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Martin Geiger, Richard Hule

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Contact address of the editor:
Research platform "Empirical and Experimental Economics"
University of Innsbruck
Universitaetsstrasse 15
A-6020 Innsbruck
Austria
Tel: + 43 512 507 7171
Fax: + 43 512 507 2970
E-mail: eeecon@uibk.ac.at

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Correlation and Coordination Risk*

Martin Geiger[†] Richard Hule[‡]

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Abstract

We study the potential role of correlated refinancing abilities among different countries for the disruption of government bond markets in a currency union. Following Morris and Shin (2004) we use a global games framework and model the simultaneous investment decision into two assets, which are subject to correlated fundamental states, as a coordination problem with correlated imperfect information. Based on this model we evaluate the role of information about one country for the coordination of creditors of another country. We find, however, that the contagious effects on the price of debt precipitated through correlation are modest. Hence, assuming that investors behave as modeled in the global game, we conclude that correlated fundamentals that precipitate informational spillovers appear to be unlikely to play a major role for e.g. the disruption of some Eurozone government bond markets in the aftermath of the recent financial and economic crisis.

Keywords: Government bond refinancing, global games, creditor coordination, currency union

JEL codes: D82, G12

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[†]Department of Economics, University of Innsbruck, Universitätsstrasse 15, A-6020 Innsbruck, Austria, Phone: +43 (0)512 507 71024, E-Mail: Martin.Geiger@uibk.ac.at

[‡]Department of Economics, University of Innsbruck, Universitätsstrasse 15, A-6020 Innsbruck, Austria, Phone: +43 (0)512 507 71020, E-Mail: Richard.Hule@uibk.ac.at

1 Introduction

News about one country can affect the refinancing costs of another one. This particularly applies to bonds of countries that share common characteristics and strong ties as it is the case for Eurozone members, and even more so, for the South-West Eurozone Periphery, namely Greece, Ireland, Italy, Portugal and Spain (see e.g. Metiu, 2012; Beetsma et al., 2013; Mink and de Haan, 2013; Giordano et al., 2013). Spreads of these countries do not only depend on country-specific characteristics[1] and an international risk factor[2], but also on news about foreign countries[3]. Such interdependencies are also a major motivation for European policies such as financial assistance measures for countries that experienced a so-called sovereign debt crisis to calm the markets and to prevent 'contagion'.[4] Contagion might be particularly disruptive in the Eurozone since the introduction of the common currency has led to high degrees of convergence and co-movement of government bond yields in the Eurozone.[5]

Why should yields in a currency union be correlated? And more specifically, why should news about one country affect the refinancing conditions of another country? On the demand side, investors have to estimate the probability of default of a single country in order to price the bond. If the fundamental states underlying the refinancing ability of different countries are correlated, investors will use information about one country to estimate the probability of default of the other country. From this point of view, it is plausible that the correlation of yields in government bond markets in the Eurozone are associated with correlation of fundamental states and correlation of news about them.[6]

In this paper we discuss an investment model that allows us to study the coordination of agents processing correlated information about assets that are subject to correlated fundamen-

[1]Country characteristics that potentially affect spreads are e.g. outstanding government debt and government deficits, inflation, the current account, the real effective exchange rate, economic growth (Aizenman et al., 2013; De Grauwe and Ji, 2013).

[2]It is quite common to use the VIX index that is representing the implied volatility of S&P 500 stock market index options, to proxy global risk aversion (De Santis, 2012; Beetsma et al., 2013).

[3]In the literature, news about other countries is e.g. proxied by changes in sovereign ratings (De Santis, 2012) or events associated with high price fluctuations (Metiu, 2012; Mink and de Haan, 2013).

[4]Examples are the Outright Monetary Transactions (OMT) Program and the establishment of the European Stability Mechanism (ESM).

[5]Pagano and von Thadden (2004) as well as Manganelli and Wolswijk (2009) argue that as the Euro was introduced, the exchange rate risk more or less vanished and identify this as a the major driver of convergence of government bond yields. Sims (2012), on the other hand, argues, that the provisions (treating bonds of European countries similarly) for collateral that is accepted by the ECB are the major cause for the convergence of bond yields.

[6]There are several reasons which may increase correlation due to the monetary union. Firstly, monetary policy is delegated to a central authority and the option of nominal exchange rate devaluation is no longer available. Secondly, the institutional architecture of currency unions generates a number of contingent liabilities among the participating countries. Thirdly, it is often argued that business cycle correlation increases once countries join a currency union (Frankel and Rose, 1998; de Haan et al., 2008; Enders et al., 2013).

tal states. We follow the seminal contribution by Morris and Shin (2004), who model debt refinancing as a coordination game using the global games approach to get a unique equilibrium.[7] We adapt the Morris and Shin (2004) model to the case of sovereign debt refinancing in currency unions. Thus, we introduce a two-asset coordination game and study the simultaneous investments decision into two assets, A and B, that are subject to correlated fundamentals. The intuition of the model is that correlation of the fundamental states implies that signals about one asset may also be informative about the other asset and vice versa. This way our set up incorporates informational interdependencies. Moreover, the extension of the Morris and Shin (2004) framework to a second dimension constitutes a novel methodological contribution to the literature.

Using our model we analyse how information about one country affects the price of debt of the other country and vice versa. This allows us to assess the potential role of correlated fundamental states for the disruption of government bond markets during the recent European sovereign debt crisis. Specifically, our model depicts a specific channel of simultaneous contagious effects of information in the context of financial markets.[8] It does not capture direct spillovers such as e.g. wealth effects transmitted through a common creditor or direct linkages among fundamental states that become active once a country defaults (see e.g. Goldstein and Pauzner, 2004; Trevino, 2015). Spillovers of this kind are discussed in a complementary literature where spillovers are typically modeled as sequential games. E.g., Manz (2002, 2010) studies contagion among firms that are subject to common fundamentals. The performance of firms affects other firms by altering the behavior of creditors. Goldstein and Pauzner (2004) elaborate a model of self-fulfilling financial crises. Two countries exhibit independent fundamentals and a group of investors is engaged in both countries. Since the occurrence of a crisis in one country reduces the wealth of investors, the investment behavior of these investors with respect to the second country is affected by the events in the first country. As a consequence, a crisis in the second country becomes more likely. Similarly, Dasgupta (2004) focuses on capital relations among banks. Goldstein and Pauzner (2005) discuss the mutual repercussions of interdepen-

[7] Global games were first studied by Carlsson and van Damme (1993) and further popularized by Morris and Shin (1998), who applied the global games refinement in a macroeconomic context. They can be applied to wide range of decision problems where coordination risk and incomplete information is involved. In such settings, the agents' payoffs depend on the actions of others as well as on an economic fundamental which is not perfectly observable.

[8] There is not one single unambiguous definition of contagion but rather disagreement on which interdependencies and spill-overs qualify as contagion and which do not. Most papers, however, agree that the transmission of a shock through non-traditional channels such as trade, banking relations, or investment flows constitutes contagion (see e.g. Forbes (2012) for a comprehensive discussion of the literature). In this broad sense, the informational interdependencies we investigate in this this paper may be referred to contagion.

dencies between banking and currency crises. These studies, however, avoid the difficulty of two-dimensional thresholds by setting up the game sequentially and solving it with backward induction.

To evaluate the effects of contagion we consider the ex ante probability of default of the assets, which determines the price of debt in our model. The asset defaults if the fundamental state is not large enough to sustain partial foreclosure. Assuming that agents play switching strategies, we get conditions for the critical fundamental state, which is in our case a function dividing the two-dimensional fundamental state space into a solvency and a default region. We show that, under weak assumptions on the parameters, the equilibrium in switching strategies is a Nash equilibrium. Given the function for the critical fundamental state and the probability density of the fundamental states one can calculate the probability of default, and hence, the price of debt. To evaluate the contagious effects we consider the changes in the price of debt of one asset caused by changes in (*i*) public signals, in (*ii*) the amount of correlation of the fundamental states and private signals, and (*iii*) in the precision of the public and private signals. The somewhat surprising result of the analysis is that the contagious effects transmitted through correlated fundamentals of one asset onto another are generally very moderate. This does not mean, however, that agents do not use the additional information about the asset. Rather, the contagious effect through optimal processing of correlated information on the price of debt is small.

Supporting this intuition, our first main observation is that, ceteris paribus, a change in the public signal about asset B has no effect on the price of debt of asset A in our setting. The reason is that a change in the public signal about asset B shifts both, the critical fundamental state function as well as the distribution of the fundamental states leaving the mass of the distribution located in the default region constant.

Our second main observation is that under certain circumstances, i.e. when precision of signals are identical for both assets and the correlation is the same for the fundamental states and private information, information about one asset does not influence the agents' estimate of the default risk of the other asset. In such cases, the agent optimally behaves as if she does not consider information about the second asset. Notably, for this special case, the two-dimensional decision problem boils down to the one-dimensional case discussed in Morris and Shin (2004) because B-dimensional parameters cancel from the investment strategy. Hence, the critical fundamental state function which divides the fundamental state space into solvency and default regions for one asset is constant in the public signal and in the informational precision of the other asset, and it is constant in the correlation of fundamental states and private information.

The third main observation is that for cases that do not represent the special case, changes in the correlation of the fundamental state and in the correlation of private signals as well as changes in the precision of private and public signals about asset B do exert effects on the price of debt of asset A, but the effects are modest. In numerical experiments, the maximum change in the price of debt precipitated by changes in the correlations is not more than 0.16 percentage points and changes in the informational precision about asset B do not affect the price of asset A by more than 0.07 percentage points. In contrast, changes in precision of information about asset A are much more decisive and can trigger up to 25 percentage point variation in the price of debt of asset A for the numerical experiments we conduct.

Overall, our results lead us to the conclusion that contagion of the kind of informational spillovers captured by our model are not likely to explain a large portion of risk premia for government bonds of currency union members. Thus, we conclude that these spillovers only played a minor role for disruption of government bond markets that occurred in the wake of the recent economic and financial crisis. Evidently, this indicates that while the rational processing of correlated information appears to be unlikely to cause contagious disruption in these markets, one may have to consider other disruptive sources such as direct spillovers, e.g. transmitted through wealth effects, or non-rational behavior involved in the coordination of creditors.

The remainder of the paper is organized as follows: First, we present the two-dimensional model in Section 2 and discuss the equilibrium and the existence of a unique equilibrium in our setting in Section 3. In Section 4 we present some analytical results and we discuss a number of numerical experiments to quantify the effects of informational contagion in Section 5. Section 6 concludes the paper.

2 The Model

In our model, agents face idiosyncratic uncertainty about economic fundamentals which determine the willingness of governments to pay creditors. In an individual decision problem, payoffs are determined by one's own actions and the state of the world. Hence, when the agent receives a message which rules out some states of the world, she can simply disregard these states of the world. The same does not apply to a setting where payoffs are conditional on both, fundamentals as well as the beliefs of others, as it is the case in the type of coordination problem we model: "Since my payoff depends on your actions and your actions are motivated by your beliefs, I care about the range of possible beliefs you may hold (Morris and Shin, 2001)".

We apply this very general idea of a coordination problem with uncertain fundamentals to the case where the agent faces the investment decision into two assets where fundamentals are correlated. We assume that agents are risk neutral and maximize their expected payoff, i.e. the total expected payoff is the sum of the expected payoff of each asset. Since we are primarily interested in the contagious effects of information, we rule out direct spillovers, i.e., the payoff of one asset does not depend on the agents' decision on the other asset and the payoff of one asset is independent of the realization of default or solvency of the other asset.

At an interim stage, creditors can review their investment; i.e. they decide whether or not to rollover. Following Morris and Shin (2004), the face value of the investments is fixed and normalized to 1. But creditors only receive the face value when the debtor is solvent and willing to pay at maturity. The values of the assets at maturity, v_A and v_B, determine the willingness to pay creditors. They depend on an unobservable fundamental state $\theta = (\theta_A, \theta_B)$ and on the portion of creditors who foreclose, l_A and l_B, weighted by a measure for the severity of disruption of partial foreclosure which we denote z_A and z_B:

$$v_A(\theta_A, l_A) = \begin{cases} \geq 1 & \text{if} \quad l_A z_A \leq \theta_A \\ 0 & \text{if} \quad l_A z_A > \theta_A \end{cases}, \; v_B(\theta_B, l_B) = \begin{cases} \geq 1 & \text{if} \quad l_B z_B \leq \theta_B \\ 0 & \text{if} \quad l_B z_B > \theta_B \end{cases}.$$

The debtor is solvent in case the value is at least 1. In case of solvency, creditors are paid back in full.

2.1 The Two-Asset Game: SB2

We now define the corresponding two asset coordination game SB2. A continuum of creditors invests into two assets, asset A and asset B. At the interim stage, creditors can either seize a collateral (i.e. the outside option), λ_A and $\lambda_B \in (0, 1)$, which is strictly larger than 0 and strictly lower than the face value of the asset, or rollover. The face value of the asset is 1 in case of solvency and 0 otherwise. For the game SB2, payoffs for one player for the respective combinations of actions and outcomes are shown in Table 1. The payoff for project A, u_A, is the first entry in the sum of each cell, while the payoff for project B, u_B, is the second one.

The information structure for the game is as follows: The unobservable fundamental state θ is distributed with density g_θ and mean y. All agents know the mean y, i.e. receive a common public signal, $y = (y_A, y_B)$. In addition, each agent gets an idiosyncratic private signal, $x_i = \left(x_A^i, x_B^i\right)$ with probability density f_x. Private signals x_i are independently distributed with density f_x and mean θ.

All agents know the distribution of θ and that of the signals x_i.
We assume a the bivariate normal distribution for the distribution of the fundamental state and private signals, because they are conjugate, incorporate correlation, and are intuitively appealing to model the case of informational dispersion. Hence, we assume that agents know that θ is normally distributed with known mean y and known covariance matrix Σ_{public}.

$$\theta = \begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix} \sim N\left(\begin{pmatrix} y_A \\ y_B \end{pmatrix}, \Sigma_{public}\right)$$

with covariance matrix

$$\Sigma_{public} = \begin{pmatrix} \frac{1}{\alpha_A} & \frac{\rho_{public}}{\sqrt{\alpha_A}\sqrt{\alpha_B}} \\ \frac{\rho_{public}}{\sqrt{\alpha_A}\sqrt{\alpha_B}} & \frac{1}{\alpha_B} \end{pmatrix},$$

and

$$x_i = \begin{pmatrix} x_A^i \\ x_B^i \end{pmatrix} \sim N\left(\begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix}, \Sigma_{private}\right)$$

with covariance matrix

$$\Sigma_{private} = \begin{pmatrix} \frac{1}{\beta_A} & \frac{\rho_{private}}{\sqrt{\beta_A}\sqrt{\beta_B}} \\ \frac{\rho_{private}}{\sqrt{\beta_A}\sqrt{\beta_B}} & \frac{1}{\beta_B} \end{pmatrix}.$$

The precisions of the information about asset A and B are denoted α_A and α_B (precision of the public signals) β_A and β_B (precision of the private signals), ρ_{public} denotes correlation of the fundamental states and $\rho_{private}$ is the correlation of the private signals.

We assume that agents update their information by computing the conditional distribution of the fundamental state θ given their private signal x_i.[9] The density of θ conditional on x_i is notated as $g_{\theta|x_i}$.

$$\theta|x_i \sim N\left(\begin{pmatrix} \xi_A^i \\ \xi_B^i \end{pmatrix}, \Sigma_{conditional}\right)$$

[9]This is equivalent to using Bayes rule. For a discussion of Bayes rule applied to bivariate normal distributions see Lehmann and Casella (1998) and DeGroot (2004).

where the mean of θ conditional on x_i is:

$$\begin{pmatrix} \xi_A^i \\ \xi_B^i \end{pmatrix} = \left(\Sigma_{public}^{-1} + \Sigma_{private}^{-1}\right)^{-1} \left(\Sigma_{public}^{-1} y + \Sigma_{private}^{-1} x_i\right)$$

and the conditional covariance matrix of θ is

$$\Sigma_{conditional} = \left(\Sigma_{public}^{-1} + \Sigma_{private}^{-1}\right)^{-1}.$$

2.2 Best Response in the game SB2 and the One-Asset-Game SB1$_\mathbf{A}$

A strategy for player i is a decision rule which maps each realization of the private signal x_i to an action for asset A, $a_A^i(x_i)$ and to an action for asset B, $a_B^i(x_i)$. In other words, actions are conditional on the player's type, that is determined by the private signal.

$$s_i = \left(a_A^i(x_i), a_B^i(x_i)\right) \text{ where } a_A^i, a_B^i \in \{0, 1\}.$$

The fraction of foreclosers is determined by the investment strategy (depending on the idiosyncratic signal x, which is informative about the fundamental state θ). We assume symmetric behavior of the investors, i.e. they follow identical investment strategies a_A and a_B. Because of the law of large numbers the portion of creditors that foreclose is:

$$l_A = 1 - E\left(1_{a_A}(x)\right) = 1 - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_x\left(x_A, x_B, \theta_A, \theta_B\right) 1_{a_A}(x) \mathrm{d}x_A \mathrm{d}x_B,$$
$$l_B = 1 - E\left(1_{a_B}(x)\right) = 1 - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_x\left(x_A, x_B, \theta_A, \theta_B\right) 1_{a_B}(x) \mathrm{d}x_A \mathrm{d}x_B,$$

where

$$1_{a_A}(x) = \begin{cases} 1 & \text{if} \quad a_A(x) = 1 \\ 0 & \text{if} \quad a_A(x) = 0 \end{cases}, \text{ and } 1_{a_B}(x) = \begin{cases} 1 & \text{if} \quad a_B(x) = 1 \\ 0 & \text{if} \quad a_B(x) = 0 \end{cases}.$$

Given our assumptions it is quite obvious that the game SB2 can be analysed by looking at each asset separately. Therefore, we introduce the games SB1$_A$ and SB1$_B$, which consider only the separate investments into asset A and asset B. The setup of SB1$_A$ is exactly the same as that of SB2, i.e. there are two countries, and the information structure is identical. The only difference is that the agent holds only the asset A (and asset B for SB1$_A$ respectively) and decides whether or not to roll over or foreclose on the asset. The corresponding payoffs are shown in Table 2.

Proposition 1. *The strategies $a^*_{i,A}$ and $a^*_{i,B}$ are the Nash equilibrium strategies for the game SB2 if and only if $a^*_{i,A}$ is a Nash equilibrium strategy for the game $SB1_A$ and $a^*_{i,B}$ is a Nash equilibrium strategy for the game $SB1_B$.*

In a Nash equilibrium of SB2, player i invests if and only if the expected payoff from investing exceeds the expected payoff from not investing into the respective asset:

$$a^*_{i,A}(x_i) = 1 \text{ if } E(u_A(1, l_A, x_i)) \geq E(u_A(0, l_A, x_i)), \text{ else } a^*_{i,A}(x_i) = 0,$$
$$a^*_{i,B}(x_i) = 1 \text{ if } E(u_B(1, l_B, x_i)) \geq E(u_B(0, l_B, x_i)), \text{ else } a^*_{i,B}(x_i) = 0.$$

These conditions are exactly the Nash equilibrium conditions for $SB1_A$ and $SB1_B$ respectively, as l_A is independent of $a^*_{i,B}(x_i)$ and l_B is independent of $a^*_{i,A}(x_i)$.

Hence, we can consider the game SB2 as two seperate games. In the exposition of the model, we focus on the game SB1$_A$ but all conclusions are applicable to the game SB1$_B$ and thus generalize to the game SB2.

3 Equilibrium of the Game SB1$_A$

In this section we investigate the equilibrium of the game Subscript SB1$_A$. As a first step, we dicuss the equilibrium in switching strategies. We then show that the equilibrium in switching strategies prevails under more general assumptions.

3.1 Equilibrium in Switching Strategies

To solve for the equilibrium in our model, we generalize the argument of Morris and Shin (2004) to the two-dimensional case and assume that agents play switching strategies. In switching strategies, the agent computes a critical signal and a corresponding critical fundamental state for the respective asset, and either rolls over or forecloses, depending on whether her own private signal x_i is above or below the critical signal which she uses as cutoff. The critical signal is a function x_A^{crit} that divides the signal space into an acceptance and a denial region:

$$a^*_{i,A}(x_i) = \begin{cases} 1 & \text{if} \quad x^i_A \geq x^{crit}_A(x^i_B) \\ 0 & \text{if} \quad x^i_A < x^{crit}_A(x^i_B) \end{cases} .$$

To compute the equilibrium in switching strategies, observe first that – because of the law of large numbers and our assumption of an infinite number of investors – the solvency state for θ is deterministic. If all investors play the switching strategy with the same cutoff, there is a function θ_A^{crit} which maps each θ_B to a critical value of the fundamental state for asset A such that the asset is on the margin of success and failure in this state. Recall that the asset is successful in case the fundamental state is large enough to sustain partial foreclosure. At the margin of success and failure for each θ_B the critical value $\theta_A^{crit}(\theta_B)$ is exactly equal to $z_A l_A$ and therefore solves the following equation:

$$\forall \theta_B : \theta_A^{crit}(\theta_B) = z_A \underbrace{\int_{-\infty}^{\infty} \int_{-\infty}^{x_A^{crit}(x_B)} f_x \left(x_A, x_B, \theta_A^{crit}(\theta_B), \theta_B\right) \mathrm{d}x_A \mathrm{d}x_B}_{l_A} . \tag{C1}$$

Note that θ_A^{crit} summarizes the impact of partial foreclosure of all other agents from the perspective of one agent.

Simultaneously, in equilibrium, at the critical value of the signal $x_A^{crit}(x_B)$, the creditor is indifferent between foreclosure and rollover such that the expected return on the investment conditional on the private information is equal to the outside option:

$$\begin{aligned} \forall x_B : \lambda_A &= E\left(u_A\left(1, l_A, x\right)\right) \qquad \text{(C2)} \\ &= \int_{-\infty}^{\infty} \int_{\theta_A^{crit}(\theta_B)}^{\infty} g_{\theta|x}\left(\theta_A, \theta_B, \xi_A\left(x_A^{crit}\left(x_B\right), x_B\right), \xi_B\left(x_A^{crit}\left(x_B\right), x_B\right)\right) \mathrm{d}\theta_A \mathrm{d}\theta_B . \end{aligned}$$

Definition (equilibrium in switching strategies of the game SB1$_A$). The functions θ_A^{crit} and x_A^{crit} are an equilibrium in switching strategies if and only if they fulfill equations C1 and C2.

The two corresponding equilibrium equations have to be solved for the two critical functions, θ_A^{crit} and x_A^{crit}. In general, this can only be done numerically.

Figure 1 illustrates the critical functions and visualizes the game for one arbitrary state of the world. The ellipses illustrate the bivariate normal distributions of θ and x_i. The left panel shows the critical fundamental state function, θ_A^{crit}, which divides the fundamental state space into the default and solvency region for θ. The left panel shows the acceptance and denial region (divided by the critical private signal function, x_A^{crit}) for private signals. If θ is to the right of θ_A^{crit} ($\theta \geq z_A l_A$) the asset pays out creditors because sufficiently many creditors have received a sufficiently good signal – a signal which is to the right of x_A^{crit}. On the other hand, in case θ is to the left of θ_A^{crit}, the fraction of creditors who foreclose is too large such that the country defaults. Figure 1 illustrates the solvency case for one arbitrary random realization of the fundamental state and a subset of corresponding private signals. Signals for creditors who foreclose are represented by hollow dots.

Under the assumption of risk neutral agents the price of debt, p_A, in our model is represented by the ex ante probability of the asset to be successful such that creditors are paid back in full:

$$\forall \theta_B : p_A = Prob\left(\theta_A \geq \theta_A^{crit}(\theta_B)\right) = \int_{-\infty}^{\infty} \int_{\theta_A^{crit}(\theta_B)}^{\infty} g_\theta \, \mathrm{d}\theta_A \mathrm{d}\theta_B.$$

3.2 Existence and Uniqueness of the Equilibrium

To analyse the properties of the Nash equilibrium we proceed in three steps. First, we show that there are equilibria in switching strategies as long as cutoffs converge to a solution for the equilibrium conditions C1 and C2. Second, we argue that if cutoffs converge to the same function pair x_A^{crit}–θ_A^{crit} regardless of the initial cutoff in the switching strategies, there is a unique equilibrium in switching strategies. Finally, we show that in these cases the switching strategy around x_A^{crit} survives the iterated deletion of dominated strategies which establishes the unique equilibrium in switching strategies as a unique Nash equilibrium.

Lemma 2. *There exists a range of parameter values of the public and private signal such that the conditional mean of the fundamental state of asset A, ξ_A, is non-decreasing in the private signal about asset A, x_A.*

Proof. To check whether there is a parametrization such that the ξ_A is non-decreasing in x_A we consider the first derivative, ξ_A:

$$\xi_A' = \frac{\beta_A\left(\alpha_B + \beta_B\right) - \sqrt{\alpha_A\alpha_B\beta_A\beta_B}\rho_{private}\rho_{public} - \beta_A\beta_B\rho_{public}^2}{\alpha_A\left(\alpha_B + \beta_B - \alpha_B\rho_{private}^2\right) - 2\sqrt{\alpha_A\alpha_B\beta_A\beta_B}\rho_{private}\rho_{public} + \beta_A\left(\alpha_B + \beta_B - \beta_B\rho_{public}^2\right)}$$

The denominator is positive for all admissible parameters ($\alpha_A > 0$, $\alpha_B > 0$, $\beta_A > 0$, $\beta_B > 0$, $0 < \rho_{public} < 1$, $0 < \rho private < 1$) whereas this is not necessarily the case for the numerator. Nevertheless, there is large set of parameters for which the numerator is positive such that $\xi_A' > 0$. Irrespective of the correlation parameters, in particular all parameters for which there exists a unique equilibrium in the one-dimensional case for asset A and asset B respectively ($z_A\alpha_A \leq \sqrt{2\pi\beta_A}$ and $z_B\alpha_B \leq \sqrt{2\pi\beta_B}$; see Morris and Shin (2004)) yield $\xi_A' > 0$. ■

If the conditions of Lemma 1 are fulfilled, we can determine the switching strategy by a sequential process, whose intuition is illustrated in Figure 2. We start by assuming that investors always roll over (or never) in which case $\theta^{crit} \equiv 0$. An individual agent will choose a switching strategy x as a best response, which will lead to a new critical θ, and so on. Based on this idea we can derive the following proposition:

Proposition 2. *If ξ_A, is non-decreasing in the private signal about asset A, x_A, and if a pair of functions, x_A^{crit} and θ_A^{crit}, exists, that is a unique solution of equations C1 and C2, then a unique Nash equilibrium for SB1$_A$ exists. In this case the Nash equilibrium is the equilibrium in switching strategies with the solutions x_A^{crit} and θ_A^{crit}.*

Proof. We define two operator sT_θ and T_x as follows:

$$
\begin{aligned}
&T_\theta : L_x \to L_\theta : x_A^i \to \theta_A^i \text{ with} \\
&\qquad \forall \theta_B : \theta_A^i(\theta_B) = z_A \int_{-\infty}^{\infty} \int_{-\infty}^{x_A^i(x_B)} f_x\left(x_A, x_B, \theta_B \theta_A^i, \theta_B\right) \mathrm{d}x_A \mathrm{d}x_B, \\
&T_x : L_\theta \to L_x : \theta_A^{i-1} \to x_A^i \text{ with} \\
&\qquad \forall x_B : \lambda_A = \int_{-\infty}^{\infty} \int_{\theta_A^{i-1}(\theta_B)}^{\infty} g_{\theta|x}\left(\theta_A, \theta_B, \xi_A\left(x_A^i(x_B), x_B\right), \xi_B\left(x_A^i(x_B), x_B\right)\right) \mathrm{d}\theta_A \mathrm{d}\theta_B.
\end{aligned}
$$

Intuitively, given the strategy x_A^i, T_θ calculates the corresponding threshold for default, θ_A^i, and, given the critical fundamental state function θ_A^{i-1}, T_x calculates the corresponding best response strategy x_A^i.

We define the function pair $\underline{\theta}_A^i$, $\underline{x}_A^i$ for all i inductively.
Starting with $\underline{x}_A^0 \equiv -\infty$ and $\underline{\theta}_A^0 = T_\theta\left(\underline{x}_A^0\right) \equiv 0$, we define $\underline{x}_A^i := T_x\left(\underline{\theta}_A^{i-1}\right)$, $\underline{\theta}_A^i := T_\theta\left(\underline{x}_A^i\right)$.
Analogously, starting with $\overline{x}_A^0 \equiv \infty$ and $\overline{\theta}_A^0 \equiv 1$, we define $\overline{x}_A^i := T_x\left(\overline{\theta}_A^{i-1}\right)$, $\overline{\theta}_A^i := T_\theta\left(\overline{x}_A^i\right)$.

In a first step of the proof we show by induction that $\underline{\theta}_A^i \leq \underline{\theta}_A^{i+1}$, $\underline{x}_A^i \leq \underline{x}_A^{i+1}$ and $\overline{\theta}_A^i \geq \overline{\theta}_A^{i+1}$, $\overline{x}_A^i \geq \overline{x}_A^{i+1}$. To show this we first consider the sequence initialized from the left where $\underline{x}_A^0 \equiv -\infty$ and $\underline{\theta}_A^0 = T_\theta\left(\underline{x}_A^0\right) \equiv 0$.

For the base case we discuss $T_\theta\left(\underline{x}_A^i\right)$ for $i = 0$ and $i = 1$ as well as $T_x\left(\underline{\theta}_A^{i-1}\right)$ for $i = 1$.

Consider $T_\theta\left(\underline{x}_A^0\right)$.
As $\underline{x}_A^0 \equiv -\infty$ it follows that $\forall i : a_A^i(x_i) = 1$ (i.e. no coordination problem) and default only occurs if $\theta_A \leq 0$ implying that the cutoff is $\forall \theta_B : \underline{\theta}_A^0(\theta_B) = 0$. This result can also be seen directly from T_θ: Since there is no mass of distribution within the integration domain $[-\infty, \infty]$, $\forall \theta_B : \underline{\theta}_A^0(\theta_B) = 0$.

Next we look at $T_x\left(\underline{\theta}_A^0\right)$.

For $i = 1$ the lower integration limit in T_x is $\theta_A^0(\theta_B) = 0$. For fixed x_B, a change in x_A shifts the mean of the conditional distribution but leaves the covariance matrix unchanged. Due to the properties of the bivariate normal distribution there exists a unique conditional mean $\xi^1 = (\xi_A^1, \xi_B^1) > -\infty$ such that a λ_A-portion of the conditional density $g_{\theta|x}$ with mean ξ^1 is above the integration limit $\underline{\theta}_A^0(\theta_B) = 0$. As $\xi_A^1 > -\infty$ and since ξ_A depends linearly on x_A, we get $\underline{x}_A^1 > -\infty$.

For the exposition of the logic of the sequence we also discuss $T_\theta\left(\underline{x}_A^1\right)$. For fixed θ_B, there is a positive density of f_x within the integration domain $[-\infty, \underline{x}_A^1(x_B)]$. Hence, we have that $0 < \underline{\theta}_A^1(\theta_B)$ because the integral is larger 0. This is obviously true for normal distribution.

Next we consider the inductive step. We have to prove that $\underline{\theta}_A^i \leq \underline{\theta}_A^{i+1}$ and $\underline{x}_A^i \leq \underline{x}_A^{i+1}$.

We look at $T_x\left(\underline{\theta}_A^i\right)$.

For fixed x_B we have from the induction hypothesis $\forall\theta_B : \underline{\theta}_A^{i-1}(\theta_B) \leq \underline{\theta}_A^i(\theta_B)$, the integration limit shifts to the right and thus the value the integral decreases below λ_A. Therefore, in order to have a constant λ_A-portion of the density located above the lower integration limit, the mean of the conditional density $g_{\theta|x}$, $\xi = (\xi_A, \xi_B)$ corresponding to the critical value $\underline{x}_A^{i+1}(x_B)$, has to shift to the right too, i.e $\xi_A\left(\underline{x}_A^{i+1}(x_B), x_B\right) > \xi_A\left(\underline{x}_A^i(x_B), x_B\right)$.

From our assumption that ξ_A is non-decreasing in x_A we get that $\underline{x}_A^{i+1}(x_B)$ has to increase as well for all x_B, which proves $\underline{x}_A^i \leq \underline{x}_A^{i+1}$.

Now we consider $T_\theta\left(\underline{x}_A^{i+1}\right)$.

From the first part of the proof we thus get $\underline{x}_A^i(x_B) \leq \underline{x}_A^{i+1}(x_B)$.

We fix θ_B. Clearly, the equation used in T_θ does not hold anymore for $\underline{\theta}_A^i$ because the integration limit shifted outwards such that

$$\underline{\theta}_A^i(\theta_B) < z_A \int_{-\infty}^{\infty} \int_{-\infty}^{\underline{x}_A^i(x_B)} f_x\left(x_A, x_B, \underline{\theta}_A^i(\theta_B), \theta_B\right) \mathrm{d}\theta_A \mathrm{d}\theta_B.$$

Hence, to compensate, $\underline{\theta}_A^{i+1}(\theta_B)$ has to shift outwards to let the right hand side in $T_\theta\left(\underline{x}_A^i\right)$, the integral, decreases. Note that at the same time the left hand side increases. The new solution $\underline{\theta}_A^{i+1}(\theta_B)$ is where the shift in $\underline{\theta}_A^{i+1}(\theta_B)$ is sufficient to make the equation hold again.

This means that we have proven the inductive step and hence, $\underline{\theta}_A^i \leq \underline{\theta}_A^{i+1}$ and $\underline{x}_A^i \leq \underline{x}_A^{i+1}$. Analogously, it holds that $\overline{\theta}_A^i \geq \overline{\theta}_A^{i+1}$, $\overline{x}_A^i \geq \overline{x}_A^{i+1}$.

Because for each θ_B, $\underline{\theta}_A^i(\theta_B)$ is defined on the compact set $0 \leq \theta_A \leq 1$ and the sequence is monotonously increasing from the left (and monotonously increasing from the right), we have that the sequence converges pointwisely to a function pair $\underline{\theta}_A^{\lim} = \lim_{n\to\infty} \underline{\theta}_A^n$, $\underline{x}_A^{\lim} = \lim_{n\to\infty} \underline{x}_A^n$. As $T_\theta\left(\underline{x}_A^{\lim}\right) = \underline{\theta}_A^{\lim}$ and $T_x\left(\underline{\theta}_A^{\lim}\right) = \underline{x}_A^{\lim}$ the function pair is an equilibrium in switching strategies.

To complete the proof we have to show that the switching strategy using $\underline{x}_A^{\lim}$ survives iterated deletion of dominated strategies. As $n \to n+1$, observe that because of the induction hypothesis, all strategies remaining after n iterations always foreclose for $x < \underline{x}_A^n$. As $\underline{x}_A^{n+1}$ is the best response to the corresponding critical border $\underline{\theta}_A^n$, it dominates all remaining strategies that are not foreclosing in the region between $\underline{x}_A^n$ and $\underline{x}_A^{n+1}$. The same logic applies for the sequence initiated from above where $\overline{\theta}_A^0 \equiv 1$ (corresponding to $\overline{x}_A^0 \equiv \infty$).

Finally, if the solution x_A^{crit} for $T\left(x_A^{crit}\right) = T_x\left(T_\theta\left(x_A^{crit}\right)\right)$ is unique (i.e. $\underline{x}_A^{\lim} = \overline{x}_A^{\lim}$ and $\underline{\theta}_A^{\lim} = \overline{\theta}_A^{\lim}$), then the only strategy surviving iterated deletion of dominated strategies (from left and from the right) is the switching strategy for x_A^{crit} and therefore is the unique Nash equilibrium. ■

4 Some analytic results

We now present some analytic results. Some of them are important on their own, while some are also helpful for the intuition behind the numerical results.

4.1 The best response $\mathbf{x_A^{crit}}$ to a constant fundamental state function $\theta_\mathbf{A}^\mathbf{crit}$

Recall that in order to get a solution for x_A^{crit} we have to solve:

$$\lambda_A = \int_{-\infty}^{\infty} \int_{\theta_A^{crit}(\theta_B)}^{\infty} g_{\theta|x}\left(\theta_A, \theta_B, \xi_A\left(x_A^{crit}\left(x_B\right), x_B\right), \xi_B\left(x_A^{crit}\left(x_B\right), x_B\right)\right) \mathrm{d}\theta_A \mathrm{d}\theta_B.$$

We define a function m^λ:

$$\xi_B \to m^\lambda\left(\xi_B\right) \text{ with } \lambda_A = \int_{-\infty}^{\infty} \int_{\theta_A^{crit}(\theta_B)}^{\infty} g_{\theta|x}\left(\theta_A, \theta_B, m^\lambda\left(\xi_B\right), \xi_B\right) \mathrm{d}\theta_A \mathrm{d}\theta_B.$$

Obviously, m^λ depends on θ_A^{crit}, the outside option λ_A, and $\Sigma_{conditional}$, but we suppress these arguments for notational convenience if they are not needed.

The solution $x_A^{crit}\left(x_B\right)$ is then found by the intersection of the curves defined by m^λ and the conditional mean ξ:

$$\xi_A\left(x_A^{crit}\left(x_B\right), x_B\right) = m^\lambda\left(\xi_B\left(x_A^{crit}\left(x_B\right), x_B\right)\right).$$

In case of a constant critical function $\theta_A^{crit}(\theta_B) = \theta^0$, it is obvious that also m^λ is constant: $m^\lambda(\xi_B) = m^0$. We can then easily solve $\xi_A\left(x_A^{crit}(x_B), x_B\right) = m^0$.

Proposition 3. *In case of a constant critical function $\theta_A^{crit}(\theta_B) = \theta^0$, $x_A^{crit}(x_B) = c_0 y_A + c_1 m^0 + c_2(x_B - y_B)$, where c_0, c_1 are constants depending on the precision of the signals only, while c_2 in addition depends on the correlations.*

We can get several insights from Proposition 3:

i) If additionally, ξ_A does not depend on x_B, x_A^{crit} is a constant. This will lead to the special case considered below.

ii) These results hold in particular for the start of our iteration in the solution (because $\theta_A \equiv 0$ or 1). From the simulations of the numerical results one gets the intuition, that this first step already shows the slope of the critical signal functions in equilibrium, with some non-linearities added because of the feedback of θ_A^{crit}.

Specifically, in this first step a change in ρ_{public} and $\rho_{private}$ only rotates x_A^{crit} (in opposite directions) around the point $(x_B - y_B)$ because c_0 and c_1 do not depend on them. As this translates into a (approximate) rotation of θ_A^{crit}, one may expect a small effect of a change in the correlations on the price of debt. Figure 3 illustrates the effects of changes of either ρ_{public} or $\rho_{private}$ on θ_A^{crit} (Panel A) and x_A^{crit} (Panel B). Based on a benchmark case, we either increase ρ_{public} or $\rho_{private}$ by 0.1. Starting from the left ($\theta_A \equiv 0$), the Figure shows the first and the last step of the sequence.

4.2 No influence of $\mathbf{y_B}$ (no contagion)

We next state an invariance result for the solution x_A^{crit}:

Lemma 2. *Let $s = (s_A, s_B)$ be a translation vector and explicitly show the parameter y in the operator T_x of the proof of Proposition 2: If $x_A^{crit} = T_x\left(\theta_A^{crit}; y\right)$, $x_A^{crit,transl}(x_B) = x_A^{crit}(x_B + s_B) - s_A$ and $\theta_A^{crit,transl}(\theta_B) = \theta_A^{crit}(\theta_B + s_B) - s_A$ then we have $x_A^{crit,transl} = T_x\left(\theta_A^{crit,transl}; y - s\right)$.*

Proof. If θ_A^{crit} is translated to $\theta_A^{crit,transl}$, then obviously, m^λ is shifted to $m^{\lambda,transl}$ with $m^{\lambda,transl}(\xi_B) = m^\lambda(\xi_B + s_B) - s_A$. As one can easily show, the conditional mean ξ is also translated to $\xi_A^{transl}(x - s; y - s) = \xi_A(x; y) - s$.

Therefore, $x_A^{crit,transl}$ solves:

$$\begin{aligned}
\xi_A^{transl}\left(x_A^{crit,transl}(x_B), x_B\right) &= \xi_A\left(x_A^{crit,transl}(x_B) + s_A, x_B + s_B\right) \\
&= \xi_A\left(x_A^{crit}(x_B + s_B), x_B + s_B\right) - s_A \\
&= m^\lambda\left(\xi_A\left(x_A^{crit}(x_B + s_B), x_B + s_B\right)\right) - s_A \\
&= m^\lambda\left(\xi_A\left({x_A}^{crit,transl}(x_B) + s_A, x_B + s_B\right)\right) - s_A \\
&= m^\lambda\left(\xi_A^{transl}\left(x_A^{crit,transl}(x_B), x_B\right) + s_B\right) - s_A \\
&= m^{\lambda,transl}\left(\xi_A^{transl}\left(x_A^{crit,transl}(x_B), x_B\right)\right).
\end{aligned}$$

This shows that $x_A^{crit,transl}$ is the result of T_x as desired. ■

The intuition of the proposition is easy: If the publicly known signal y is shifted, this changes the conditional mean, but leaves the conditional covariance matrix unchanged. Thus, if the critical fundamental state function is shifted by the same vector, then the solution for the critical signal is also just shifted.

One can derive an analogous result for different changes of parameters, however for general transformations one has to be careful about the description of the region $R = \{\theta : \theta_A^{crit}(\theta_B) < \theta_A\}$ in terms of the new variables. The main problem is that e.g. a rotation of a general critical function θ_A^{crit} may no longer be invertible in the new coordinates. The fundamental problem with invariance in equation C1 is that the solution is confined to the interval $0 \leq \theta_A \leq z_A$. We will show this by looking at the effect of a translation.

Lemma 3 *A translation of x_A^{crit} in the direction of x_B shifts the critical function θ_A^{crit} by the same amount. A translation in the direction of x_A leads to a smaller shift in the same direction because part of the response is compensated by the left-hand side of the equation.*

Proof. Clear. ■

Proposition 4 (no contagion). *Let us explicitly show the dependency of the solution to (C1) and (C2) from y_B: x_A^{crit,y_B} and θ_A^{crit,y_B}.*
We then have: $x_A^{crit,y_B}(x_B) = x_A^{crit,0}(x_B - y_B)$ and $\theta_A^{crit,y_B}(\theta_B) = \theta_A^{crit,0}(\theta_B - y_B)$. As a consequence we get that the price of debt is independent of y_B since the mass of the distribution above the critical border is constant.

Proof. From Lemma 3 above we get that if θ^{crit} is shifted then the critical signal fulfills the equation from above. From equation C1 (which does not depend on y_B) we get exactly the shifted equation for θ. ■

4.3 The Special Case: Constant Critical Functions

Before we proceed to the numerical experiments to quantify informational contagion we investigate a special case, namely that of identical precision parameters for asset A and B, and identical correlation for private information and the fundamental state. This can be thought of as a border case, because we basically assume that – perhaps precipitated by the introduction of the currency union – precision parameters for both dimensions are identical and also public and private correlation have converged.

Proposition 5. *If the precisions of the public and the private signal are identical for both assets and the private and public correlation is the same, we can show that the critical functions* x_A^{crit} *and* θ_A^{crit}*, as well as* x_B^{crit} *and* θ_B^{crit}*, are constant in* $SB1_A$ *and* $SB1_B$*, and the respective game is identical to the one-dimensional case.*

Proof. For the special case we have to show that the critical functions are constant. Hence, we have to show that for the special case, the equilibrium solutions do not depend on x_B and θ_B in SB1$_A$ and do not depend on x_A and θ_A in SB1$_B$. Since in the special case SB1$_A$ and SB1$_B$ are symmetric, we confine our attention to the game SB1$_A$.

Note that in equilibrium, the critical signal x_A^{crit} only enters the equation C2 via the mean of the conditional distribution, which links the two equilibrium conditions. Thus, we consider the mean of the conditional distribution of θ, $\xi = (\xi_A, \xi_B)$, for the special case. For ξ_A to be independent of x_B and ξ_B to be independent of x_A it is necessary and sufficient that $\alpha_A = \alpha_B$, $\beta_A = \beta_B$, and $\rho_{public} = \rho_{private}$. Plugging in these conditions, reduces ξ_A and ξ_B to:

$$\xi_A = \frac{x_A \beta_B + y_A \alpha_B}{\alpha_A + \beta_A},\ \xi_B = \frac{\beta_A x_B + \alpha_A y_B}{\alpha_A + \beta_A}.$$

We can use this result to simplify the equilibrium conditions and plug the the reduced expressions into densities in equations C1 and C2 for the game $SB1_A$:

$$f_x = \frac{1}{2\pi\sqrt{\frac{1-\rho_{private}^2}{\beta_A^2}}} exp\left(\frac{1}{2\left(\rho_{private}^2-1\right)}\beta_A\left(-2\theta_A\theta_B\rho_{private} - 2x_A\left(\theta_A + \rho_{\text{private}}\left(x_B - \theta_B\right)\right)\right.\right.$$

$$\left.\left.+2\theta_A x_B\rho_{private} + \theta_A^2 + x_A^2 + \theta_B^2 - 2\theta_B x_B + x_B^2\right)\right)$$

$$g_{\theta|x} = \frac{1}{2\pi\sqrt{\frac{1-\rho_{private}^2}{(\alpha_A+\beta_A)^2}}} \exp\left(-\frac{1}{2\left(\rho_{private}^2-1\right)}\left(\left(\theta_B - \frac{\beta_A x_B + \alpha_A y_B}{\alpha_A+\beta_A}\right)\left(-\alpha_A\theta_B - \beta_A\theta_B\right.\right.\right.$$

$$\left.+\beta_A x_B + \alpha_A y_B + \alpha_A\theta_A\rho_{private} + \beta_A\theta_A\rho_{private} - \beta_A x_A\rho_{private} - \alpha_A y_A\rho_{private}\right)$$

$$+\left(\theta_A - \frac{\beta_A x_A + y_A\alpha_A}{\alpha_A+\beta_A}\right)\left(-\alpha_A\theta_A - \beta_A\theta_A + \alpha_A\theta_B\rho_{private} + \beta_A\theta_B\rho_{private}\right.$$

$$\left.\left.\left.-\beta_A x_B\rho_{private} - \text{yB}\alpha_A\rho_{private} + \beta_A x_A + y_A\alpha_A\right)\right)\right)$$

When we now integrate over the densities, under the special case conditions, it turns out that θ_B and x_B cancel out:

$$\lambda_A = \Phi\left(\sqrt{\alpha_A+\beta_A}\left(\alpha_A y_A + \beta_A x_A^{crit} - \left(\alpha_A+\beta_A\right)\theta_A\right)\right),$$

$$\theta_A^{crit} = z_A\Phi\left(\frac{1}{\sqrt{\beta_A}}\left(x_A^{crit} - \theta_A^{crit}\right)\right),$$

where Φ is the cumulative distribution function of the univariate normal distribution. Obviously, the equilibrium conditions for the special case do not depend on θ_B and x_B and therefore, the functions solving the equations are constant as stated in the proposition. Moreover, B-dimensional precision parameters cancel out from the equilibrium conditions. ■

Notably, the two equilibrium conditions we derive for the special case are identical to the equilibrium conditions for the one-dimensional case, which is discussed in Morris and Shin (2004). This result implies that for the special case, the game SB1_A is identical to the one-dimensional game and hence, information about asset B does not affect the probability of default, and hence the price of debt of asset A.

In addition, as long as the correlations are the same, we obviously have the following result as the critical functions are independent of the correlation.

Proposition 6. *For the special case, the equilibrium is independent of the correlation.*

This means that we have found a constellation of parameters such that the increase of correlation due to the currency union has no impact on the decisions of the agents and thus, no impact on the price of debt, and contagion is ruled out. Obviously, it is not clear how relevant this case is because it assumes that informational precision is identical for both assets and also correlations are equal – though the currency union might promote such a synchronization.

5 Numerical Experiments

To quantify the contagious effects of news precipitated through correlated fundamentals further, we now consider numerical experiments for parameterizations of the model which do not represent the special case.

We use the operators defined in the proof of Proposition 2 to iteratively solve equations C1 and C1. Starting with $x_A^{i-1}(x_B) = -1000$, we solve equation C1 on a grid of θ_B-values $\{\theta_B^1, ..., \theta_B^n\}$. We can then interpolate the pointwise solutions linearly to attain a first the solution of C1, $\theta_A^i(\theta_B)$. In the second step we use $\theta_A^i(\theta_B)$ to solve equation C2 on a grid of x_B-values $\{x_B^1, ..., x_B^n\}$ and interpolate the pointwise solutions to obtain $x_A^i(x_B)$. We initialize a second sequence from the right where $x_A^{i-1}(x_B) = 1000$ and iterate both sequences until they converge to the same function pair x_A^{crit} and θ_A^{crit}. For all numerical experiments discussed below we obtain unique equilibria.

To focus on the effects of variations of parameters from the second dimension, we consider parameters which depart considerably from the special case discussed above. Intuitively, we expect impacts of the B-dimension to be more pronounced, the farther we depart from the special case. Moreover, signals about asset B are more precise compared to signals about asset A in the baseline which should increase the weight of B-dimensional information. Also, we have chosen a low public signal about Asset B to capture the case of negative spillovers. The rest of the parametrization is chosen such that the values and the resulting yield (approximately 6.5 percentage points) are plausible for the refinancing problem of currency union members.

In each experiment, we vary one parameter while all parameters are held constant. The baseline values are shown in Table 3. The outside option ($\lambda_A = 0.5$) as well as the measure for the severity of disruption ($z_A = 1$) are constant throughout all experiments. We look at changes in the informational precision, and changes in the correlation of both, the fundamental states and the private signals, to assess whether changes in these parameters bring about effects on the price of asset A. The intuition for contagious effects is that changes in these parameters affect the weighting of the public and private signals. In particular, we are interested in changes in correlation and precision parameters of the B-dimensional information but we also show results for the variation in the A-dimensional precision parameters to have a benchmark to compare the variation in the B-dimensional parameters with.

We vary the parametrization in 11 equidistant steps while all other parameters are held constant at the values shown in Table 3. The range of the variation in the parameters is shown in Table 4. We report the maximum difference in the price of debt due to the parameter variation, $\max\left(p_A^{j,k}\right) - \min\left(p_A^{j,k}\right)$ where $j \in \{1, ..., 11\}$ are the equidistant steps and k denotes the respective experiment. Also, we show how the differences in the price of debt translate into differences in yields and indicate $\max\left(yield_A^{j,k}\right) - \min\left(yield_A^{j,k}\right)$, where $yield_A^{j,k} = \frac{1}{p_A^{j,k}} - 1$. In particular, we want to investigate the contagious effects on the price of debt (i) through changes in changes in the correlation of either the fundamental states of private signals and (ii) through changes in the informational precision of public and private signals.

Recall that the price of debt is affected by either (i) changes in the density of the fundamental state θ, g_θ, or (ii) changes of the critical fundamental state function, θ_A^{crit}. The parameters α_A, α_B and ρ_{public} change the density of the distribution of the fundamental state, while the density does not depend on β_A, β_B and $\rho_{private}$. θ_A^{crit}, on the other hand, depends on all parameters that are varied in the numerical experiments. Figure 4 shows θ_A^{crit} as well as g_θ for some numerical experiments to illustrate the effects of the variation in the parameters. Each panel shows three experiments: The left column shows θ_A^{crit} and g_θ for the first step of the respective parameter variation, the second column always shows the baseline case with the parameters shown in Table 3 and the third column shows the last step of the respective parameter variation.

Changes in either ρ_{public} or $\rho_{private}$ exert modest effects on the price of debt of asset A and the potential for disruption in our experiments is 0.16 percentage points at most in case of the variation in ρ_{public} and 0.15 percentage points in case of the variation in $\rho_{private}$. Changes in ρ_{public} are shown in Panel A of Figure 4. They affect both, the slope and the size of the elliptical distribution of θ. As ρ_{public} increases, the minor axes becomes smaller such that the ellipse becomes narrower. But since θ_A^{crit} mainly rotates around the major axes, the effects of the parameter variation on the slope are at least partly offset and the compression of the ellipse is mostly ineffective for changing the price of debt. Changes in $\rho_{private}$ shown in Panel B of Figure 4 only rotate θ_A^{crit} while the density g_θ remains unchanged. θ_A^{crit} again rotates roughly around the major axis of the elliptical distribution of θ and hence, the effects on the price of debt are rather modest.

Contagious effects precipitated through changes in the informational precision about asset B (α_B ,β_B) are even smaller and amount to 0.07 percentage points on the price of debt in case of α_B and 0.05 percentage points in case of β_B. The reason why changes in α_B do not lead to major changes in the price of debt are twofold since they effect both, g_θ as well as θ_A^{crit}. The changes of the density of fundamental state caused by changes in α_B mainly result in a contraction of the minor axes (i.e. the elliptical distribution becomes narrower), which leaves the portion of the distribution in the solvency region rather constant. The critical function rotates roughly around the major axes, leaving the fraction of the distribution above θ_A^{crit} rather constant. Changes in β_B only affect θ_A^{crit}. Also in this case, the mass of the distribution above θ_A^{crit} remains relatively constant despite the rotation of θ_A^{crit} since the function mainly rotates around the main axis.

In Table 4 we also show effects of variations in the A-dimensional precision parameters α_A, β_A, and of the public signals y_A, y_B. The effects of α_A, β_A are much more pronounced compared to their B-dimensional counterparts. Variations in y_A naturally have the largest effects. As we would expect from the analytical results, variations in the public signal about asset B have no effects on the price of debt.

6 Conclusion

In order to study public debt refinancing in currency unions we model the simultaneous investment decision into two assets where fundamental states are subject to positive correlation. To this end we set up a two-dimensional global game which allows us to study unique equilibria. Using this model we discuss the contagious effects of information about one country onto the price of debt of another country. Interestingly, and perhaps also surprisingly, the contagious channels we study in this paper are not likely to be a major source of the disruption of government bond markets in the aftermath of the recent economic and financial crisis. Assuming that agents are risk neutral and play Nash, adverse contagious effects precipitated by bad news about one country are not primarily transmitted through the formation of beliefs about the refinancing ability of another country. There are some cases where the agent even optimally behaves as if she does not consider information about the state of one country when taking the investment decision into another country. Moreover, a change in the public signal about one country does generally not affect the price of debt of another country. However, correlations of fundamental states and private signals as well as the precision of information about one country do affect the price of debt of another country in most cases. Nevertheless, these effects are relatively weak. In turn, this indicates that other disruptive sources of contagion such as, inter alia, direct spillovers, e.g. transmitted through wealth effects, or non-rational behavior involved in the coordination of creditors may be more important in causing the disruption of government bond markets.

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Tables

Table 1: Payoffs $SB2$

	solvency of A and B	solvency of A, default of B	default of A, solvency of B	default of A and B
	$l_A z_A \leq \theta_A$ $l_B z_B \leq \theta_B$	$l_A z_A \leq \theta_A$ $l_B z_B > \theta_B$	$l_A z_A > \theta_A$ $l_B z_B \leq \theta_B$	$l_A z_A > \theta_A$ $l_B z_B > \theta_B$
invest in both	$1+1$	$1+0$	$0+1$	$0+0$
invest in A not B	$1+\lambda_B$	$1+\lambda_B$	$0+\lambda_B$	$0+\lambda_B$
invest in B not A	λ_A+1	λ_A+0	λ_A+1	λ_A+0
not invest in both	$\lambda_A+\lambda_B$	$\lambda_A+\lambda_B$	$\lambda_A+\lambda_B$	$\lambda_A+\lambda_B$

Table 2: Payoffs $SB1_A$

	solvency of A	default of A
	$l_A z_A \leq \theta_A$	$l_A z_A > \theta_A$
invest in A	1	0
not invest	λ_A	λ_A

Table 3: Parametrization

	Precision of public signal	Precision of private signal	Correlation of fund. states	Correlation of private signals	Public signals
	α_A, α_B	β_A, β_B	ρ_{public}	$\rho_{private}$	y_A, y_B
Asset A	0.5	5	0.8	0.5	2.5
Asset B	1	10			0

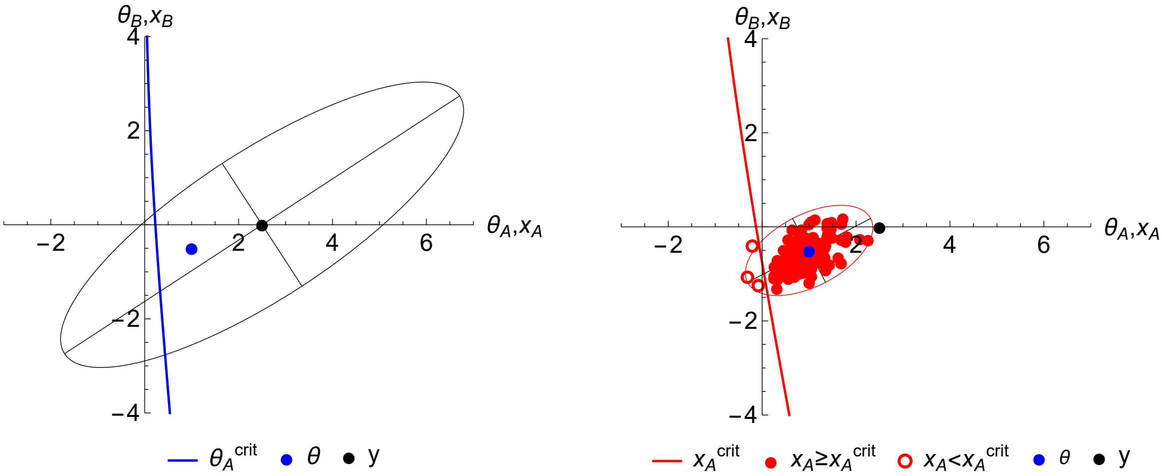
Table 4: Numerical Experiments

Parameter	$\rho_{private}$	ρ_{public}	α_A	α_B	β_A	β_B	y_A	y_B
from	0.01	0.01	0.1	0.5	2.5	5	0	0
to	0.99	0.99	1.1	2	7.5	15	5	5
steps	0.098	0.098	0.1	0.15	0.5	1	0.5	0.5
$Max(p_A^{j,k}) - Min(p_A^{j,k})$	0.0016	0.0015	0.2531	0.0007	0.008	0.0005	0.6505	0
$Max(yield_A^{j,k}) - Min(yield_A^{j,k})$	0.0018	0.0017	0.3443	0.0008	0.009	0.0006	1.863	0

Figures

Figure 1: Illustration of the Game $SB1_A$

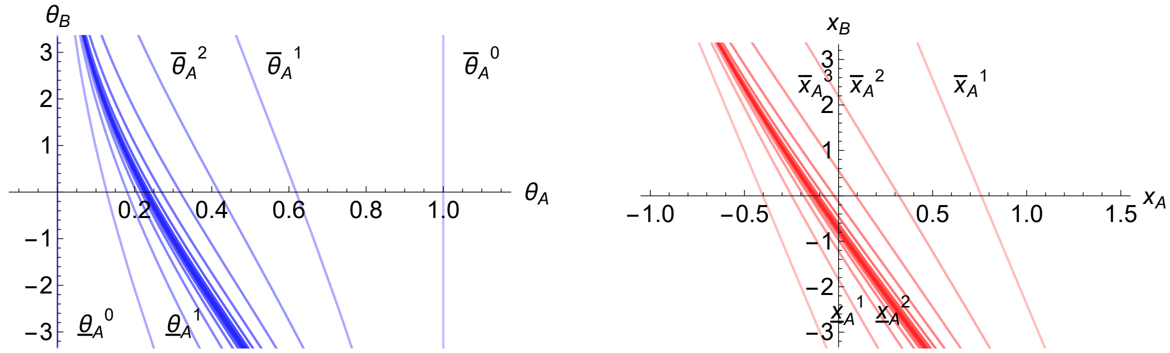
Panel A: Solvency and Default Regions | Panel B: Acceptance and Denial Regions



Notes: The density of θ, g_θ, is represented by the 99 percent confidence ellipse (black). The density of x, f_x, is represented by the 99 percent confidence ellipse (red). Default and solvency regions are illustrated in Panel A. Realizations of θ to the right of θ_A^{crit} lead to a payout for the investment into asset A while relations to the left of θ_A^{crit} correspond to no payout. Panel B shows acceptance and denial regions. Agents with realizations of private signals x_i to the right of x_A^{crit} rollover while agents signal to the left of x_A^{crit} foreclose. The parametrization is as in the baseline discussed in Section 5

Figure 2: Convergence to θ_A^{crit} and x_A^{crit}

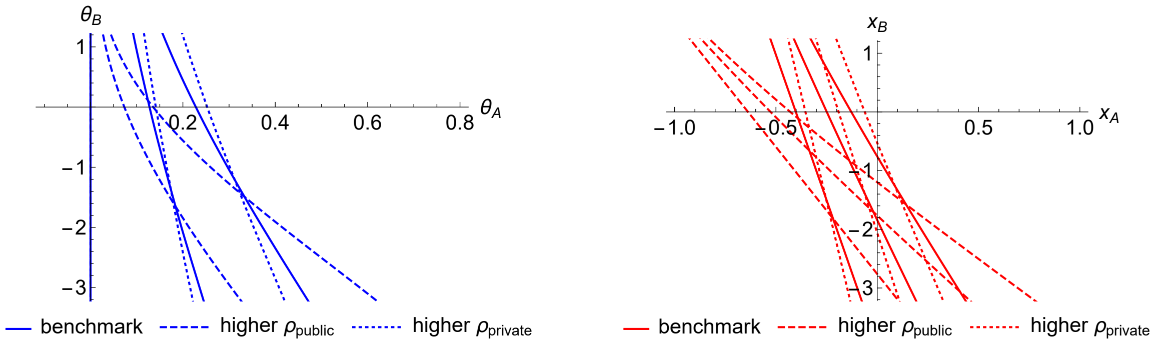
Panel A: Convergence to θ_A^{crit} | Panel B: Convergence to x_A^{crit}



Notes: As the sequence of the best response iteration progresses, the colors of the critical functions get more intense.

Figure 3: The correlations ρ_{public} and $\rho_{private}$ and the critical functions θ_A^{crit} and x_A^{crit}

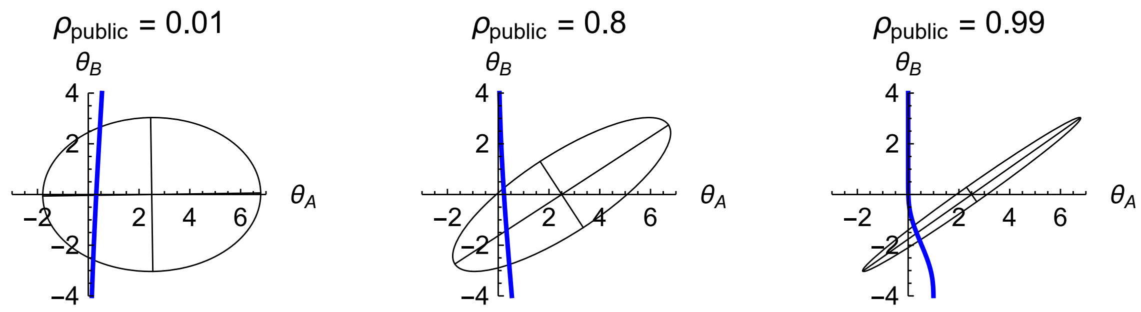
Panel A | Panel B



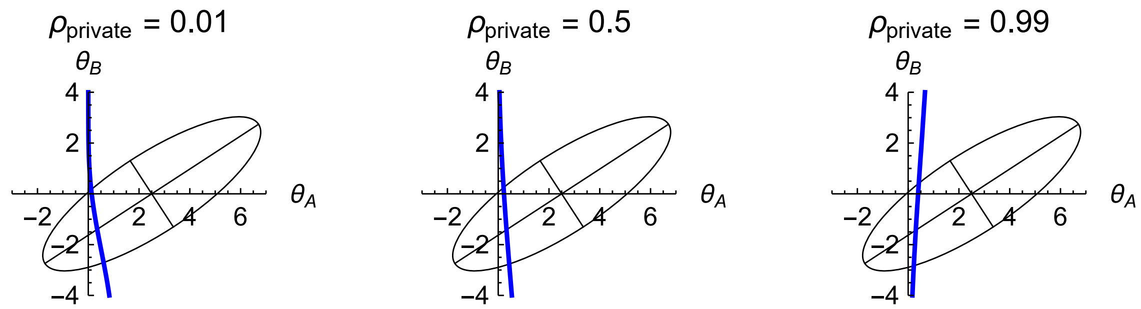
Notes: Starting from the left ($\theta_A \equiv 0$), the Figure shows the first and the last step of the sequence. The parametrization for the benchmark is as in the baseline discussed in Section 5; for this case $\rho_{public} = 0.8$ and $\rho_{private} = 0.5$. To illustrate how higher correlations affect the shape of θ_A^{crit} and x_A^{crit}, we increase either ρ_{public} or $\rho_{private}$ by 0.1.

Figure 4: θ_A^{crit} and g_θ

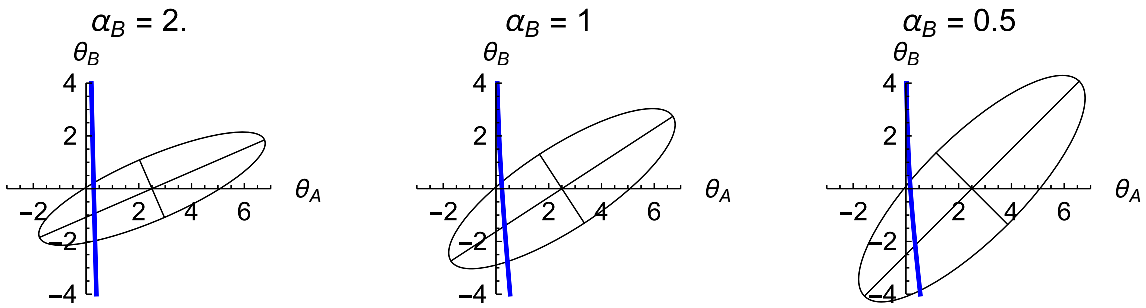
Panel A: Variation in ρ_{public}



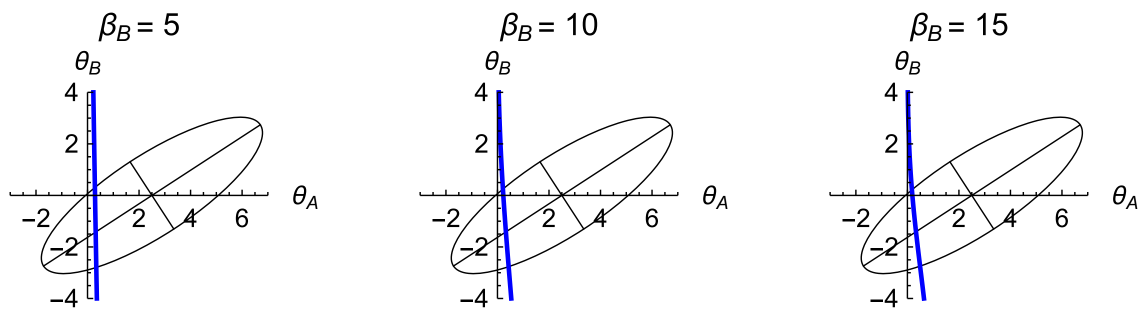
Panel B: Variation in $\rho_{private}$



Panel C: Variation in α_A



Panel D: Variation in β_B



Notes: The blue line indicates θ_A^{crit}. The density of θ, g_θ, is represented by the 99 percent confidence ellipse.

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Correlation and coordination risk

Abstract
We study the potential role of correlated refinancing abilities among different countries for the disruption of government bond markets in a currency union. Following Morris and Shin (2004) we use a global games framework and model the simultaneous investment decision into two assets, which are subject to correlated fundamental states, as a coordination problem with correlated imperfect information. Based on this model we evaluate the role of information about one country for the coordination of creditors of another country. We find, however, that the contagious effects on the price of debt precipitated through correlation are modest. Hence, assuming that investors behave as modeled in the global game, we conclude that correlated fundamentals that precipitate informational spillovers appear to be unlikely to play a major role for e.g. the disruption of some Eurozone government bond markets in the aftermath of the recent financial and economic crisis.

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