright prohibition of any incentives paid by product providers. For instance, as of January 1st 2013, the new rules of the UK's financial regulator do not allow financial advisors to receive commissions.[12] This is meant to steer the market towards consumers paying directly for advice.[13] Also, in this paper

[9]Misra and Nair (2011) observe such behavior for the salesfoce of a seller of contact lenses, while Larkin (2014) shows this for enterprise software sales. Using data from the pharmaceutical industry, Kishore et al. (2013) show that an early fulfillment of a sales target dampens further sales. Tzioumis and Gee (2013) have recently studied the remuneration of mortgage officers, showing a spike in sales and a notable reduction in processing time at the end of the month. On the respective "gaming" see theoretically Holmstrom and Milgrom (1987).

[10]Chung et al. (2013) also put their model to data, testing a combination of different bonuses. See also Basu et al. (1985) for an early model of salesforce compensation plans in the marketing literature.

[11]This difference is particularly evident in our model variant where customers arrive simultaneously. Instead, in models where an agent is asked to exert effort on multiple projects simultaneously, the optimal compensation is indeed to pay the agent if and only if all projects are successful (Laux 2001), as this is the most informative state.

[12]More precisely, this restriction applies to the sale of investment products such as pensions, annuities, and unit trusts. Notably credit products, such as mortgages, but also insurance policies are not affected. When consumers naively fail to perceive the possibility of potentially biased advice, Inderst and Ottaviani (2012b) have shown that a general cap on commissions could be warranted. In fact, without such interference, they show that only commissions but not lump-sum fees will arise in equilibrium.

[13]Gravelle (1994) examines two different schemes, a fee-for-advice and a commission, separately, in a market with advice where firms are in perfect competition and only advisors can exploit a fraction of

we do not consider policies aimed at mandatory disclosure. For instance, in the European Union, since January 2008 the disclosure of commissions on retail financial products has become mandatory.[14] Still another dimension of contract design and policy is the potential (mandatory) deferral of compensation, as analyzed in Hoffman et al. (2016).[15]

Relation to the antitrust debate on loyalty rebates and the Industrial Organization literature on (mixed) bundling. In our model, an advisor does not take ownership of firms' products but only makes recommendations. For this he is remunerated through the respective commissions and bonuses. Suppose now instead that the advisor would take ownership of the respective products and accordingly acts as an intermediary. In this case, the respective commissions of our model would be akin to a per-unit margin whereas bonuses would be akin to a lump-sum wholesale price reduction that is triggered when a certain sales target is met. These are the characteristics of so-called retroactive loyalty rebates, which have been intensively scrutinized in the antitrust literature. A notable example is the European Commission's case against the bonus system of British Airways with respect to travel agencies.[16] Other well-known cases include that of the tyre manufacturer Michelin. Against the claim of antitrust authorities that such incentives have the object or at least the effect of foreclosing the market to notably smaller rivals or newcomers, economists have been quick to point out possible efficiencies in providing incentives, akin to the analysis in Innes (1990). Interestingly, both of these well-known cases, involving travel agencies and garages, relate to products where final consumers typically rely on advice. Our analysis may thus strengthen the case of authorities against such nonlinear incentives, as even when these are provided by firms that are equally well positioned to capture market share, they risk biasing advice.

customers. He demonstrates that it is not necessarily true that welfare is higher under the fee-for-advice scheme than under the commission. In Stoughton et al (2011), they allow for both the fee-for-advice and the commission ("rebate") in a different market for financial advice where advisors are now in perfect competition while firms (portfolio managers) are instead able to exploit customers (investors), and their result supports regulation to ban rebates in the presence of the fee-for-advice.

[14] Markets in Financial Instruments Directive (MiFID). In the US, already in November 2008 the US Department of Housing and Urban Development imposed stricter disclosure requirements for third-party brokers in the mortgage market.

[15] Inderst (2015) provides a short survey of literature dealing with such policies. It also contains a first analysis on non-linear incentives, albeit the exercise there restricts the advisor to apply one and the same advice ("threshold") strategy to all consumers. This does not allow to analyze the key question posed in this paper, namely how non-linear incentives make advice given to one customer contingent on the advice given to another customer.

[16] This case was decided in 1999 and the decision was finally upheld in 2007 by the European Court of Justice.

To push this further, our model with simultaneous arrival of customers turns out to be isomorphic to models used in the Industrial Organization literature on (mixed) bundling. There, multi-product firms face a consumer who may want to purchase one or more of a firms' different products, and firms may charge both individual prices as well as prices for bundles. The discount that is offered with a bundle is akin to the bonus that the advisor obtains in our model.[17] In this literature, our main case with a sequential arrival of customers has not yet been analyzed.[18]

2 The Model

We consider a model of firms' competition in providing incentives for advisors. Our key departure from the existing literature is that we analyze the role and possible regulation of nonlinear incentives for an advisor, and our focus is consequently on the analysis of the incentive scheme and its interaction with the suitability of advice. For this purpose, we consider two firms or product providers $n = A, B$, that wish to sell their products through a single advisor. We stipulate that firms sell only indirectly through the advisor, though we provide conditions for when non-advised sales would not arise in equilibrium. Firms' per-unit production costs are equal to c_n and the respective prices are p_n.[19] Normalizing the utility of customers from not purchasing to zero, the utility of a given customer j from purchasing either product depends on a binary state variable $\theta_j \in \{A, B\}$ which captures the product's suitability, as follows: The customer derives utility $v_h > 0$ if the product matches the state and utility v_l otherwise, with $v_l < v_h$ (where v_l may be negative). This set-up follows Inderst and Ottaviani (2009).

Advisor. A key feature of our model is that the advisor has private information about the suitability of either product for a given customer. As in Inderst and Ottaviani (2012a,b), we directly work with the advisor's posterior beliefs. Based on his private information, the advisor's posterior belief is given by $q_j = \Pr(\theta_j = A) \in [0, 1]$, which captures the likelihood that product A is more suitable. A priori, q_j is i.i.d. with CDF $G(q_j)$ and density $g(q_j) > 0$ on $[0, 1]$.[20] From this, the prior beliefs of all parties are thus $\Pr(\theta_j = A) = \int_0^1 qg(q)dq$ and

[17]From this perspective, the analyses in McAfee et al. (1988) and Armstrong and Vickers (2010) are closest to ours.

[18]This analogy between the models is decribed in detail in the Online Appendix.

[19]Given our interest in situations in which private contracting through warranties fails, we rule out payments from or to customers that are contingent on the realized utility.

[20]Hence, for convenience only we thus suppress the respective signal-generating technology.

$\Pr(\theta_j = B) = \int_0^1 (1-q)g(q)dq$.

The advisor is motivated both by the (yet to be specified) incentive payments and by a preference for the suitability of his advice. He derives utility w_h if the purchased product is suitable and w_l ($< w_h$) otherwise. When a customer does not purchase any of the two products, we denote the advisor's utility (gross of any payments) by w_0. Assuming that $w_0 < w_l$, the advisor always recommends one of the two considered products, and thereby firms are always in competition through compensation.[21] When making recommendations, the difference $w = w_h - w_l$ captures the advisor's concern for the suitability of his recommendation. Despite his concern for the suitability of his advice, the advisor might make biased recommendations so as to receive higher remuneration from the firms, as described next. We assume that w is not too high and the advisor is sufficiently responsive to pay schemes to make incentives worthwhile for firms. Below we will define a threshold for which this assumption holds.

Compensation. Firms observe the advisor's total sales of their products and can thus make their incentive schemes contingent on that number s_n. Incentives schemes are thus given by a discrete function, $F_n : \mathbb{N}_0 \to \mathbb{R}$, which must satisfy $F_n(s_n) \geq 0$ for any given $s_n \in \mathbb{N}_0 = \{0, 1, 2, \dots\}$ due to limited liability.[22] We assume that the advisor has an outside option of value zero. It then follows that firms have no incentives to pay a base salary, so that $F_n(0) = 0$.

To study the implications and regulation of nonlinear incentives in what follows, it is sufficient to restrict attention to the case with at most two customers, so that $i = 1, 2$. Then, we can conveniently express compensation as follows. Define f_n as a per-unit "commission" and b_n as a lump-sum "bonus", albeit we do not restrict it to be non-negative, so that $F_n(1) = f_n$ and $F_n(2) = 2f_n + b_n$.

Timing. To close the model, we next specify the timing for our baseline analysis.

$t = 1$: Firms simultaneously offer incentives (f_n, b_n), which the advisor can accept or reject.

$t = 2$: Firms simultaneously set product prices p_n.

[21] This corresponds to the "full coverage" assumption in the terminology of Hotelling competition. Without the full-coverage assumption, the analyzed problem boils down to *either* two separate monopoly problems *or* the same problem as we analyze here.

[22] Note that $F_n(s_n) \geq 0$ implies that firm n can not force the advisor to hand over any compensation received from the other firm.

$t = 3$: The advisor provides advice to the first customer, who then decides whether or not to purchase a single product (and if so, which one). Subsequently, upon arrival of the second customer, this is repeated. The order of arrival is randomly assigned and denoted by $i = 1, 2$, and customers do not know whether they are first or second in line. The advisor privately observes product suitability q_i for the i-th arriving customer, and advice is provided by sending a message $m_i \in \{A, B\}$.

All payoffs are realized after $t = 3$. We abstract from discounting and assume that all parties are risk-neutral. We next fill some remaining gaps in the specification of the model and then comment on subsequent variations.

In our baseline analysis, we suppose that customers can not observe the incentive schemes, so that they have to form rational expectations. For this we assume that they hold passive beliefs out of equilibrium. For stage $t = 3$, we focus on pure strategy equilibria in which advice is informative at $t = 3$. Ignoring the payoff-equivalent outcome in which messages are swapped, we will show that in equilibrium the i-th arriving customer follows the advisor's respective "recommendation" through message $m_i = A$ or B. Figure 1 illustrates the model.

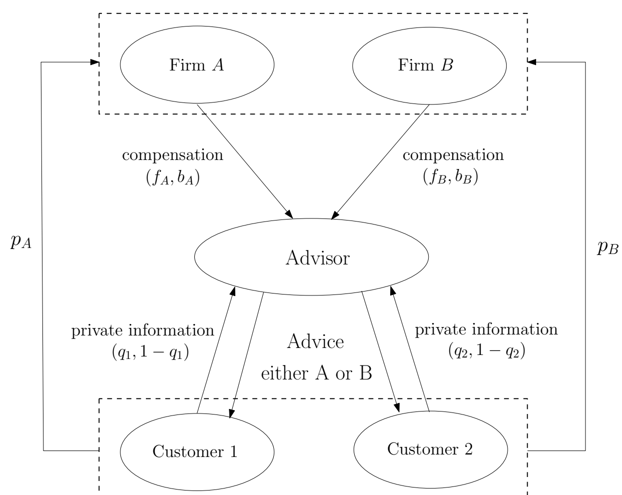

Figure 1: Illustration of the model.

We now impose restrictions that ensure that (i) the market opens up, (ii) advice is necessary, and (iii) firms are in competition. We also invoke assumptions that subsequently ensure uniqueness. To do so and to streamline the exposition, we focus in what follows on the case where product providers are symmetric, with $c_n = c$ and with G being symmetric around the (common) prior belief $q = 1/2$, $G(q) = 1 - G(1-q)$. Note from the Introduction that in models with only a single customer, such symmetry ensures that advice is unbiased, regardless of the size of (in equilibrium symmetric) incentives. Focusing on the case with symmetry makes thus transparent where implications differ when advisors face several customers and product providers can offer non-linear incentives. To ensure uniqueness we make the standard assumption that the hazard rate, $g(q)/(1-G(q))$, is increasing in $q \in [0,1]$. Together with symmetry of G, this implies that the reverse hazard rate, $g(q)/G(q)$, is decreasing in q, so that

$$\frac{d}{dq}\frac{g(q)}{1-G(q)} > 0 \qquad \text{and} \qquad \frac{d}{dq}\frac{g(q)}{G(q)} < 0. \tag{1}$$

Denote now $v_A(q) = qv_h + (1-q)v_l$ and $v_B(q) = (1-q)v_h + qv_l$. Requiring

$$\int_0^1 v_A(q)g(q)dq = \int_0^1 v_B(q)g(q)dq = \frac{v_l + v_h}{2} < c \tag{2}$$

ensures that advice is essential for selling either product. We subsequently invoke a second assumption that ensure that the market with advice opens up.

Plan of the Analysis. In the following sections we decompose the analysis as follows. Section 3 derives the optimal compensation, holding thereby fixed prices p_n and assuming that customers indeed follow the advisor's recommendation. In Section 4 we endogenize prices and fully characterize the unregulated equilibrium. Section 5 compares this to the regulated equilibrium outcome when firms can only use linear incentive schemes and thus $b_n = b = 0$. Section 6 extends all our insights to the case of simultaneous arrivals of customers. Section 7 concludes. The Online Appendix contains various alternative extensions of the analysis, on which we comment whenever this is appropriate.

3 Optimal Nonlinear Compensation

3.1 Preliminary Results

In this section, we first consider stage $t = 3$, where the advisor makes recommendations to customers. For now we postulate that these recommendations are followed. Subsequently we show that this must indeed hold in an equilibrium of the full game (intuitively, as the respective prices p_n will be chosen accordingly in $t = 2$).

Advice. We first consider the pattern of advice for the customer who arrives second. When he has sold product A to the first customer, the advisor anticipates that he receives an expected payoff equal to $f_A + b_A + q_2 w + w_l$ from recommending product A (through sending message $m_2 = A$) to the second customer, whose observed product suitability is $q_2 \in [0, 1]$, and $f_B + (1 - q_2)w + w_l$ from recommending product B (through sending message $m_2 = B$). Comparing payoffs yields the threshold

$$\bar{q}_2^A = \frac{1}{2} - \frac{1}{2w}(f_A - f_B + b_A),$$

such that the advisor prefers $m_2 = A$ if $q_2 \geq \bar{q}_2^A$ and $m_2 = B$ otherwise. The subscript 2 in $\bar{q}_2^A$ stands for the advice cutoff applied to the second arriving customer, and the superscript A indicates that the advisor has already sold product A to the first customer.

Similarly, having sold product B to the first customer, the advisor anticipates that he receives an expected payoff equal to $f_A + q_2 w + w_l$ from recommending product A to the second customer, and $f_B + b_B + (1 - q_2)w + w_l$ from recommending product B. Comparing those two payoffs yields the threshold

$$\bar{q}_2^B = \frac{1}{2} - \frac{1}{2w}(f_A - f_B - b_B),$$

such that the advisor prefers $m_2 = A$ if $q_2 \geq \bar{q}_2^B$ and $m_2 = B$ otherwise. Here the superscript B in $\bar{q}_2^B$ indicates that the advisor has sold product B to the first customer. Note that we restrict the exposition to interior thresholds. To deal with corner solutions, we set $\bar{q}_2^A = 0$ if $w \leq f_A - f_B + b_A$ and $\bar{q}_2^A = 1$ if $w \leq -(f_A - f_B + b_A)$. Similarly, we set $\bar{q}_2^B = 0$ if $w \leq f_A - f_B - b_B$ and $\bar{q}_2^B = 1$ if $w \leq -(f_A - f_B - b_B)$.

Next we consider the pattern of advice for the first arriving customer. Let

$$Z(\bar{q}_2^A) = \int_0^{\bar{q}_2^A} (f_B + (1 - q)w + w_l)g(q)dq + \int_{\bar{q}_2^A}^1 (f_A + b_A + qw + w_l)g(q)dq$$

and

$$Z(\bar{q}_2^B) = \int_0^{\bar{q}_2^B} (f_B + b_B + (1 - q)w + w_l)g(q)dq + \int_{\bar{q}_2^B}^1 (f_A + qw + w_l)g(q)dq.$$

When recommending product A (through sending message $m_1 = A$) to the first customer, whose observed product suitability is $q_1 \in [0, 1]$, the advisor realizes

$$f_A + q_1 w + w_l + Z(\bar{q}_2^A),$$

and by recommending product B (through sending $m_1 = B$)

$$f_B + (1 - q_1)w + w_l + Z(\bar{q}_2^B).$$

Comparing payoffs yields the threshold

$$\bar{q}_1 = \frac{1}{2} - \frac{1}{2w}\left(f_A - f_B + Z(\bar{q}_2^A) - Z(\bar{q}_2^B)\right),$$

such that the advisor prefers $m_1 = A$ if $q_1 \geq \bar{q}_1$ and $m_1 = B$ otherwise. The subscript 1 in $\bar{q}_1$ stands for the advice cutoff applied to the first customer. To deal with corner solutions, we set $\bar{q}_1 = 0$ if $w \leq f_A - f_B + Z(\bar{q}_2^A) - Z(\bar{q}_2^B)$ and $\bar{q}_2^A = 1$ if $w \leq -(f_A - f_B + Z(\bar{q}_2^A) - Z(\bar{q}_2^B))$.

We can now characterize the pattern of advice for any given compensation (f_n, b_n).

Lemma 1 *When customers follow his recommendation, the advisor's optimal recommendation is characterized as follows:*

$$(m_1, m_2) = \begin{cases} (A, A) & \textit{if } q_1 \in [\bar{q}_1, 1] \textit{ and } q_2 \in [\bar{q}_2^A, 1], \\ (A, B) & \textit{if } q_1 \in [\bar{q}_1, 1] \textit{ and } q_2 \in [0, \bar{q}_2^A], \\ (B, A) & \textit{if } q_1 \in [0, \bar{q}_1] \textit{ and } q_2 \in [\bar{q}_2^B, 1], \\ (B, B) & \textit{if } q_1 \in [0, \bar{q}_1] \textit{ and } q_2 \in [0, \bar{q}_2^B]. \end{cases}$$

In what follows, it will be inconsequential how we resolve cases of indifference as these are zero-probability events. To illustrate Lemma 1, consider first a symmetric compensation scheme (f, b). In the case of no bonus ($b = 0$), as then $\bar{q}_2^A = \bar{q}_2^B = 1/2$ holds and so also $\bar{q}_1 = 1/2$, the advisor recommends product A to the i-th arriving customer (sends message $m_i = A$) if $q_i \geq 1/2$ and B otherwise, irrespective of the order of their arrival. Figure 2 illustrates this pattern of advice. In contrast, with $b > 0$ advice cutoffs are given by $\bar{q}_2^A = 1/2 - b/(2w) = 1 - \bar{q}_2^B \in (0, 1/2)$ and by $\bar{q}_1 = 1/2$ given symmetry.

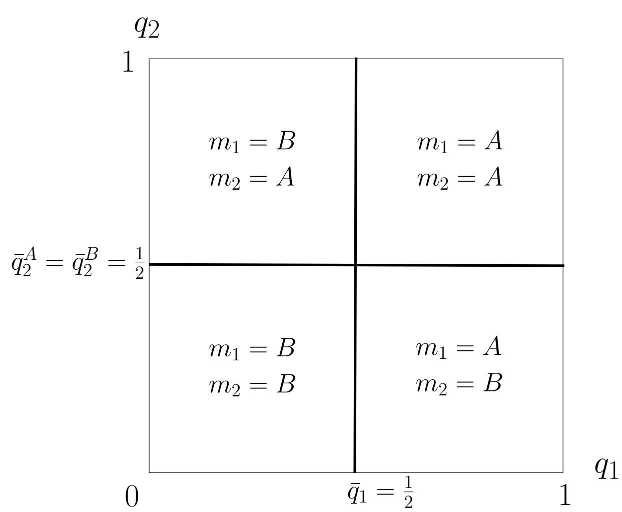

Figure 2: Pattern of advice without bonus.

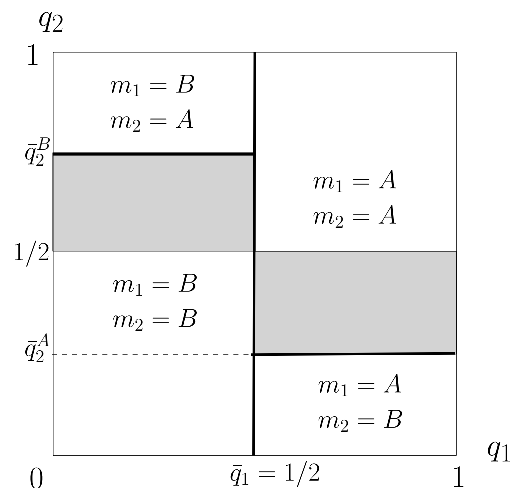

Figure 3: Pattern of advice with bonus.

This pattern of advice with a strictly positive bonus is illustrated in Figure 3. There, the gray-colored regions represent the effect of the bonus on advice, compared to Figure 2, as the advisor's recommendation to the second customer depends on his recommendation to the first customer.

With symmetry, we have that $\bar{q}_1 = 1/2$, so that the first recommendation is not biased in either direction. This would be different under asymmetry, and it is useful for our subsequent derivation of an equilibrium to express $\bar{q}_1$ more generally. Equation (3) captures how the advisor's anticipation of his subsequently chosen cutoffs, $\bar{q}_2^A$ and $\bar{q}_2^B$, also affects his first advice.

Lemma 2 *If* $(\bar{q}_2^A, \bar{q}_2^B) \in (0,1)^2$,

$$\bar{q}_1 = \bar{q}_2^A + \int_{\bar{q}_2^A}^{\bar{q}_2^B} G(q)dq. \tag{3}$$

Proof. See the Appendix.

Firm profits. For any given advice cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B) \in [0,1]^3$, define by

$$\Pr(1) = G(\bar{q}_1)(1 - G(\bar{q}_2^B)) + (1 - G(\bar{q}_1))G(\bar{q}_2^A)$$

the probability that the advisor makes recommendations for different products. Note that this is always common to both firms, so that $\Pr(1)$ is independent of $n = A, B$. Similarly, define by

$$\Pr_A(2) = (1 - G(\bar{q}_1))(1 - G(\bar{q}_2^A))$$

the probability that the advisor recommends product A to both customers, and by

$$\mathrm{Pr}_B(2) = G(\bar{q}_1)G(\bar{q}_2^B)$$

the probability that the advisor recommends product B to both customers. We denote by

$$\mathrm{S}_n = \mathrm{Pr}(1) + 2\mathrm{Pr}_n(2)$$

expected total sales for firm n. For given compensation (f_n, b_n) and product price p_n, for $n = A, B$, expected profits are written as

$$\pi_n = \mathrm{S}_n(p_n - c_n - f_n) - \mathrm{Pr}_n(2)b_n. \tag{4}$$

In light of the subsequent analysis, it is now helpful to briefly analyze how compensation affects firm profits. There are two first-order effects of the marginal increase in firm n's incentive component $x \in \{f_n, b_n\}$: a profit gain by the increase in sales and a profit loss by the increase in expected compensation. Differentiating firm n's profit (4) with respect to f_n and b_n yields the respective marginal profits

$$\frac{\partial \pi_n}{\partial f_n} = \mathrm{S}_n^f(p_n - c_n - f_n) - \mathrm{Pr}_n^f(2)b_n - \mathrm{S}_n \tag{5}$$

and

$$\frac{\partial \pi_n}{\partial b_n} = \mathrm{S}_n^b(p_n - c_n - f_n) - \mathrm{Pr}_n^b(2)b_n - \mathrm{Pr}_n(2), \tag{6}$$

where we denote by $\mathrm{Pr}^x(1)$ and $\mathrm{Pr}_n^x(2)$ the respective partial derivatives of the single-unit sale $\mathrm{Pr}(1)$ and of the two-unit sales $\mathrm{Pr}_n(2)$ with respect to $x_n \in \{f_n, b_n\}$, and by $\mathrm{S}_n^x = \mathrm{Pr}^x(1) + 2\mathrm{Pr}_n^x(2)$ the partial derivatives of total sales S_n.

Welfare. Before continuing the analysis, we define welfare. We suppose that w arises from a penalty imposed on the sale of an unsuitable product, and the advisor is concerned only about this penalty. If such penalties are monetary transfers, given that parties are risk neutral they do not enter social welfare. For any given advice cutoff $\bar{q} \in \{\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B\}$, we denote by

$$\mathbb{E}[v \mid \bar{q}] = \int_0^{\bar{q}} v_B(q)g(q)dq + \int_{\bar{q}}^1 v_A(q)g(q)dq$$

a given customer's expected gross utility from following advice when she anticipates that the advisor applies the cutoff rule based on $\bar{q}$. Taking account of the random order of customer arrivals, the expected gross utility ("suitability of advice") from following advice is

$$U = (1/2)\left(\mathbb{E}[v \mid \bar{q}_1] + G(\bar{q}_1)\mathbb{E}[v \mid \bar{q}_2^B] + (1 - G(\bar{q}_1))\,\mathbb{E}[v \mid \bar{q}_2^A]\right), \tag{7}$$

which will be useful also subsequently when determining the equilibrium price. As the expected product cost per customer is always c, expected welfare per customer is $W = U - c$. This is clearly maximized when the advisor sets the threshold $q^{FB} = 1/2$ with each customer, advising A if and only if $q_i \geq q^{FB}$. Put differently, welfare is maximized when the advisor treats all customers equally with $\bar{q}_1 = \bar{q}_2^A = \bar{q}_2^B = q^{FB}$. In what follows we refer to any other (equilibrium) outcome as biased.

3.2 The Impossibility of Unbiased Advice without Regulation

Our objective is to show that linear incentive schemes for both firms are never part of any equilibrium without regulation. We argue to a contradiction and thus sppose that firms set $b_n = b = 0$. In this case, advice cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$ should all be equal, which we denote by $\bar{q} \in (0,1)$.[23] Consider now firm n's marginal profits with respect to the commission and the bonus $x_n \in \{f_n, b_n\}$. We first examine the effects of the marginal increases in f_n and b_n on the advice cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B) \in (0,1)^3$.

Lemma 3 *At $\bar{q} = \bar{q}_1 = \bar{q}_2^A = \bar{q}_2^B \in (0,1)$, which holds when $b_n = 0$ for both firms,*

$$\frac{\partial \bar{q}_2^A}{\partial f_n} = \frac{\partial \bar{q}_2^B}{\partial f_n} = \frac{\partial \bar{q}_1}{\partial f_n} = \begin{cases} -\frac{1}{2w}, & \text{if } n = A, \\ \frac{1}{2w}, & \text{if } n = B, \end{cases} \tag{8}$$

and

$$\left(\frac{\partial \bar{q}_2^A}{\partial b_n}, \frac{\partial \bar{q}_2^B}{\partial b_n}, \frac{\partial \bar{q}_1}{\partial b_n} \right) = \begin{cases} \left(-\frac{1}{2w}, 0, -\frac{(1-G(\bar{q}))}{2w} \right), & \text{if } n = A, \\ \left(0, \frac{1}{2w}, \frac{G(\bar{q})}{2w} \right), & \text{if } n = B. \end{cases} \tag{9}$$

Proof. See the Appendix.

We comment on the respective derivatives (8) and (9). The marginal shifts of advice cutoffs are inversely related to the advisor's liability w: The advisor is less responsive to incentives as w increases. Intuitively, in (9) a bonus only affects the cutoff in the second period if the respective firm's product has been recommended in the first period (while otherwise $\frac{\partial \bar{q}_2^B}{\partial b_n} = 0$ for $n = A$ and $\frac{\partial \bar{q}_2^A}{\partial b_n} = 0$ for $n = B$). Note also that the effect that the bonus has on the first period cutoff $\bar{q}_1$ depends on the anticipated likelihood with which the advisor expects to recommend the same product in the second period (that is, $\frac{\partial \bar{q}_1}{\partial b_n} = -\frac{(1-G(\bar{q}))}{2w}$ for $n = A$ and $\frac{\partial \bar{q}_1}{\partial b_n} = \frac{G(\bar{q})}{2w}$ for $n = B$). Using (8) and (9), we can evaluate the effect of the marginal increase in the incentive component $x \in \{f_n, b_n\}$ on expected

[23] The advice cutoff $\bar{q}$ should lie in the open interval $(0,1)$, as otherwise the advisor would always recommend a particular firm's product to customers, which contradicts assumption (2).

sales, $S_n^x = \Pr^x(1) + 2\Pr_n^x(2)$. The following Lemma compares the effect of a higher bonus (at $b_n = 0$) wth that of a higher commission:

Lemma 4 *At $\bar{q} = \bar{q}_1 = \bar{q}_2^A = \bar{q}_2^B \in (0,1)$, which holds when $b_n = 0$, we have that*

$$S_n^b = \begin{cases} (1 - G(\bar{q}))S_A^f, & \text{if } n = A, \\ G(\bar{q})S_B^f, & \text{if } n = B. \end{cases} \tag{10}$$

Proof. See the Appendix.

We next illustrate expression (10). Figure 4 illustrates this relationship for $n = A$. The left panel of the figure shows the resulting change in the pattern of advice when firm A marginally increases its commission f_A by $\varepsilon > 0$, whereas the right panel shows the respective marginal change when increasing its bonus b_A by the same amount. At $b_n = b = 0$, the two pictures illustrate why the marginal impact of starting to pay a bonus is exactly equal to the marginal impact of further increasing the commission multiplied with the likelihood with which the advisor will recommend the respective product (that is, $1 - G(\bar{q})$ for $n = A$ and $G(\bar{q})$ for $n = B$, as given by (10)).

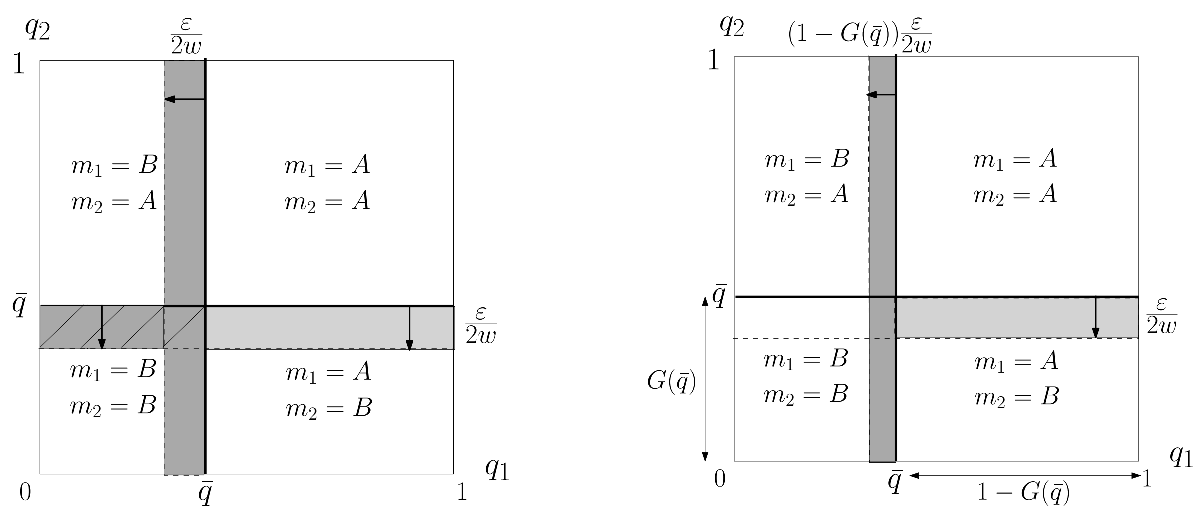

Figure 4: Effect of a marginal increase in commmission (left panel) and bonus (right panel), both at $b_n = 0$.

We now make use of the derived expressions to conduct the following argument. Starting from the considered situation with $b_n = b = 0$, we show that any of the two firms would then however strictly benefit from paying a strictly positive bonus (while possibly reducing its commission). We first show this formally and then provide an intuition. To show this formally, we consider for a given firm n a marginal increase in its bonus and, at the same time, a marginal decrease in its commission, so that total expected sales remain unchanged. We then show that along this gradient the firm's profit strictly increases, contradicting the optimality of $b_n = 0$.

Consider thus marginal adjustments $(df_n, db_n) \in \mathbb{R}^2$ such that total expected sales remain unchanged, that is,

$$\mathrm{S}_n^f df_n + \mathrm{S}_n^b db_n = 0. \tag{11}$$

Applying (10) to (11), we can derive a relationship between these marginal adjustments (df_n, db_n) in the absence of bonuses ($b_n = b = 0$).

Lemma 5 *For any given $n = A, B$, consider marginal adjustments $(df_n, db_n) \in \mathbb{R}^2$ defined by (11). If $b_A = b_B = 0$, (df_n, db_n) must satisfy*

$$df_n = \begin{cases} -(1 - G(\overline{q}))db_A, & \text{if } n = A, \\ -G(\overline{q})db_B, & \text{if } n = B. \end{cases} \tag{12}$$

This result is an immediate implication of Lemma 4. Geometrically, it can be easily seen as follows. Take the right-hand panel of Figure 4. Consider now the light-gray-colored area (of length $(1 - G(\overline{q}))$ and thickness $\frac{\varepsilon}{2w}$). If we were to stretch this area along the whole horizontal line, its thickness would be exactly $\frac{\varepsilon}{2w}$. Comparing this rearranged figure now to the left-hand side of Figure 4, we immediately see that, at $b_n = b = 0$, the impact of a marginal increase in the bonus is, for firm A, exactly $1 - G(\overline{q})$ times that of increasing the commission (and for firm B, $G(\overline{q})$ that of increasing the commission).

Starting from the respective expressions (5) and (6), consider next the total derivative of firm profits

$$d\pi_n = -\mathrm{S}_n df_n - \mathrm{Pr}_n(2) db_n, \tag{13}$$

where, at $b_n = 0$, we omitted the term $-(\mathrm{Pr}_n^f(2)df_n + \mathrm{Pr}_n^b(2)db_n)b_n$, which then clearly equals zero. Since all advice cutoffs are equal to $\bar{q}$, we have $\mathrm{Pr}(1) = 2G(\bar{q})(1 - G(\bar{q}))$, $\mathrm{Pr}_A(2) = (1 - G(\bar{q}))^2$, and $\mathrm{Pr}_B(2) = G(\bar{q})^2$, so that $\mathrm{S}_A = 2(1 - G(\bar{q}))$ and $\mathrm{S}_B = 2G(\bar{q})$. Evaluating the term when (df_n, db_n) satisfies (12), expression (13) finally becomes

$$d\pi_n = db_n \mathrm{Pr}_n(2).$$

In other words, when firm n starts paying a bonus and reduces its commission so that total expected sales remain unchanged, this strictly increases profits with derivative $\frac{d\pi_n}{db_n} = \mathrm{Pr}_n(2)$, which is the likelihood with which the advisor will recommend two times firm n's product. We have thus arrived at the following result:

Proposition 1 *Nonlinear incentives are part of any unregulated equilibrium when compensation is positive, i.e., there is no equilibrium in which compensation is positive but* $b_n = b = 0$.

Though we have not yet fully characterized an unregulated equilbrium, including equilibrium prices p_n, Proposition 1 already implies that the unregulated outcome must be inefficient: The presence of a bonus distorts the second-period recommendation, as it wrongly makes the advisor's recommendation to the second customer contingent on his first recommendation.

Discussion. Recall that at $b_n = b = 0$ the impact of a marginally higher bonus on sales is, for firm A, exactly $1 - G(\overline{q})$ of that of increasing the commission. The positive impact on increasing sales must be traded off with the respective costs of higher compensation, notably the higher "inframarginal" rent for the advisor, which is the higher compensation in cases where this did not sway the advisor's recommendation. *If* the higher bonus was only paid with likelihood $1-G(\overline{q})$ times that of the higher commission, the two changes in compensation would be equally profitable. However, the higher bonus is effectively paid relatively *less* often.[24]

We now provide an additional intuition by appealing to a slightly changed model setup. Suppose that a firm would indeed only use commissions, but make these contingent on when the respective sale took place. That is, firm A would pay f_A^1 when a product is sold to the first customer and f_A^2 when a product is sold to the second customer (and $f_A^1+f_A^2$ when both customers bought product A). We do not find this realistic, but this more flexible way to use commissions helps us to make the following argument more transparent. In fact, we show that despite this greater flexibility in using commissions, commissons are never used exclusively.

When both commissions are chosen optimally, then also the choice of f_A^2 satisfies the respective first-order condition: The costs of paying one Euro more commission whenever the advisor recommends A to customer 2 just balances the incremental benefits from increasing the likelihood of selling to customer 2. Instead of marginally raising f_A^2, consider now the introduction of a marginal bonus b_A (say, equal to one Euro as well). Take its impact on recommendations when the advisor has already recommended A to the first customer. Note that the suitability of a given product is distributed independently across

[24]Precisely, as the higher commission is paid with likelihood $2(1-G(\overline{q}))G(\overline{q})+2(1-G(\overline{q}))^2 = 2(1-G(\overline{q}))$ and the higher bonus with likelihood $(1 - G(\overline{q}))^2$, the respective ratio is $\frac{1}{2}(1 - G(\overline{q})) < 1 - G(\overline{q})$.

customers, so that the distribution of q_2 *conditional* on a recommendation of A to customer 1 is the same as the unconditional distribution. This implies that with respect to the advisor's recommendation to customer 2, the effects of a marginal bonus trade off in exactly the same way as those of a marginal increase in f_A^2. From this perspective alone, introducing a bonus would not dominate an increase in f_A^2, as considered beforehand. However, the analysis so far neglects the impact that a bonus has on the recommendation to customer 1: The bonus tilts the advisor's recommendation towards product A in those cases where the advisor was previously (just) indifferent, as he now expects to earn in this case $b_A > 0$ from the second customer, at least with some probability. This leads to additional sales without any additional costs when a bonus is introduced, so, starting from $b_n = 0$, the bonus is more cost-effective than the commission.[25]

3.3 Characterization of the Optimal Non-linear Compensation

We now derive the optimal nonlinear incentive scheme for a symmetric equilibrium, with $f > 0$ and $b > 0$. As we show later, such an equilibrium is unique and exists whenever w is not too large. Below we characterize the equilibrium compensation as well when w is larger, and we derive the respective thresholds for w that separate the different cases. Note also that for now we take $p_n = p$ as given, which is derived below, as part of the full equilibrium characterization.

Under symmetric compensation (f, b), advice cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$ satisfy $\bar{q}_1 = 1/2$ and $\bar{q}_2^A = 1 - \bar{q}_2^B \in (0, 1/2)$.[26] Note that the advice cutoffs are no longer all equal, unlike in the case of no bonus ($b = 0$). From symmetry, we have that $\Pr_A(2) = (1/2)(1 - G(\bar{q}_2^A)) = (1/2)G(\bar{q}_2^B) = \Pr_B(2)$, which we simply denote by $\Pr(2)$. Albeit advice cutoffs are no longer all equal, as when $b = 0$, with symmetric market shares the following result perfectly mirrors that in Lemma 5:

Lemma 6 *For any given symmetric compensation $(f_n, b_n) = (f, b)$ with $0 < b < w$, consider marginal adjustments $(df_n, db_n) \in \mathbb{R}^2$ defined by (11), so that total sales remain constant. Then, (df_n, db_n) must satisfy*

$$df_n = -\frac{1}{2} db_n. \tag{14}$$

[25] As $b_A = 0$ to start with, there are no additional costs to be considered. If instead $b_A > 0$, though, the positive bonus needs to be paid with a higher likelihood also when the first recommendation threshold shifts. This is why the argument for why $b_n > 0$ does not generally imply that *only* $b_n > 0$ while $f_n = 0$ (though, as we subsequently show, this is the case when w is sufficiently high).

[26] When $(f_n, b_n) = (f, b)$ with $f > 0$ and $b > 0$, advice cutoffs $\bar{q}_2^A$ and $\bar{q}_2^B$ simplify to $1/2 - b/(2w)$ and $1/2 + b/(2w)$, respectively. Using (3), this leads to $\bar{q}_1 = (1/2)(\bar{q}_2^A + \bar{q}_2^B) = 1/2$ due to $\bar{q}_2^A = 1 - \bar{q}_2^B$.

Proof. See the Appendix.

The way we now characterize the optimal compensation is by invoking the first-order condition along the gradient of the profit function where (11) is satisfied. To do so, note first the total derivative

$$d\pi = -\left(\mathrm{Pr}^f(2)df + \mathrm{Pr}^b(2)db\right)b - \mathrm{S}df - \mathrm{Pr}(2)db,$$

where we have again dropped the subscript due to symmetry. Noting now that[27]

$$\mathrm{Pr}(1) = G(\bar{q}_2^A), \quad \mathrm{Pr}(2) = (1/2)(1-G(\bar{q}_2^A)) \quad \text{and} \quad \mathrm{S} = \mathrm{Pr}(1) + 2\mathrm{Pr}(2) = 1,$$

while requiring that $df = -\frac{1}{2}db$ with constant sales, we have the total derivative

$$d\pi = \frac{1}{2}\left[\left(\mathrm{Pr}^f(2) - 2\mathrm{Pr}^b(2)\right)b + \mathrm{Pr}(1)\right]db.$$

Along the considered gradient, where (14) is satisfied, this must be zero at an optimal compensation choice. Rearranging yields the requirement

$$\left(2\mathrm{Pr}^b(2) - \mathrm{Pr}^f(2)\right)b = \mathrm{Pr}(1). \tag{15}$$

Equation (15) has a straightforward intuition. For this recall that we presently derive a necessary condition along the gradient where, as we marginally adjust compensation, total expected sales for the considered firm n remain unchanged. We know that this requires $df_n = -\frac{1}{2}db_n$, so that the left-hand side of (15) is exactly equal to the marginal increase in the probability with which a bonus will be paid multiplied by the prevailing bonus, i.e., it represents the increase in the advisor's rent. At an optimum, along the considered gradient, this must be equal to the rent that is saved when, as the bonus is increased, the commission is reduced and the respective reduction is no longer paid for single-unit sales (the right-hand side). Working with (15), we now substitute for the respective derviatives, where we obtain (cf. the subsequent proof) that

$$2\mathrm{Pr}^b(2) - \mathrm{Pr}^f(2) = \left(\frac{1}{2w}\right)g(\bar{q}_2^A)\left(1-G(\bar{q}_1)\right) = \frac{g(\bar{q}_2^A)}{4w}, \tag{16}$$

where the last step uses, in a symmetric equilibrium, that $1-G(\bar{q}_1) = 1/2$. Note also that $\mathrm{Pr}(1) = G(\bar{q}_2^A)$, so that we obtain from (15):

[27] More precisely, $\mathrm{Pr}(1) = (1-G(\bar{q}_1))G(\bar{q}_2^A) + G(\bar{q}_1)(1-G(\bar{q}_2^B)) = G(\bar{q}_2^A)$ due to $\bar{q}_1 = 1/2$, $\bar{q}_2^A = 1-\bar{q}_2^B$, and symmetry of G around 1/2, and where $\mathrm{Pr}(2) = \mathrm{Pr}_A(2) = (1-G(\bar{q}_1))(1-G(\bar{q}_2^A)) = (1/2)(1-G(\bar{q}_2^A)) = G(\bar{q}_1)G(\bar{q}_2^B) = \mathrm{Pr}_B(2)$, so that $\mathrm{S}_n = \mathrm{S} = 1$.

Lemma 7 *In a symmetric equilibrium with $f_n = f > 0$ and $b_n = b > 0$, the optimal choice of the bonus must satisfy*

$$b = 4\frac{G(\bar{q}_2^A)}{g(\bar{q}_2^A)}w. \tag{17}$$

Proof. See the Appendix.

Combining condition (17) with $\bar{q}_2^A = 1/2 - b/(2w)$, which again uses that $f_n = f$ and $b_n = b$, we can now pin down the equilibrium advice cutoff $\bar{q}_2^A$ as follows:

Lemma 8 *In a symmetric equilibrium with $f_n = f > 0$ and $b_n = b > 0$, the advice cutoff for the second customer, conditoinal on that A was recommended to the first customer, is uniquely determined by*

$$\bar{q}_2^A = \frac{1}{2} - 2\frac{G(\bar{q}_2^A)}{g(\bar{q}_2^A)}. \tag{18}$$

The respective advice cutoff when B was recommended to the first customer, is symmetric and given by $\bar{q}_2^B = 1 - \bar{q}_2^A$.

Proof. See the Appendix.

Equation (18) has two main implications. First, we have that $\bar{q}_2^A < 1/2$ and $\bar{q}_2^B > 1/2$, so that advice is always biased in an unregulated equilibrium. Second, the two cutoffs are independent of the agent's liability w. Before we return to a closer discussion of the second observation, we first complete the characterization of the optimal compensation. We do so by considering the first-order condition for the commission f_n: $\frac{d\pi_n}{df_n} = 0$. Letting

$$H(\bar{q}_2^A) = \frac{G(\bar{q}_2^A)}{g(\bar{q}_2^A)} + \frac{1}{4g(1/2)(1 - G(\bar{q}_2^A))^2 + g(\bar{q}_2^A)}, \tag{19}$$

we can derive the following:

Lemma 9 *In a symmetric equilibrium with $f_n = f > 0$ and $b_n = b > 0$, given $\bar{q}_2^A$ from (18), the optimal commission satisfies*

$$f = p - c - 2wH(\bar{q}_2^A), \tag{20}$$

where $H(\bar{q}_2^A)$ is defined by (19).

Proof. See the Appendix.

We know from (17) that the ratio b/w is constant in w, so are all advice cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$ and consequently also $H(\bar{q}_2^A)$, and therefore the commission f in (20) decreases with the size of w, while the bonus b in (17) increases. A firm thus adjusts its commission and bonus to the size of w, seeking to balance out the cost-effectiveness between them, which induces a unique pattern of advice (through a unique advice cutoff $\bar{q}_2^A = 1 - \bar{q}_2^B$ with $\bar{q}_1 = 1/2$), irrespective of the advisor's liability w, as long as both the commission and the bonus are positive. We now turn to this condition. The optimal commission in (20) is indeed positive as long as w is below a certain level, otherwise it equals zero. We denote by w_f^* the respective threshold, which is given by

$$w_f^* = \frac{p - c}{2H(\bar{q}_2^A)}, \tag{21}$$

where $\bar{q}_2^A$ is uniquely determined by (18) while we turn to a characterization of the equilibrium price p below. We can now characterize the optimal nonlinear incentive scheme when $f_n > 0$ and $b_n > 0$.

Proposition 2 *Suppose that the advisor's liability w is below the threshold w_f^* defined by (21). Then, there is a unique symmetric equilibrium compensation scheme, giving rise to unique advice cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$, which are all independent of w. The commission f decreases with the size of w while the bonus b increases. Precisely, $\bar{q}_1 = 1/2$, $\bar{q}_2^A = 1 - \bar{q}_2^B$ is determined by (18), and (f, b) solve (20) and (17), respectively.*

The result that the pattern of advice and thereby also the size of the bias are independent of the advisor's liability may at first seem counterintuitive, given that this implies a strictly higher expected liability payment. In fact, if we were to hold the compensation fixed, this bias would be reduced as w increases. However, at least as long as $w < w_f^*$ firms react to the increased liability by increasing the bonus. They reduce the commmission at the same time, reacting to the fact that the advisor becomes less responsive to monetary incentives. The intuition for the firms' overall response follows directly from the way how we derived the optimal compensation, that is from firms' optimal trade-off between the two instruments. For this trade-off, w only plays a role in affecting the sensitivity of the respective cutoffs. But as long as both incentive components are still used, i.e., both $f_n > 0$ and $b_n > 0$, the sensitivity of the respective cutoffs effectively cancels out when trading off db_n with df_n. The implications for policy are immediate: At least as long as $w < w_f^*$, the advisor's bias is not affected by the size of the advisor's liability. Product providers just step up their bonuses!

Before we close the model by also deriving equilibrium prices, we note that this stark implication however ceases to hold when w is sufficiently large. Once w reaches the threshold $w_f^* > 0$, setting a positive commission is no longer profitable and firms will react to a change in w by reducing their still positive bonus, which then has an immediate implication for the respective advice cutoffs. More explicitly, with $f = 0$ and symmetry, the first-order-condition with respect to the bonus is then

$$\mathrm{S}^b(p-c) - \mathrm{Pr}^b(2)b = \mathrm{Pr}(2).$$

From this we can derive, after substition, an implicit expression for b, which we however relegate to the proof of the subsequent proposition. With this at hand, we can derive the size of w at which the bonus too becomes zero, which, for the respective price p, is at

$$w_b^* = 2g(1/2)(p-c). \tag{22}$$

Lemma 10 *Let w_f^* and w_b^* be the thresholds of w defined by (21) and (22), respectively. If $w \in (w_f^*, w_b^*)$, the optimal commission is zero and the optimal bonus is given by (A.5). For $w \geq w_b^*$ compensation is zero.*

Proof. See the Appendix.

Lemma 10 will next be employed to finally derive the characterization of the unregulated equilibrium outcome.

4 Unregulated Equilibrium

So far we have taken prices as given to determine the optimal incentive scheme. In this section, we derive the full equilibrium of the game. To this aim, we first provide a definition of the equilibrium. We then prove existence of a unique equilibrium, and finally return to a comparative analysis.

To derive customers' conditional valuations, we define their beliefs about the (non-observed) compensation by $(\hat{f}_n, \hat{b}_n)$ and the corresponding rationally anticipated advice cutoffs by $\hat{\mathbf{q}} = (\hat{q}_1, \hat{q}_2^A, \hat{q}_2^B)$. For some arbitrary advice cutoff $\hat{q} \in \{\hat{q}_1, \hat{q}_2^A, \hat{q}_2^B\}$, let

$$\mathbb{E}[v_A \mid \hat{q}] \equiv \frac{1}{1-G(\hat{q})} \int_{\hat{q}}^1 v_A(q)g(q)dq \quad \text{and} \quad \mathbb{E}[v_B \mid \hat{q}] \equiv \frac{1}{G(\hat{q})} \int_0^{\hat{q}} v_B(q)g(q)dq$$

be the customer's *conditional* expected valuation for products A and B, respectively. Taking account of sequential arrivals in a random order, each customer anticipates that the expected valuation conditional on being recommended product $n(= A, B)$ would be

$$\mathbb{E}[v_n \mid \hat{\mathbf{q}}] \equiv \frac{1}{2} \left[\mathbb{E}[v_n \mid \hat{q}_1] + G(\hat{q}_1)\mathbb{E}[v_n \mid \hat{q}_2^B] + (1 - G(\hat{q}_1))\, \mathbb{E}[v_n \mid \hat{q}_2^A] \right]. \tag{23}$$

Given passive beliefs, in equilibrium each firm optimally extracts the full conditional valuation, so that $p_n = \mathbb{E}[v_n \mid \hat{\mathbf{q}}]$.

An **unregulated equilibrium** is characterized by a tuple of firm strategies (f_n, b_n, p_n), the advisor's cutoff strategy $\mathbf{q} = (\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$, and customer beliefs about compensation $(\hat{f}_n, \hat{b}_n)$, which give rise to the expected cutoffs $\hat{\mathbf{q}} = (\hat{q}_1, \hat{q}_2^A, \hat{q}_2^B)$, so that:
i) Incentive schemes must be optimal, given prices p_n, i.e., (f_n, b_n).
ii) Prices must be optimal and so satisfy $p_n = \mathbb{E}[v_n \mid \hat{\mathbf{q}}]$.
iii) The advisor makes optimal recommendations, implying that $\mathbf{q} = (\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$ are given by Lemma 1.
iv) Beliefs are rational as $(\hat{f}_n, \hat{b}_n) = (f_n, b_n)$ and thereby also $\hat{\mathbf{q}} = \mathbf{q}$.

We now impose an assumption that ensures that the market for advice opens up. Specifically, we make this assumption slightly stronger to ensure that we are in the interesting case where both $f > 0$ and $b > 0$ can indeed arise (for sufficiently small w). Taking the advice cutoffs as uniquely characterized in Lemma 8, that is $\mathbf{q} = (\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$, we stipulate that

$$\mathbb{E}[v_n \mid \mathbf{q}] > c. \tag{24}$$

Given this choice of $\mathbf{q} = (\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$, we now substitute this into $w_f^* = (p - c)/(2H(\bar{q}_2^A))$ from (21). Likewise, taking $w_b^* = 2g(1/2)(p - c)$ from (22), we substitute the respective conditional valuations evaluated at the unbiased cutoffs $\mathbf{q} = (1/2, 1/2, 1/2)$, that is

$$p = \mathbb{E}[v_n \mid \mathbf{q} = (1/2, 1/2, 1/2)] = 2\int_{1/2}^{1} v_A(q)g(q)dq = 2\int_{0}^{1/2} v_B(q)g(q)dq.$$

Using these thresholds, we have the following result:

Proposition 3 *When $w < w_f^*$, there exists a unique unregulated equilibrium where firms pay both positive commissions and bonuses $f_n = f > 0$ and $b_n = b > 0$, as characterized in Proposition 2. Advice is biased and the bias is also not mitigated when liability w (marginally) increases. When liability is instead high with $w \geq w_b^*$, firms do not provide incentives to the advisor. In the intermediate case, where $w_f^* \leq w < w_b^*$, we have $f_n = f = 0$ and $b_n = b > 0$, as in Lemma 10. Then, as w increases, advice becomes less biased (as always $\bar{q}_1 = 1/2$, while $1/2 - \bar{q}_2^A > 0$ and $\bar{q}_2^B - 1/2 > 0$ strictly decrease).*

Proof. See the Appendix.

As w changes, this does not affect the suitability of advice and consequently also not the maximum price that firms can charge as long as $w < w_f^*$. In our concluding remarks we argue why setting an arbitrarily high liability may not be a realistic solution, also as it risks having various unwanted consequences.

Example 1: Uniform Distribution. Suppose that G is the uniform distribution and that the exogenous parameters are given as $(v_h, v_l, c) = (1, 0, 0.6)$. Figure 5 illustrates the optimal nonlinear incentive scheme (f, b). As seen in the left panel, the commission f decreases with the size of w while the bonus b increases if w is below the threshold $w_f^*(\approx 0.17)$. When w is higher, the bonus b decreases as w further increases, up to the threshold $w_b^*(= 0.3)$, from which on all compensation is zero. Accordingly, the advice cutoff $\bar{q}_2^A = 1 - \bar{q}_2^B$ shifts as described in the right panel of the figure.[28]

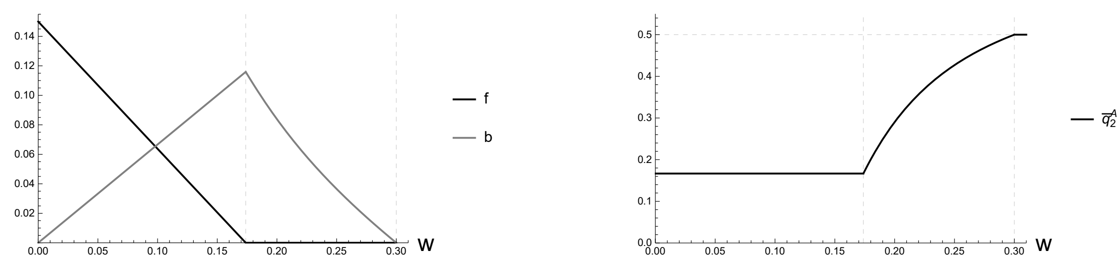

Figure 5: Uniform example.

Example 2: Truncated Normal Distribution. We have shown that the pattern of advice is invariant to w when $w < w_f^*$. The second key characteristic of the advisor, next to w, is the degree to which he is better informed than customers. Holding all else constant, as the advisor becomes better informed, this should increase welfare. But this ignores a potential adjustment of compensation, which may have a countervailing effect.

To analyze this, we take another standard example for the functional specification of $G(q)$. We consider a normal distribution with mean $1/2$ and variance $\sigma > 0$ and we let $G(q)$, which captures the advisor's (better) information, be its truncated distribution, restricted to $q \in [0, 1]$. Then as σ increases, resulting in a mean-preserving spread for the posterior distribution, the advisor unambiguously becomes better informed about products' suitability.[29]

[28]See the Appendix for all derivations.

[29]Put differently, supposing that some original distribution $G_1(q)$ was captured by the respective obser-

Proposition 4 *Suppose $G(q)$ is a truncated normal distribution with mean $1/2$. Then as the advisor becomes better informed, as reflected in a higher variance of the distribution of posterior beliefs, product providers increase their bonus and the advice cutoff for the second customer becomes more biased away from the first-best, that is $1/2 - q_2^A = q_2^B - 1/2 > 0$ increases. The equilibrium adjustment of non-linear compensation thus reduces the benefits of the advisor's better information.*

Proof. See the Appendix.

Discussion. Before we move on and introduce regulation, we offer some additional remarks on the derivation of the unregulated equilibrium outcome. When $w \leq w_f^*$ each product provider makes profits of

$$\pi = 2w\left[\Pr(1)H(\bar{q}_2^A) + 2\Pr(2)\left(\frac{1}{4g(1/2)(1-G(\bar{q}_2^A))^2 + g(\bar{q}_2^A)}\right)\right],$$

where all terms in the bracket are positive for any given $\bar{q}_2^A \in (0, 1/2)$ and independent of w. Therefore, $\pi_n = \pi$ is an increasing function of w (and, in fact, proportional). Firms are better off when the agent's liability increases as this effectively reduces competition. In fact, an increase in w is akin to greater differentiation in a standard (Hotelling) game of price competition, albeit in our game firms compete with (non-linear) incentives.[30] As customers' utility is not affected by w when $w < w_f^*$, the advisor's payoff must be strictly decreasing in w. While the advisor does not change his recommendations when his liability increases, he is thus worse off in two ways. First, he incurs higher (expected) liability costs for given recommendations. Second, while firms adjust their compensation so as to ensure that his recommendation remains unchanged, firms' overall incentives to steer advice decrease as advice becomes less responsive to monetary incentives. Note at this point that we implicitly assume throughout the analysis that the advisor's participation constraint is satisfied. Clearly, when a very high w drives out all compensation, this is only the case when the advisor earns some additional payoff, e.g., from other business with the respective customer. If not, then high liability may drive out advice.[31]

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vation of some signal s_1 (of arbitrary dimensionality), when the advisor can observe, in addition, another signal s_2, then the transformation from $G_1(q)$ to $G_2(q)$ can always be expressed as a mean-preserving spread.

[30] In a standard Hotelling game, firms' profits would be proportional to customers' (constant) "transportation costs", which corresponds to parameter w in our framework.

[31] To keep the advisor otherwise in business, product providers may have to pay him a fixed compensation, and as the advisor is not the employee of a single product provider in our model, firms will clearly try to free-ride (as now their fixed compensation become complements in inducing the advisor to participate).

5 Regulated Market Equilibrium

As discussed in the Introduction, regulatory authorities may intervene by imposing restrictions on the contingencies of incentive pay. In what follows, we suppose that such regulation restricts the considered compensation schemes (f_n, b_n) to be proportional to sales, thus banning bonus payments $(b_n = 0)$. Under such incentives, the advisor adopts a common threshold $\bar{q} \in [0, 1]$ for all customers, recommending product B if product suitability q_i for customer $i(= 1, 2)$ is below $\bar{q}$ and product A otherwise. Receiving a payoff $f_A + q_i w + w_l$ from recommending product A (through sending message $m_i = A$) and $f_B + (1 - q_i)w + w_l$ from recommending product B (through sending message $m_i = B$), the respective cutoff satisfies

$$\bar{q} = \frac{1}{2} - \frac{1}{2w}(f_A - f_B). \tag{25}$$

To deal with corner solutions, we set $\bar{q} = 0$ when $w \leq f_A - f_B$ and $\bar{q} = 1$ when $w \leq -(f_A - f_B)$.

Lemma 11 *When customers follow his recommendation, with regulated linear incentives the advisor's optimal recommendation simplifies as follows:*

$$m_i = \begin{cases} A, & \text{if } q_i \in [\bar{q}, 1], \\ B, & \text{if } q_i \in [0, \bar{q}]. \end{cases}$$

In the absence of bonuses, profits of firm n are $\mathrm{S}_n(p_n - c_n - f_n)$, which simplifies to

$$\pi_n = \begin{cases} 2(1 - G(\bar{q}))(p_A - c_A - f_A), & \text{if } n = A, \\ 2G(\bar{q})(p_B - c_B - f_B), & \text{if } n = B, \end{cases}$$

where $\bar{q}$ is given by (25). Solving the profit maximization problem, firm A sets

$$f_A^R = p_A - c_A - 2w\frac{1 - G(\bar{q})}{g(\bar{q})},$$

provided that this is strictly positive (otherwise $f_A^R = 0$) and not above $f_B^R + w$ (otherwise $f_A^R = f_B^R + w$), and similarly firm B sets

$$f_B^R = p_B - c_B - 2w\frac{G(\bar{q})}{g(\bar{q})},$$

provided that this is strictly positive (otherwise $f_B^R = 0$) and not above $f_A^R + w$ (otherwise $f_B^R = f_A^R + w$). Under symmetry, for given $p_n = p$, we have

$$f^R = p - c - \frac{w}{g(1/2)}. \tag{26}$$

In equilibrium, p is set equal to a customer's conditional valuation $\mathbb{E}[v_n \mid \mathbf{q}]$ evaluated at $\mathbf{q} = (\bar{q}, \bar{q}, \bar{q})$ with $\bar{q} = 1/2$; cf. more formally Proposition 5 below. We define

$$w^* = g(1/2)(p - c), \tag{27}$$

noting that w^* is below w_b^* defined by (22).

Proposition 5 *There exists a unique regulated equilibrium in which the optimal commission $f_n = f^R$ is given by (26) if w is below the threshold defined by (27), and zero otherwise. Advice is always unbiased, so that suitability of advice is always strictly higher than in the unregulated case, as long as firms make use of incentive pay there (as w is below the threshold defined by (22)).*

Proof. See the Appendix.

Directly interfering with firms' (admissible) compensation thus leads to the efficient outcome. Before we point to potential drawbacks of such interference, we show how all our results extend to the case where customers arrive simultaneously.

6 Extension to Simultaneous Advice

6.1 Modifying the Model

So far, we have supposed that customers will arrive in a sequential order. We now consider the case of simultaneous arrival. The key difference is that the advisor now observes product suitability for customers at the same time and can likewise make both recommendations simultaneously. The subsequent analysis of the unregulated outcome is kept short as results mirror those under sequential arrival.

Advice. When recommending different products through message $(m_1, m_2) = (A, B)$, the advisor's expected payoff equals

$$f_A + f_B + q_1 w + (1 - q_2)w + 2w_l,$$

and when recommending the same product to both customers through message $(m_1, m_2) = (A, A)$, the payoff equals

$$2f_A + b_A + q_1 w + q_2 w + 2w_l.$$

Comparing these two payoffs yields the threshold

$$q^* = \frac{1}{2} - \frac{1}{2w}(f_A - f_B + b_A),$$

such that the advisor prefers (A, A) if $q_2 \geq q^*$ and (A, B) otherwise. Note that the subscripts in m_1 and m_2 now correspond to the customers' identity $j = 1, 2$ (unlike in the case of sequential advice, where this is related to the order of arrival). Similarly, by considering the two payoffs

$$f_A + f_B + (1 - q_1)w + q_2 w + 2w_l$$

from sending $(m_1, m_2) = (B, A)$ and

$$2f_B + b_B + (1 - q_1)w + (1 - q_2)w + 2w_l$$

from $(m_1, m_2) = (B, B)$, we have

$$q^{**} = \frac{1}{2} - \frac{1}{2w}(f_A - f_B - b_B),$$

such that the advisor prefers (B, A) if $q_2 \geq q^{**}$ and (B, B) otherwise. Also, the advisor prefers (A, A) over (B, A) if $q_1 \geq q^*$ and (B, A) otherwise, and (A, B) over (B, B) if $q_1 \geq q^{**}$ and (B, B) otherwise. Considering finally the payoffs from sending $(m_1, m_2) = (A, A)$ and $(m_1, m_2) = (B, B)$, we have the threshold

$$\overline{q}_2(q_1) = 1 - q_1 - \frac{1}{2w}(2(f_A - f_B) + b_A - b_B),$$

such that the advisor prefers message (A, A) if $q_2 \geq \overline{q}_2(q_1)$ and (B, B) otherwise. The thereby obtained thresholds $(q^*, q^{**}, \overline{q}_2(q_1))$ fully characterize the advisor's optimal recommendation for any incentive scheme (f_n, b_n).[32]

Lemma 12 *When customers follow his recommendation, the advisor's optimal recommendation with simultaneous arrivals is characterized as follows:*

$$(m_1, m_2) = \begin{cases} (A, A) & \text{if } (\forall i = 1, 2)\ q_i \in [q^*, 1] \text{ and } q_2 \geq \overline{q}_2(q_1), \\ (B, B) & \text{if } (\forall i = 1, 2)\ q_i \in [0, q^{**}] \text{ and } q_2 \leq \overline{q}_2(q_1), \\ (A, B) & \text{if } q_1 \in [q^{**}, 1],\ q_2 \in [0, q^*], \text{ and } q_1 \geq q_2, \\ (B, A) & \text{if } q_1 \in [0, q^*],\ q_2 \in [q^{**}, 1], \text{ and } q_1 \leq q_2. \end{cases}$$

[32]To deal with corner solutions, define for ease of exposition the following: $q^* = 0$ if $w \leq f_A - f_B + b_A$ and $q^* = 1$ if $w \leq -(f_A - f_B + b_A)$; $q^{**} = 0$ if $w \leq f_A - f_B - b_B$ and $q^{**} = 1$ if $w \leq -(f_A - f_B - b_B)$; and $\overline{q}_2(q_1) = 0$ if $2w(1 - q_1) \leq 2(f_A - f_B) + b_A - b_B$ and $\overline{q}_2(q_1) = 1$ if $2wq_1 \leq -(2(f_A - f_B) + b_A - b_B)$.

To illustrate Lemma 12, consider again a symmetric compensation scheme (f, b). In the case of no bonus ($b = 0$) and consequently $q^* = q^{**} = 1/2$, the advisor recommends product A to customer $j = 1, 2$ (sends message $m_j = A$) if $q_j \geq 1/2$ and B otherwise, irrespective of the *other* customer's product suitability. Figure 6 illustrates this (unbiased) pattern of advice.

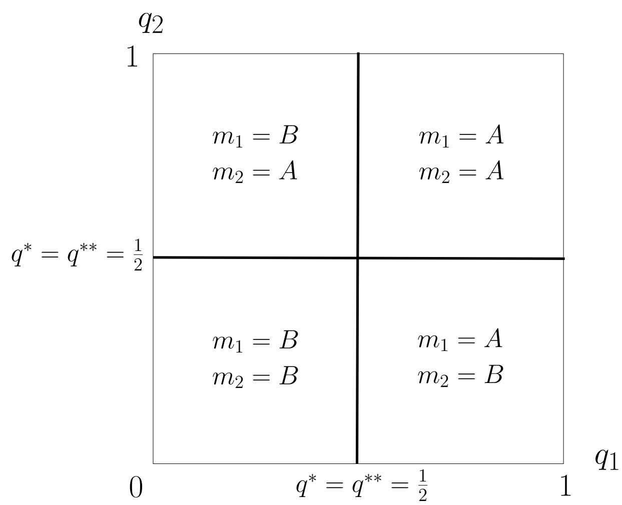

Figure 6: Pattern of advice without bonus (simultaneous arrival).

Figure 7 illustrates the case of (biased) advice with a strictly positive bonus. The difference to the case without a bonus is represented by the shaded areas: In these cases, the bonus again creates a wedge between the efficent outcome and the advisor's actual recommendations.

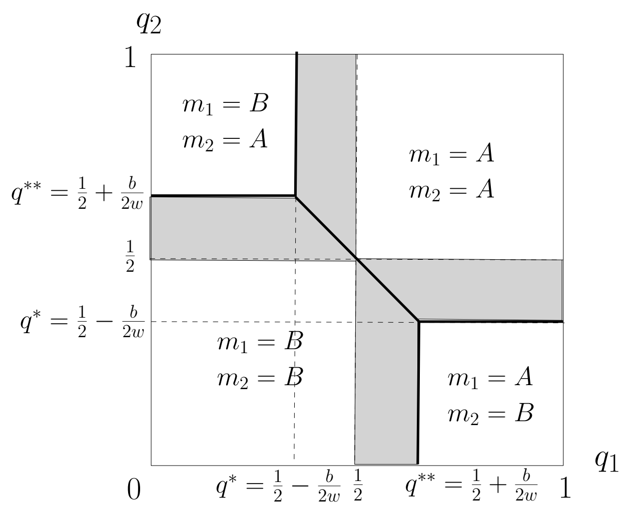

Figure 7: Pattern of advice with bonus (simultaneous arrival).

Firm profits. Define by

$$\Pr(1) = 2G(q^*)\left(1 - G(q^{**})\right)$$

the probability that the advisor makes recommendations for different products, which is common to both firms. Similarly, define by

$$\Pr_A(2) = \left(1 - G(q^*)\right)\left(1 - G(q^{**})\right) + \int_{q^*}^{q^{**}} \left(1 - G(\overline{q}_2(q_1))\right) g(q_1) dq_1$$

the probability that the advisor recommends product A to both customers, and by

$$\Pr_B(2) = G(q^*)G(q^{**}) + \int_{q^*}^{q^{**}} G(\overline{q}_2(q_1))g(q_1)dq_1$$

the probability that the advisor recommends product B to both customers. For given compensation (f_n, b_n) and product price p_n, for $n = A, B$, expected profits are written in the same way as in the case of sequential advice, replacing probabilities $\Pr(1)$ and $\Pr_n(2)$ with the ones defined above.

6.2 Analysis

For any given $q^* \in (0, 1/2)$, define now

$$H(q^*) = \frac{G(q^*)}{g(q^*)} + \frac{1}{2(G(q^*)g(q^*) + \int_{q^*}^{1-q^*} g^2(q)dq)}. \tag{28}$$

We first state our result and then comment on its derivation. For brevity's sake we omit the specification of equilibrium prices $p_n = p = \mathbb{E}[v_n \mid \mathbf{q}]$.

Proposition 6 *Consider the case with simultaneously arriving customers. Generally, $b_n = b = 0$ is again not an equilibrium when there is positive compensation. If there exists an equilibrium where both the commission and the bonus are strictly positive, $f > 0$ and $b > 0$, then it is again unique and characterized as follows. The advice cutoff $q^* = 1 - q^{**} \in (0, 1/2)$ is uniquely determined by*

$$q^* = \frac{1}{2} - \frac{G(q^*)}{g(q^*)}, \tag{29}$$

the optimal bonus by

$$b = 2\frac{G(q^*)}{g(q^*)}w$$

and the optimal commission by

$$f = p - c - wH(q^*).$$

There, as the advisor cares more about the suitability of his recommendations (higher w), incentives become steeper, as b increases and f decreases, while the pattern of advice and thus suitability of advice remain unchanged.

Proof. See the Appendix.

This extends the two main results from the unregulated equilibrium with sequential arrival: (i) the optimality of non-linear incentives (Proposition 1) and (ii) the characterization of the optimal nonlinear incentive scheme (Proposition 2).

7 Concluding Remarks

To conclude, we stress two insights from our analysis, which both apply with sequential and simultaneous arrival of customers. First, when firms want to steer advisors' recommendations, they use (volume-contingent) bonuses, which lead to biased advice: Such incentives make the recommendation given to any given customer implicitly contingent on other recommendations, which should not be the case from a welfare perspective. Second, at least when the agent's liability is not too high, imposing stricter liability does not affect this bias: Product providers fully counteract the agent's higher liability by sufficiently stepping up the bonus. Directly interfering with firms' incentives, namely by requiring that these are proportional to sales rather than contingent on certain sales targets, leads to unbiased advice, while higher liability alone may have zero impact.

A first caveat to this conclusion may be that imposing high liability would also lead to unbiased advice, at least when this fully drives out product providers' compensation: When the agent becomes sufficiently unresponsive to incentives, these no longer arise in equilibrium. While in our setting there is indeed no reason for why imposing such strict liability may be counterproductive, in practice this should be different. For instance, a higher liability may lead to the exit of advisors, increasing the remaining advisors' market power. Advisors may also face obstacles in generating revenues directly from customers. In fact, there may be reasons for why in many markets the outcome has not converged to one where (only) customers directly pay for advice and where inducements from product providers are absent, albeit we must leave such extensions to future work.

But imposing linear incentives may, in some circumstances, also be counterproductive, and policymakers should at least be aware of such a possibility. We show in the Online Appendix that when products differ in costs, so that efficiency requires asymmetric market shares, prescribing linear incentives that do not allow for a bonus may backfire as the market share of the more efficient product is inefficiently reduced. Also, the imposition of linear incentives can reduce welfare when it lowers the advisor's overall compensation (per-customer), as this could stifle incentives to acquire customers in the first place. This may be harmful particularly for products where customers typically exhibit considerable inertia (such as pensions and savings plans). Extending the analysis to this would combine our model of advice with one of effort provision as in Innes (1990). Stifling effort provision would have a first-order effect as even without regulation, effort provision would not likely be first best for various reasons, notably as with common agency there is a public good problem in the provision of incentives. A full analysis of such an extended model is also left to future research. Finally, note that in this paper we do not discuss the imposition of liability on product providers. The imposition of such a derived liability is also left to future work.

A Appendix

Proof of Lemma 2. We derive equation (3) for any given (f_n, b_n) and $(\bar{q}_2^A, \bar{q}_2^B) \in (0,1)^2$. For this we can write $Z(\bar{q}_2^A) - Z(\bar{q}_2^B)$ as $b_A - 2w \int_{\bar{q}_2^A}^{\bar{q}_2^B} G(q)dq$, by which the right-hand side becomes $\bar{q}_2^A + \int_{\bar{q}_2^A}^{\bar{q}_2^B} G(q)dq$, thus (3). **Q.E.D.**

Proof of Lemma 3. We derive both (8) and (9) when $\bar{q} = \bar{q}_1 = \bar{q}_2^A = \bar{q}_2^B \in (0,1)$. In doing so, we focus on firm A and first examine the effects of the marginal increases in f_A and b_A on the advice cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B) \in (0,1)^3$, which leads to (8) and (9) in case of $n = A$. The same argument applies to the case of $n = B$, leading to the remaining part of (8) and (9).

Consider a marginal increase in f_A. This leads to a downward shift of the advice cutoffs $\bar{q}_2^A$ and $\bar{q}_2^B$ by

$$\frac{\partial \bar{q}_2^A}{\partial f_A} = \frac{\partial \bar{q}_2^B}{\partial f_A} = -\frac{1}{2w},$$

which follows from differentiating both $\bar{q}_2^A$ and $\bar{q}_2^B$ with respect to f_A. This further leads to the downward shift of the advice cutoff $\bar{q}_1$ by

$$\frac{\partial \bar{q}_1}{\partial f_A} = G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial f_A} + \left(1 - G(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial f_A} = \frac{\partial \bar{q}_2^B}{\partial f_A} = \frac{\partial \bar{q}_2^A}{\partial f_A} = -\frac{1}{2w},$$

where $\bar{q}_1$ is described by (3) due to Lemma 2 and we have used $G(\bar{q}_2^A) = G(\bar{q}_2^B)$ at $\bar{q}_2^A = \bar{q}_2^B = \bar{q}$.

Next consider a marginal increase in b_A. This leads to a downward shift of the advice cutoff $\bar{q}_2^A$ alone, while $\bar{q}_2^B$ remains unchanged, precisely

$$\frac{\partial \bar{q}_2^B}{\partial b_A} = 0 \quad \text{and} \quad \frac{\partial \bar{q}_2^A}{\partial b_A} = -\frac{1}{2w} = \frac{\partial \bar{q}_2^A}{\partial f_A}.$$

This further leads to a downward shift of the advice cutoff $\bar{q}_1$ by

$$\frac{\partial \bar{q}_1}{\partial b_A} = G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial b_A} + \left(1 - G(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial b_A} = \left(1 - G(\bar{q})\right)\frac{\partial \bar{q}_2^A}{\partial f_A},$$

where the last equality follows from $\partial \bar{q}_2^B / \partial b_A = 0$, $\partial \bar{q}_2^A / \partial b_A = -1/(2w)$, and $\bar{q}_2^A = \bar{q}$. **Q.E.D.**

Proof of Lemma 4. We derive (10) using the marginal shifts of advice cutoffs $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B)$ given by (8) and (9) in Lemma 3. We now focus on firm A. Recall that

$$\begin{aligned} \mathrm{S}_A &= \Pr(1) + 2\Pr_A(2) \\ &= G(\bar{q}_1)(1 - G(\bar{q}_2^B)) + (1 - G(\bar{q}_1))G(\bar{q}_2^A) + 2(1 - G(\bar{q}_1))(1 - G(\bar{q}_2^A)). \quad \text{(A.1)} \end{aligned}$$

Differentiating (A.1) with respect to $x \in \{f_A, b_A\}$ yields $\mathrm{S}_A^x = \mathrm{Pr}^x(1) + 2\mathrm{Pr}_A^x(2)$, which can be written as

$$\begin{aligned}
\mathrm{S}_A^x &= -\left(G(\bar{q}_2^B) + 1 - G(\bar{q}_2^A)\right) g(\bar{q}_1)\frac{\partial \bar{q}_1}{\partial x} - G(\bar{q}_1)g(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial x} - (1 - G(\bar{q}_1))\, g(\bar{q}_2^A)\frac{\partial \bar{q}_2^A}{\partial x} \\
&= -g(\bar{q})\left(\frac{\partial \bar{q}_1}{\partial x} + G(\bar{q})\frac{\partial \bar{q}_2^B}{\partial x} + (1 - G(\bar{q}))\,\frac{\partial \bar{q}_2^A}{\partial x}\right) \\
&= \begin{cases} -2g(\bar{q})\frac{\partial \bar{q}_2^A}{\partial x}, & \text{if } x = f_A, \\ -2\,(1 - G(\bar{q}))\, g(\bar{q})\frac{\partial \bar{q}_2^A}{\partial x}, & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the first equality follows from $\bar{q}_1 = \bar{q}_2^A = \bar{q}_2^B = \bar{q}$ and the second from (8) and (9). This leads to $\mathrm{S}_A^b = (1 - G(\bar{q}))\mathrm{S}_A^f$ and thus corresponds to (10) in case of $n = A$. The same argument applies to $n = B$, yielding the remaining part of (10). **Q.E.D.**

Proof of Lemma 6. We show first that

$$\frac{\partial \bar{q}_1}{\partial f_n} = 2\frac{\partial \bar{q}_1}{\partial b_n} = 2G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial f_n} = 2\left(1 - G(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial f_n}.$$

We know from Lemma 3 that $(\partial \bar{q}_2^B/\partial x, \partial \bar{q}_2^A/\partial x)$ for $x \in \{f_n, b_n\}$ are given by (8) and (9), independent of $(\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B) \in (0,1)^3$.

As in the proof of Lemma 3, we focus on firm A and consider a marginal shift of the advice cutoff $\bar{q}_1$. Differentiating $\bar{q}_1$, defined by (3), with respect to f_A and evaluating it at $\bar{q}_2^A = 1 - \bar{q}_2^B$ yields

$$\frac{\partial \bar{q}_1}{\partial f_A} = G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial f_A} + \left(1 - G(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial f_A} = 2G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial f_A} = 2\left(1 - G(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial f_A},$$

where the second and third equalities follow from both $G(\bar{q}_2^A) = 1 - G(\bar{q}_2^B)$ by symmetry of G and $\partial \bar{q}_2^A/\partial f_A = \partial \bar{q}_2^B/\partial f_A$ by (8). Similarly,

$$\frac{\partial \bar{q}_1}{\partial b_A} = G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial b_A} + \left(1 - G(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial b_A} = \left(1 - G(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial f_A} = G(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial f_A},$$

where the second equality follows from both $\partial \bar{q}_2^B/\partial b_A = 0$ and $\partial \bar{q}_2^A/\partial b_A = \partial \bar{q}_2^A/\partial f_A$ by (9) and the third from both $G(\bar{q}_2^B) = 1 - G(\bar{q}_2^A)$ and $\partial \bar{q}_2^A/\partial f_A = \partial \bar{q}_2^B/\partial f_A$ by (8). Taken together, at $\bar{q}_2^A = 1 - \bar{q}_2^B$ we have $\partial \bar{q}_1/\partial f_A = 2(\partial \bar{q}_1/\partial b_A) = 2G(\bar{q}_2^B)(\partial \bar{q}_2^B/\partial f_A) = 2(1 - G(\bar{q}_2^A))(\partial \bar{q}_2^A/\partial f_A)$, which completes the argument for $n = A$. The same argument applies to $n = B$.

Next, we derive the derivatives of sales. Considering again first firm A, $\mathrm{S}_A^x = \mathrm{Pr}^x(1) +$

$2\mathrm{Pr}_A^x(2)$ for $x \in \{f_A, b_A\}$ can be written as

$$\begin{aligned}
\mathrm{S}_A^x &= -\left(G(\bar{q}_2^B) + 1 - G(\bar{q}_2^A)\right) g(\bar{q}_1)\frac{\partial \bar{q}_1}{\partial x} - G(\bar{q}_1) g(\bar{q}_2^B)\frac{\partial \bar{q}_2^B}{\partial x} - (1 - G(\bar{q}_1))\, g(\bar{q}_2^A)\frac{\partial \bar{q}_2^A}{\partial x} \\
&= -2\left(1 - G(\bar{q}_2^A)\right) g(1/2)\frac{\partial \bar{q}_1}{\partial x} - \frac{1}{2} g(\bar{q}_2^A)\left(\frac{\partial \bar{q}_2^B}{\partial x} + \frac{\partial \bar{q}_2^A}{\partial x}\right) \\
&= \begin{cases} -\left(4\left(1 - G(\bar{q}_2^A)\right)^2 g(1/2) + g(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial f_A}, & \text{if } x = f_A, \\ -\frac{1}{2}\left(4\left(1 - G(\bar{q}_2^A)\right)^2 g(1/2) + g(\bar{q}_2^A)\right)\frac{\partial \bar{q}_2^A}{\partial f_A}, & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the first equality follows from $\bar{q}_1 = 1/2$, $\bar{q}_2^A = 1 - \bar{q}_2^B$, and symmetry of G around $1/2$ with $g(q) = g(1-q)$ for any given $q \in [0,1]$ and the second from (8) and (9), by which

$$\frac{\partial \bar{q}_2^B}{\partial f_A} = \frac{\partial \bar{q}_2^A}{\partial f_A} = \frac{\partial \bar{q}_2^A}{\partial b_A}, \frac{\partial \bar{q}_2^B}{\partial b_A} = 0, \text{ and } \frac{\partial \bar{q}_1}{\partial f_A} = 2\frac{\partial \bar{q}_1}{\partial b_A} = 2(1 - G(\bar{q}_2^A))\frac{\partial \bar{q}_2^A}{\partial f_A}. \tag{A.2}$$

This leads to $S_A^f = 2S_A^b$. Similarly, we can consider firm B and derive $S_B^f = 2S_B^b$. Taken together, we have thus shown that, using symmetry,

$$\mathrm{S}^f = 2\mathrm{S}^b. \tag{A.3}$$

The final step is now to consider again marginal adjustments (df_n, db_n) satisfying (11), so that total sales S_n remain unchanged, i.e., with symmetry $\mathrm{S}^f df + \mathrm{S}^b db = 0$. The assertion follows then immediately from substitution. **Q.E.D.**

Proof of Lemma 7. Consider firm A. Using (A.2) at $\bar{q}_1 = 1/2$ and $\bar{q}_2^A = 1 - \bar{q}_2^B$, we can derive $\mathrm{Pr}^x(1)$ and $\mathrm{Pr}_A^x(2) = \mathrm{Pr}^x(2)$ for $x \in \{f_A, b_A\}$ and $s = 1, 2$ as follows:

$$\begin{aligned}
\mathrm{Pr}^x(1) &= \frac{\partial \bar{q}_1}{\partial x} g(\bar{q}_1)\left(1 - G(\bar{q}_2^B) - G(\bar{q}_2^A)\right) - \frac{\partial \bar{q}_2^B}{\partial x} g(\bar{q}_2^B) G(\bar{q}_1) + \frac{\partial \bar{q}_2^A}{\partial x} g(\bar{q}_2^A)(1 - G(\bar{q}_1)) \\
&= \begin{cases} 0, & \text{if } x = f_A, \\ -\frac{g(\bar{q}_2^A)}{4w}, & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the second equality follows from (i) $1 - G(\bar{q}_2^B) = G(\bar{q}_2^A)$ and $g(\bar{q}_2^B) = g(\bar{q}_2^A)$ as $\bar{q}_2^A = 1 - \bar{q}_2^B$ and G is symmetric around $1/2$ and (ii) $\partial \bar{q}_2^A / \partial f_A = \partial \bar{q}_2^B / \partial f_A = \partial \bar{q}_2^A / \partial b_A$ and $\partial \bar{q}_2^B / \partial b_A = 0$ by (8) and (9) with $G(\bar{q}_1) = G(1/2) = 1/2$;

$$\begin{aligned}
\mathrm{Pr}_A^x(2) &= -\frac{\partial \bar{q}_1}{\partial x} g(\bar{q}_1)\left(1 - G(\bar{q}_2^A)\right) - \frac{\partial \bar{q}_2^A}{\partial x} g(\bar{q}_2^A)(1 - G(\bar{q}_1)) \\
&= \begin{cases} \frac{1}{4w}\left(4g(1/2)\left(1 - G(\bar{q}_2^A)\right)^2 + g(\bar{q}_2^A)\right), & \text{if } x = f_A, \\ \frac{1}{4w}\left(2g(1/2)\left(1 - G(\bar{q}_2^A)\right)^2 + g(\bar{q}_2^A)\right), & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the second equality follows from (i) $\partial\bar{q}_2^A/\partial f_A = \partial\bar{q}_2^A/\partial b_A = -1/(2w)$ and $\partial\bar{q}_1/\partial f_A = 2(\partial\bar{q}_1/\partial b_A) = 2(1-G(\bar{q}_2^A))(\partial\bar{q}_2^A/\partial f_A)$ by (8) and (9) and (ii) $G(\bar{q}_1) = G(1/2) = 1/2$.

Similarly, we can derive the derivatives for firm B with respect to f_B and b_B, and then show that $\mathrm{Pr}_A^x(s) = \mathrm{Pr}_B^x(s) = \mathrm{Pr}^x(s)$ holds for $x \in \{f, b\}$ and $s = 1, 2$. Thus, we have

$$\mathrm{Pr}^f(1) = 0 > \mathrm{Pr}^b(1) = -\frac{g(\bar{q}_2^A)}{4w} = -\frac{g(\bar{q}_2^B)}{4w}$$

and

$$\begin{aligned}\mathrm{Pr}^f(1) + 2\mathrm{Pr}^f(2) &= 2(\mathrm{Pr}^b(1) + 2\mathrm{Pr}^b(2)) \\ &= -\frac{1}{2w}\left(4g(1/2)\left(1-G(\bar{q}_2^A)\right)^2 + g(\bar{q}_2^A)\right) \\ &= -\frac{1}{2w}\left(4g(1/2)\left(G(\bar{q}_2^B)\right)^2 + g(\bar{q}_2^B)\right),\end{aligned}$$

by which we can obtain

$$\mathrm{Pr}^f(2) - 2\mathrm{Pr}^b(2) = \mathrm{Pr}^b(1),$$

leading to (16). **Q.E.D.**

Proof of Lemma 8. We show the uniqueness of the advice cutoff $\bar{q}_2^A = 1/2 - b/(2w) \in (0, 1/2)$ determined by (18). The left-hand-side of the equation (18) is a bijective (or one-to-one) function of $\bar{q}_2^A \in (0, 1/2)$ and converges to $1/2$ in the limit as $\bar{q}_2^A$ approaches $1/2$ from below, while the right-hand-side of (18) is decreasing in $\bar{q}_2^A$ due to the hazard rate condition (1) and converges to $1/2$ in the limit as $\bar{q}_2^A$ goes to zero from above. Taken together, there must be a fixed point $\bar{q}_2^A \in (0, 1/2)$ such that the left-hand- and right-hand sides intersect only once, thus equation (18) holds for a unique value $\bar{q}_2^A \in (0, 1/2)$. **Q.E.D.**

Proof of Lemma 9. We derive the optimal commission given by (20). Recall that, as shown in the proof of Lemma 7, $\mathrm{Pr}^f(1) = 0$ and

$$\mathrm{Pr}^f(2) = \frac{1}{4w}\left(4g(1/2)\left(1-G(\bar{q}_2^A)\right)^2 + g(\bar{q}_2^A)\right). \tag{A.4}$$

With $\mathrm{Pr}(1) + 2\mathrm{Pr}(2) = 1$, the first-order condition with respect to f_A, evaluated at a symmetric equilibrium, can now be transformed stepwise as follows:

$$\begin{aligned} f &= p - c - \frac{1}{2}\left(b + \frac{1}{\mathrm{Pr}^f(2)}\right) \\ &= p - c - \frac{1}{2}\left(4w\frac{G(\bar{q}_2^A)}{g(\bar{q}_2^A)} + \frac{4w}{4g(1/2)\left(1-G(\bar{q}_2^A)\right)^2 + g(\bar{q}_2^A)}\right) \\ &= p - c - 2w\left(\frac{G(\bar{q}_2^A)}{g(\bar{q}_2^A)} + \frac{1}{4g(1/2)\left(1-G(\bar{q}_2^A)\right)^2 + g(\bar{q}_2^A)}\right) \\ &= p - c - 2wH(\bar{q}_2^A).\end{aligned}$$

Q.E.D.

Proof of Lemma 10. We can derive $\Pr^b(1) = -g(\bar{q}_2^A)/(4w)$ and $\Pr^b(2) = (1/(4w))(2g(1/2)(1-G(\bar{q}_2^A))^2 + g(\bar{q}_2^A))$, so that $\mathrm{S}^b = \Pr^b(1) + 2\Pr^b(2) = (1/(4w))(4g(1/2)(1-G(\bar{q}_2^A))^2 + g(\bar{q}_2^A))$. Here, $\Pr(2) = (1/2)(1-G(\bar{q}_2^A))$. With this we can then substitute to obtain

$$b = \frac{\left(4g(1/2)\left(1-G(\bar{q}_2^A)\right)^2 + g(\bar{q}_2^A)\right)(p-c) - 2w\left(1-G(\bar{q}_2^A)\right)}{2g(1/2)\left(1-G(\bar{q}_2^A)\right)^2 + g(\bar{q}_2^A)}, \tag{A.5}$$

where $\bar{q}_2^A = 1/2 - b/(2w) = 1 - \bar{q}_2^B \in (0, 1/2]$. Suppose now that $w \in (w_f^*, w_b^*)$ and that the optimal bonus satisfies (A.5). Firm A's marginal profit with respect to f_A, evaluated at $f_A = 0$, is written as

$$\begin{aligned}
& \mathrm{S}^f(p-c) - \Pr^f(2)b - \Pr(1) - 2\Pr(2) \\
= \;& 2\mathrm{S}^b(p-c) - \left(\Pr^b(1) + 2\Pr^b(2)\right)b - \Pr(1) - 2\Pr(2) \\
= \;& 2\left(\mathrm{S}^b(p-c) - \Pr^b(2)b\right) - \Pr^b(1)b - \Pr(1) - 2\Pr(2) \\
= \;& 2\Pr(2) - \Pr^b(1)b - \Pr(1) - 2\Pr(2) = -\Pr^b(1)b - \Pr(1) < 0,
\end{aligned}$$

where the first equality follows from $\mathrm{S}^f = \Pr^f(1) + 2\Pr^f(2) = 2\mathrm{S}^b$ with $\Pr^f(1) = 0$, and the third one the first-order-condition with respect to b_A. Similarly, if $w \geq w_b^*$, we can show that the marginal profit with respect to f_A, evaluated at $(f_n, b_n) = (0, 0)$ for both $n = A, B$, is negative as

$$\begin{aligned}
& \mathrm{S}^f(p-c) - \Pr(1) - 2\Pr(2) \\
= \;& 2\mathrm{S}^b(p-c) - \Pr(1) - 2\Pr(2) = 2\left(\mathrm{S}^b(p-c) - \Pr(2)\right) - \Pr(1) \\
< \;& -\Pr(1) < 0,
\end{aligned}$$

where the first inequality follows from the fact that the marginal profit with respect to b_A is negative too. At both cutoffs w_f^* and w_b^* the assertion follows as we assume strict quasiconcavity of firm profits in the two instruments. **Q.E.D.**

Proof of Proposition 3. Take first the case with $w < w_f^*$. If there exists a symmetric equilibrium with $f > 0$ and $b > 0$, we know from Lemma 9 that the equilbrium must be unique: Advice cutoffs are uniquely determined and do not depend on p, the price is in turn uniquely determined by these cutoffs, and finally the level of commissions are determined by p. By construction of w_f^*, there is also no equilibrium where $f = 0$; cf. the proof of Lemma 10. From Lemma 10 follows also the unique characterzation when $w \geq w_b^*$

and that in the intermediate range, we must have $f = 0$ and $b > 0$. As noted in the main text, what complicates the characterization in this case is that as b changes with w, so do the advice cutoffs and thus the price (that is, customers' conditional valuation). Still, that the bias must strictly decrease as w increases, follows from the following argument to a contradiction. For this we first rearrange the respective first-order condition for b to obtain, at a symmetric equilibrium,

$$\left(\Pr^b(1) + \Pr^b(2)\right)(p - c) + \Pr^b(2)(p - c - b) - \Pr(2) = 0. \tag{A.6}$$

We now evaluate this at a given (second-customer) cutoff $\bar{q}_2^A$, which, independently of w, also fixes $\bar{q}_2^B$ and p, as well as obviously $\Pr(2)$. Now suppose that (A.6) holds for given w and corresponding equilibrium b. We now consider a strictly lower w'. Evaluating this at the same $\bar{q}_2^A$, note that, first, this requires $b' < b$ and that, second, both $\Pr^b(1)+\Pr^b(2)$ and $\Pr^b(2)$ increase.[33] Invoking strict quasiconcavity of the profit function, this implies that at w', (A.6) is striclty positive when evaluated at b' so that the bias would remain unchanged. The claim follows then by invoking again strict quasiconcavity of the profit function (together with the first-order condition). **Q.E.D.**

Derivations in the Case of a Uniform Distribution. We derive the optimal incentive scheme (f, b) and its associated thresholds (w_f^*, w_b^*) when G is the uniform distribution. By Proposition 2, we can derive a unique advice cutoff $\bar{q}_2^A = 1 - \bar{q}_2^B \in (0, 1/2)$ from (18), which yields

$$\bar{q}_2^A = \frac{1}{2} - 2\bar{q}_2^A,$$

that is, $\bar{q}_2^A = 1/6$ and $\bar{q}_2^B = 1 - \bar{q}_2^A = 5/6$. Applying $\bar{q}_2^A = 1/6$ to (17) leads to the optimal bonus given by

$$b = \frac{2}{3}w.$$

To pin down the optimal commission, we derive the function $H(\bar{q}_2^A)$ defined by (19). With $\bar{q}_2^A = 1/6$, this reduces to

$$H(\bar{q}_2^A) = \bar{q}_2^A + \frac{1}{4(1 - \bar{q}_2^A)^2 + 1} = \frac{22}{51},$$

by which the optimal commission defined by (20) simplifies to

$$f = p - c - \frac{44}{51}w.$$

[33] Note that Pr(1)+Pr(2) is the likelihood that a given firm sells at least one product. The respective derivative is evaluated at a symmetric choice $b_n = b$, but it holds constant the competitor's choice b (as otherwise it would remain unchanged).

As $\mathbf{q} = (\bar{q}_1, \bar{q}_2^A, \bar{q}_2^B) = (1/2, 1/6, 5/6)$, we have

$$\begin{aligned}
\mathbb{E}[v_A \mid \mathbf{q}] &= \frac{1}{2}\left[\mathbb{E}[v_A \mid \bar{q}_1] + G(\bar{q}_1)\mathbb{E}[v_A \mid \bar{q}_2^B] + (1 - G(\bar{q}_1))\,\mathbb{E}[v_A \mid \bar{q}_2^A]\right] \\
&= \frac{1}{2}\left[2\left(\frac{3v_h + v_l}{8}\right) + \frac{1}{2}\left(\frac{3v_h + v_l}{2}\right)\right] = \frac{3v_h + v_l}{4}.
\end{aligned}$$

Thus, in the case of the uniform distribution the equilibrium price is constant and given by

$$p = \frac{3v_h + v_l}{4}.$$

The optimal commission is positive if w is below the threshold defined by (21), which is simplified to

$$w_f^* = \frac{p - c}{2H(\bar{q}_2^A)} = \frac{51}{44}(p - c) = \frac{51}{176}(3v_h + v_l - 4c).$$

Next, we consider the corner solution in which w is both above w_f^* and below w_b^* where now

$$w_b^* = 2(p - c) = \frac{1}{2}(3v_h + v_l - 4c).$$

In this case, the optimal bonus is given by (A.5), which can be written as

$$b = \frac{\left(4(1 - \bar{q}_2^A)^2 + 1\right)(p - c) - 2w(1 - \bar{q}_2^A)}{2(1 - \bar{q}_2^A)^2 + 1}.$$

We have $b = w(1 - 2\bar{q}_2^A)$. Applying this to the optimal bonus, when $f = 0$ but still $b > 0$, we can derive an equilibrium advice cutoff as the solution to the equation

$$w\left(2(1 - \bar{q}_2^A)^2 + 1\right)(1 - 2\bar{q}_2^A) = \left(4(1 - \bar{q}_2^A)^2 + 1\right)(p - c) - 2w(1 - \bar{q}_2^A).$$

Q.E.D.

Proof of Proposition 4. Consider a normal distribution with mean μ and variance $\sigma > 0$. Define the standard normal distribution and its density function by

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}dt$$

and

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2},$$

respectively. With these functions, we write a truncated normal distribution function G with support $[0, 1]$ by

$$G(q) = \frac{\Phi(\frac{q-\mu}{\sigma}) - \Phi(\frac{-\mu}{\sigma})}{\Phi(\frac{1-\mu}{\sigma}) - \Phi(\frac{-\mu}{\sigma})}$$

and its density function g by

$$g(q) = \frac{\phi(\frac{q-\mu}{\sigma})}{\sigma\left(\Phi(\frac{1-\mu}{\sigma}) - \Phi(\frac{-\mu}{\sigma})\right)},$$

where the mean of $G(q)$ is unchanged at $\mu = 1/2$ and and the variance is given by

$$\sigma^2\left[1 - \frac{\mu\phi(\frac{-\mu}{\sigma}) + (1-\mu)\phi(\frac{1-\mu}{\sigma})}{\sigma\left(\Phi(\frac{1-\mu}{\sigma}) - \Phi(\frac{-\mu}{\sigma})\right)} - \left(\frac{\phi(\frac{-\mu}{\sigma}) - \phi(\frac{1-\mu}{\sigma})}{\Phi(\frac{1-\mu}{\sigma}) - \Phi(\ \frac{-\mu}{\sigma})}\right)^2\right].$$

We first show that the inverse of the reverse hazard rate $G(q)/g(q)$ is increasing in σ.

We can write $G(q)/g(q)$ as

$$\begin{aligned}
\frac{G(q)}{g(q)} &= \sigma\frac{\Phi(\frac{q-\mu}{\sigma}) - \Phi(\frac{-\mu}{\sigma})}{\phi(\frac{q-\mu}{\sigma})} \\
&= \sigma\left(\int_{\frac{-\mu}{\sigma}}^{\frac{q-\mu}{\sigma}} e^{-\frac{1}{2}x^2}dx\right)\left(e^{\frac{1}{2}\left(\frac{q-\mu}{\sigma}\right)^2}\right) \\
&= \sigma\left(\int_{\frac{-\mu}{\sigma}}^{\frac{q-\mu}{\sigma}} e^{-\frac{1}{2}\left(x^2-\left(\frac{q-\mu}{\sigma}\right)^2\right)}dx\right) \\
&= \sigma\left(\int_{-\frac{1/2}{\sigma}}^{-\frac{1/2-q}{\sigma}} e^{-\frac{1}{2\sigma^2}\left(x\sigma+\frac{1}{2}-q\right)\left(x\sigma-\frac{1}{2}+q\right)}dx\right) \\
&= \sigma\left(\int_{\frac{1}{2}}^{\frac{1}{2}-q} e^{-\frac{1}{2\sigma^2}\left(y-\frac{1}{2}+q\right)\left(y+\frac{1}{2}-q\right)}\left(-\frac{1}{\sigma}\right)dy\right) \\
&= \int_{\frac{1}{2}-q}^{\frac{1}{2}} e^{-\frac{1}{2\sigma^2}\left(y-\frac{1}{2}+q\right)\left(y+\frac{1}{2}-q\right)}dy,
\end{aligned}$$

where the fourth equation is evaluated at $\mu = 1/2$, the fifth one follows from changing variable x to $y = -\sigma x$, and the integrand in the last equation

$$h(y \mid \sigma) \equiv e^{-\frac{1}{2\sigma^2}\left(y-\frac{1}{2}+q\right)\left(y+\frac{1}{2}-q\right)}$$

is increasing in σ for any given $y \in (\frac{1}{2}-q, 1/2]$ if $q \in (0, 1/2)$ as $(y-1/2+q)(y+1/2-q) > 0$. Note that $h(y \mid \sigma)$ converges to zero in the limit as σ goes to zero from above. Thus, we have

$$\frac{G(q)}{g(q)} = \int_{\frac{1}{2}-q}^{\frac{1}{2}} h(y \mid \sigma)dy,$$

which is increasing in σ for any given $q \in (0, 1/2)$. Using the fact that $G(q)/g(q)$ increases with the size of σ, we now show that in equilibrium as σ increases, the advice cutoff

$\bar{q}_2^A = 1-\bar{q}_2^B$ decreases while the optimal bonus increases. Since $\bar{q}_2^A \in (0,1/2)$ is determined in equilibrium by equation (18), it decreases as $G(\bar{q}_2^A)/g(\bar{q}_2^A)$ increases proportional to σ. Since the optimal bonus can be written as $b/(2w) = 1/2 - \bar{q}_2^A$ with $(f_n, b_n) = (f, b)$ and thus decreases with $\bar{q}_2^A$, b is increasing in σ. **Q.E.D.**

Proof of Proposition 5. It remains to show uniqueness of the regulated equilibrium and its existence. Consider first the case where w is below the threshold of w^* defined by (27). Note that $w_b^* = 2w^*$, where $p = \mathbb{E}[v_n \mid 1/2]$, and w^* is below w_b^*. In equilibrium, customers must hold rational beliefs with $\hat{f}_n = f^R$. Together with the pattern of advice defined by (25), this determines the equilibrium advice cutoffs $\hat{\mathbf{q}} = \mathbf{q} = (\bar{q}, \bar{q}, \bar{q})$ with $\bar{q} = 1/2$, irrespective of the size of w, as well as the price $p_n = \mathbb{E}[v_n \mid \hat{\mathbf{q}}] = \mathbb{E}[v_n \mid \mathbf{q}]$, which is common for both $n = A, B$ due to $\bar{q} = 1/2$ and symmetry of $v_A(q) = v_B(1-q)$ for any given $q \in [0,1]$, and which we denote by p. Firms set their optimal commissions $f_n = f^R$ by (26). These tuple of $(f^R, \mathbf{q}, p)$ with customers' rational beliefs of $\hat{f}_n = f^R$ and $\hat{\mathbf{q}} = \mathbf{q}$ constitutes a unique regulated equilibrium as long as $w < w^*$, otherwise $(0, \mathbf{q}, p)$ would be a unique equilibrium as a corner solution.

The existence of the equilibrium for any given $w > 0$ follows from assumption (24), by which the equilibrium price evaluated at $\mathbf{q} = (1/2, 1/2, 1/2)$ exceeds marginal cost c and therefore selling the product is profitable for the firm with equilibrium price $p = \mathbb{E}[v_n \mid 1/2]$ as

$$\mathbb{E}[v_n \mid 1/2] = \int_{1/2}^{1} v_A(q) \frac{g(q)}{1 - G(1/2)} dq = \int_0^{1/2} v_B(q) \frac{g(q)}{G(1/2)} dq > c.$$

Q.E.D.

Proof of Proposition 6. We will provide a series of lemmas to extend both the optimality of nonlinear incentives and the characterization of the optimal nonlinear incentive scheme to the case with simultaneous advice. Suppose that firms set $b_A = b_B = 0$. In this case, advice cutoffs (q^*, q^{**}) should be equal, which we denote by $\bar{q} \in (0,1)$.[34] Also, advice cutoff $\bar{q}_2(q_1)$ reduces to

$$\bar{q}_2(q_1) = 1 - q_1 - \frac{1}{w}(f_A - f_B).$$

Consider now firm n's marginal profits with respect to the commission and the bonus $x_n \in \{f_n, b_n\}$. We focus on firm A and first examine the effects of the marginal increases in f_A and b_A on the advice cutoffs $(q^*, q^{**}, \bar{q}_2(q_1)) \in (0,1)^2$.

[34] The advice cutoff $\bar{q}$ should lie in the open interval $(0,1)$, as otherwise the advisor would always recommend a particular firm's product to customers, which contradicts assumption (2).

Lemma 13 *For any given* $(q^*, q^{**}, \bar{q}_2(q_1)) \in (0,1)^2$,

$$\frac{\partial q^*}{\partial f_n} = \frac{\partial q^{**}}{\partial f_n} = \frac{1}{2}\frac{\partial \bar{q}_2(q_1)}{\partial f_n} = \begin{cases} -\frac{1}{2w}, & \textit{if } n = A, \\ \frac{1}{2w}, & \textit{if } n = B, \end{cases} \tag{A.7}$$

and

$$\left(\frac{\partial q^*}{\partial b_n}, \frac{\partial q^{**}}{\partial b_n}, \frac{\partial \bar{q}_2(q_1)}{\partial b_n}\right) = \begin{cases} \left(-\frac{1}{2w}, 0, -\frac{1}{2w}\right), & \textit{if } n = A, \\ \left(0, \frac{1}{2w}, \frac{1}{2w}\right), & \textit{if } n = B. \end{cases} \tag{A.8}$$

With (A.7) and (A.8), consider now a marginal increase in sales, $\mathrm{S}_n^x = \Pr^x(1) + 2\Pr_n^x(2)$, with $x \in \{f_n, b_n\}$.

Lemma 14 *At* $\bar{q} = q^* = q^{**} \in (0,1)$*, equation (10) holds true.*

Proof. We derive (10) using (A.7) and (A.8) in the case of $b_n = b = 0$. For now we restrict attention to firm A. By (A.7) and (A.8), $\mathrm{S}_A^x = \Pr^x(1) + 2\Pr_A^x(2)$ for $x \in \{f_A, b_A\}$ can be written as

$$\begin{aligned} \mathrm{S}_A^x &= 2\left[\begin{array}{c} -\left(1 - G(\bar{q}_2(q^*))\right) g(q^*)\frac{\partial q^*}{\partial x} - G(\bar{q}_2(q^{**}))g(q^{**})\frac{\partial q^{**}}{\partial x} \\ -\int_{q^*}^{q^{**}} g(\bar{q}_2(q_1))g(q_1)dq_1\frac{\partial \bar{q}_2(q_1)}{\partial x} \end{array}\right] \\ &= -2g(\bar{q})\left(\left(1 - G(\bar{q}_2(\bar{q}))\right)\frac{\partial q^*}{\partial x} + G(\bar{q}_2(\bar{q}))\frac{\partial q^{**}}{\partial x}\right) \\ &= \begin{cases} -2g(\bar{q})\frac{\partial q^*}{\partial f_A}, & \text{if } x = f_A, \\ -2\left(1 - G(\bar{q})\right) g(\bar{q})\frac{\partial q^*}{\partial f_A}, & \text{if } x = b_A, \end{cases} \end{aligned}$$

where the first equality follows from $\bar{q} = q^* = q^{**}$ and the second from (i) $\partial q^*/\partial f_A = \partial q^{**}/\partial f_A$ due to (A.7), (ii) $\partial q^*/\partial b_A = \partial q^*/\partial f_A$ and $\partial q^{**}/\partial b_A = 0$ due to (A.8), and (iii) $\bar{q}_2(\bar{q}) = \bar{q}$. Thus, we obtain equation (10) in case of $n = A$. The same argument applies to $n = B$, leading to the remaining part of (10). **Q.E.D.**

Consider now marginal adjustments $(df_n, db_n) \in \mathbb{R}^2$ such that total sales remain unchanged, as in (11). Applying (10) to (11), we have:

Lemma 15 *For* $n = A, B$*, consider marginal adjustments* $(df_n, db_n) \in \mathbb{R}^2$ *as defined by (11). If* $b_n = b = 0$*,* (df_n, db_n) *must satisfy equation (12).*

We next examine the total derivative of π_n with respect to the marginal adjustments (df_n, db_n) such that total sales remain unchanged as (12) holds. Taking $b_n = b = 0$ as given, the total derivative can be written as (13), which gives rise to:

Proposition 7 *Nonlinear incentives are part of any equilibrium, i.e., there is no equilibrium in which* $b_n = b = 0$.

Suppose that firms set their compensation $(f_n, b_n) = (f, b)$ with $0 < b < w$. Under symmetric compensation (f, b), the advice cutoffs (q^*, q^{**}) satisfy $q^* = 1 - q^{**} \in (0, 1/2)$, and so

$$\begin{aligned}
\Pr_A(2) &= (1 - G(q^*))(1 - G(q^{**})) + \int_{q^*}^{q^{**}} (1 - G(\bar{q}_2(q_1)))g(q_1)dq_1 \\
&= G(q^{**})G(q^*) + \int_{q^*}^{q^{**}} G(q_1)g(q_1)dq_1 \\
&= G(q^{**})G(q^*) - G(q_1)\left(1 - G(q_1)\right)\Big|_{q^*}^{q^{**}} + \int_{q^*}^{q^{**}} \left(1 - G(q_1)\right) g(q_1)dq_1 \\
&= G(q^{**})G(q^*) + \int_{q^*}^{q^{**}} G(\bar{q}_2(q_1))g(q_1)dq_1 = \Pr_B(2)
\end{aligned}$$

where the first equality follows from the definition of $\Pr_A(2)$, the second from $\bar{q}_2(q_1) = 1 - q_1$ at (f, b) and $G(\bar{q}_2(q_1)) = 1 - G(q_1)$ with symmetry of G, the third from integration by parts, the fourth from $G(q_1)\left(1 - G(q_1)\right)|_{q^*}^{q^{**}} = 0$ and $G(\bar{q}_2(q_1)) = 1 - G(q_1)$, and the last from the definition of $\Pr_B(2)$. Since $\Pr_A(2) = \Pr_B(2)$, we simply denote them by $\Pr(2)$. Furthermore, using

$$\int_{q^*}^{q^{**}} G(q_1)g(q_1)dq_1 = \frac{1}{2}\left((G(q^{**}))^2 - (G(q^*))^2\right),$$

we can rewrite

$$\Pr(2) = \frac{1}{2}\left(1 - 2(G(q^*))^2\right).$$

Lemma 16 *At* $q^* = 1 - q^{**} \in (0, 1/2)$, *equation (A.3) holds true.*

Proof. We derive (A.3) using the derivatives of (q^*, q^{**}) given by (A.7) and (A.8) when $(f_n, b_n) = (f, b)$. For now we restrict attention to firm A. With $q^* = 1 - q^{**}$, $\mathrm{S}_A^x = \Pr^x(1) + 2\Pr_A^x(2)$ for $x \in \{f_A, b_A\}$ can be written as

$$\begin{aligned}
& 2\left[\begin{array}{c} -\left(1 - G(\bar{q}_2(q^*))\right) g(q^*)\frac{\partial q^*}{\partial x} - G(\bar{q}_2(q^{**}))g(q^{**})\frac{\partial q^{**}}{\partial x} \\ -\int_{q^*}^{q^{**}} g(\bar{q}_2(q_1))g(q_1)dq_1\frac{\partial \bar{q}_2(q_1)}{\partial x} \end{array}\right] \\
= & -2\left(g(q^*)G(q^*)\left(\frac{\partial q^*}{\partial x} + \frac{\partial q^{**}}{\partial x}\right) + \int_{q^*}^{q^{**}} (g(q_1))^2 dq_1 \frac{\partial \bar{q}_2(q_1)}{\partial x}\right) \\
= & \begin{cases} -4\frac{\partial q^*}{\partial f_A}\left(g(q^*)G(q^*) + \int_{q^*}^{q^{**}} (g(q_1))^2 dq_1\right), & \text{if } x = f_A, \\ -2\frac{\partial q^*}{\partial f_A}\left(g(q^*)G(q^*) + \int_{q^*}^{q^{**}} (g(q_1))^2 dq_1\right), & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the first equality follows from $q^* = 1 - q^{**}$, $\bar{q}_2(q_1) = 1 - q_1$, and symmetry of G around 1/2 with $g(q) = g(1-q)$ for any given $q \in [0,1]$ and the second from both (i) $\partial q^*/\partial f_A = \partial q^{**}/\partial f_A = (1/2)(\partial \bar{q}_2(q_1)/\partial f_A)$ and (ii) $\partial q^*/\partial b_A = \partial q^*/\partial f_A$, $\partial q^{**}/\partial b_A = 0$, and $\partial \bar{q}_2(q_1)/\partial b_A = \partial q^*/\partial f_A$. This leads to $S_A^f = 2S_A^b$. By symmetry, $S_B^f = 2S_B^b$ holds true. Thus, we have derived (A.3). **Q.E.D.**

Consider now marginal adjustments (df_n, db_n) defined by (11) such that total sales remain unchanged. As we have symemtric incentive schemes (f, b) with $0 < b < w$, we omit subscript $n = A, B$. As in Lemma 6 we then have $df = -(1/2)db$. We use next that $\Pr(1) = 2G(q^*)(1 - G(q^{**})) = 2(G(q^*))^2$, $\Pr(2) = (1/2)(1 - \Pr(1))$, and S $= \Pr(1) + 2\Pr(2) = 1$ by symmetry, and

$$\Pr^f(2) - 2\Pr^b(2) = -G(q^*)\frac{g(q^*)}{w}.$$

Applying this, next to $df = -(1/2)db$, to the total derivative of π, we finally have that

$$d\pi = \frac{G(q^*)}{2}\left(-\frac{g(q^*)}{w}b + 2G(q^*)\right)db.$$

Lemma 17 *For any given $q^* \in (0, 1/2)$, the optimal bonus is uniquely determined by*

$$b \;=\; 2\frac{G(q^*)}{g(q^*)}w. \tag{A.9}$$

Proof. We focus on firm A. Using (A.7) and (A.8) with $q^* = 1 - q^{**}$, we derive $\Pr^x(1)$ and $\Pr_n^x(2) = \Pr^x(2)$ for $x \in \{f_A, b_A\}$ as follows:

$$\begin{aligned}
\Pr^x(1) &= 2\left(g(q^*)\left(1 - G(q^{**})\right)\frac{\partial q^*}{\partial x} - g(q^{**})G(q^*)\frac{\partial q^{**}}{\partial x}\right) \\
&= \begin{cases} 2g(q^*)G(q^*)\left(\frac{\partial q^*}{\partial f_A} - \frac{\partial q^{**}}{\partial f_A}\right) = 0, & \text{if } x = f_A, \\ 2g(q^*)G(q^*)\frac{\partial q^*}{\partial b_A} = -G(q^*)\frac{g(q^*)}{w}, & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the second equality follows from (i) $1 - G(q^{**}) = G(q^*)$, (ii) $g(q^*) = g(q^{**})$, and (iii)

$\partial q^*/\partial f_A = \partial q^{**}/\partial f_A = \partial q^*/\partial b_A = -1/(2w)$ and $\partial q^{**}/\partial b_A = 0$ due to (A.7) and (A.8);

$$\begin{aligned}
\Pr_A^x(2) &= -\left[(1-G(q^{**})) + (1-G(\bar{q}_2(q^*)))\right] g(q^*)\frac{\partial q^*}{\partial x} \\
&\quad -\left[(1-G(q^{**})) + (1-G(\bar{q}_2(q^{**})))\right] g(q^{**})\frac{\partial q^{**}}{\partial x} \\
&\quad -\int_{q^*}^{q^{**}} g(\bar{q}_2(q_1))g(q_1)dq_1\frac{\partial \bar{q}_2(q_1)}{\partial x} \\
&= -\left(2G(q^*)g(q^*)\frac{\partial q^*}{\partial x} + \int_{q^*}^{q^{**}}(g(q_1))^2 dq_1\frac{\partial \bar{q}_2(q_1)}{\partial x}\right) \\
&= \begin{cases} -2\frac{\partial q^*}{\partial f_A}\left(G(q^*)g(q^*) + \int_{q^*}^{q^{**}}(g(q_1))^2 dq_1\right), & \text{if } x = f_A, \\ -\frac{\partial q^*}{\partial f_A}\left(2G(q^*)g(q^*) + \int_{q^*}^{q^{**}}(g(q_1))^2 dq_1\right), & \text{if } x = b_A, \end{cases}
\end{aligned}$$

where the second equality follows from $q^* = 1 - q^{**}$, $\bar{q}_2(q_1) = 1 - q_1$, and symmetry of G with $g(q) = g(1-q)$ for any given $q \in [0,1]$ and the third from both (i) $\partial q^*/\partial f_A = \partial q^{**}/\partial f_A = (1/2)(\partial \bar{q}_2(q_1)/\partial f_A)$ due to (A.7) and (ii) $\partial q^*/\partial b_A = \partial q^*/\partial f_A$, $\partial q^{**}/\partial b_A = 0$, and $\partial \bar{q}_2(q_1)/\partial b_A = \partial q^*/\partial f_A$ due to (A.8).

Similarly, we can derive the same derivatives for firm B with respect to f_B and b_B, and then $\Pr_A^x(s) = \Pr_B^x(s) = \Pr^x(s)$ for any given $x \in \{f, g\}$ and $s = 1, 2$. Thus, we have

$$\Pr^f(1) = 0 > \Pr^b(1) = -G(q^*)\frac{g(q^*)}{w}$$

and

$$\Pr^f(1) + 2\Pr^f(2) = 2(\Pr^b(1) + 2\Pr^b(2)) = \frac{2}{w}\left(G(q^*)g(q^*) + \int_{q^*}^{q^{**}}(g(q_1))^2 dq_1\right),$$

by which $\Pr^f(2) - 2\Pr^b(2) = \Pr^b(1)$, leading to (A.9). **Q.E.D.**

Combining condition (A.9) with $q^* = 1/2 - b/(2w)$ under symmetric compensation (f, b), as in the proof of Lemma 8, we can pin down q^* as follows:

Lemma 18 *The advice cutoff $q^* = 1 - q^{**} \in (0, 1/2)$ is uniquely determined by*

$$q^* = \frac{1}{2} - \frac{G(q^*)}{g(q^*)}.$$

Since $\Pr^f(1) = 0$ as shown in the proof of Lemma 17 and $\Pr(1) + 2\Pr(2) = 1$ at a symmetric equilibrium, we use also that b is given by (A.9) and that

$$\Pr^f(2) = \frac{1}{w}\left(G(q^*)g(q^*) + \int_{q^*}^{q^{**}}(g(q_1))^2 dq_1\right).$$

Define $H(q^*)$ by (28). Applying both (A.9) and $\Pr^f(2)$, together with $H(q^*)$, determines the optimal commission.

Lemma 19 *At a symmetric equilibrium with* $q^* = 1 - q^{**} \in (0, 1/2)$ *determined by* $q^* = 1/2 - G(q^*)/g(q^*)$*, the optimal commission is given by*

$$f \;=\; p - c - wH(q^*), \tag{A.10}$$

where $H(q^*)$ *is defined by (28).*

Proof. We derive the optimal commission given by (20). From the proof of Lemma 7, we know that $\Pr^f(2) = (1/w)(G(q^*)g(q^*) + \int_{q^*}^{q^{**}} (g(q_1))^2 dq_1)$. Applying (A.9) and $\Pr^f(2)$, together with $H(q^*)$ defined by (28), yields

$$\begin{aligned} f &= p - c - \frac{1}{2}\left(2w\frac{G(q^*)}{g(q^*)} + \frac{w}{G(q^*)g(q^*) + \int_{q^*}^{q^{**}} (g(q_1))^2 dq_1}\right) \\ &= p - c - w\left(\frac{G(q^*)}{g(q^*)} + \frac{1}{2(G(q^*)g(q^*) + \int_{q^*}^{q^{**}} (g(q_1))^2 dq_1)}\right) \\ &= p - c - wH(q^*), \end{aligned}$$

which leads to (A.10). **Q.E.D.**

One can easily observe that the optimal commission (A.10) is positive as long as w is below a certain level, otherwise equals zero. Denote by w_f^* the respective threshold, which is given by

$$w_f^* = \frac{p - c}{H(q^*)} \tag{A.11}$$

where q^* is uniquely determined by (29). We can now characterize the optimal nonlinear incentive scheme in an interior solution.

Proposition 8 *Suppose that prices are symmetric and the advisor's concern level* w *is below the threshold* w_f^* *defined by (A.11). Then, there are unique advice cutoffs* (q^*, q^{**}) *that are independent of* w*. Precisely,* $q^* = 1 - q^{**}$ *is determined by (29), and* (f, b) *solve (A.10) and (A.9), respectively. The commission* f *decreases with the size of* w *while the bonus* b *increases.*

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Nonlinear incentives and advisor bias

Abstract
We analyze firms' competition to steer an advisor's recommendations through potentially non-linear incentives. Even when firms are symmetric, so that the overall size of compensation would not distort advice when incentives were linear, advice is biased when firms are allowed to make compensation non-linear, which they optimally do. Policies that target an advisor's liability are largely ineffective, as firms react to such increased liability by making incentives even steeper, increasing bonus payments while reducing the linear (commission) part at the same time. This observation may justify policymakers' direct interference with firms' compensation practice, as frequently observed notably in consumer finance.

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