University of Innsbruck

Working Papers
in
Economics and Statistics

Coordination in Evolving Networks with Endogenous Decay

Francesco Feri and Miguel A.Meléndez-Jiménez

2009-19

Coordination in Evolving Networks with Endogenous Decay*

Francesco Feri[†]
University of Innsbruck

Miguel A. Meléndez-Jiménez
Universidad de Málaga

July 17, 2009

Abstract

This paper studies an evolutionary model of network formation with endogenous decay, in which agents benefit both from direct and indirect connections. In addition to forming (costly) links, agents choose actions for a coordination game that determines the level of decay of each link. We address the issues of coordination (long-run equilibrium selection) and network formation by means of stochastic stability techniques. We find that both the link cost and the trade-off between efficiency and risk-dominance play a crucial role in the long-run behavior of the system.

JEL Classification: C72; C73; D83; D85
Keywords: Coordination; Networks; Risk-dominance; Stochastic stability

*A former version of this paper, entitled "Network formation with endogenous decay", was awarded the Young Economist Award by the European Economic Association in 2004.

[†]Corresponding author. Department of Public Finance, University of Innsbruck. Universitätsstrasse 15/4, 6020 Innsbruck, Austria. E-mail: Francesco.Feri@uibk.ac.at

1 Introduction

Social interactions are usually subjected to generic frictions, such as noise or delay, that reduce the benefits that agents may potentially achieve. This is the case within organizations, where the internal structure of a group usually affects the performance of its members. For instance, within a firm, a worker may not be able to perform her task until those that precede her in the production process have finished their own assignments.[1] It is also the case in communication processes. For instance, when news are transmitted between agents, the accuracy of information may sharply decrease the higher the number of times it passes from one person to another. In these situations, we generically talk of *decay*. Formally, the existence of decay implies that the benefit that an individual receives from another one is a decreasing function of the distance between them in the network. The literature on networks has usually treated the decay as exogenous.[2] However, in many real world situations, the level of decay may be affected by the agents' decisions. For instance, the rate of decay in a communication network may well depend on the quality of the device (or technology) used by each agent, or on the level of effort exerted by each agent. Moreover, the degree of coordination between two (linked) agents may also affect the decay between them. If this is the case, agents could prefer to coordinate on the same device or technology in order to exchange information. Coordination is, however, generally subjected to a trade-off between efficiency and risk-dominance, as pointed out in the next two settings.

Consider agents that, in order to create their web sites, choose between using HTML (a static markup language) and DHTML (a collection of technologies used to create interactive and animated web sites). Each agent benefits from others visiting her web site, and each additional visit may attract potential new ones (since, if the visitor likes the web site, she could inform friends about it, or even create a link to it in her own web site). If an agent uses DHTML (efficient), her web site will be of high quality (which increases her expected profit if others visit it). However, only those people using an advanced web browser will be able to open it properly. In contrast, if an agent chooses HTML (risk-dominant), her web site will be of low quality, but readable by any web browser. This example illustrates a trade-off between sophistication and compatibility. In this case, a sophisticated but non-compatible technology represents the efficient choice, whereas a more basic but compatible one represents the risk-dominant choice.[3]

[1] In this line, a worker's outcome will usually depend on the number of workmates that act before her and, also, in the time and effort that each of them exerts in her interactions.

[2] See, for instance, Bala and Goyal [1], Jackson and Wolinsky [17], Hojman and Szeidl [14], Watts [22] and Feri [9].

[3] Other example could be that of agents who advertise products through e-mail. If an agent chooses *word* (efficient) to compose her add, it will be of high quality, but only those people using *word* will be able to read it. In contrast, if an agent chooses *ascii* (risk-dominant), her add will be of low quality but readable by anyone.

The second setting is that of minimum effort games. In a minimum effort game, each agent's benefit is determined by the minimum level of effort exerted across players and by the own effort cost. In this case, as the following example illustrates, a high effort can represent the efficient choice and, a low effort, the risk-dominant one. Consider a population of agents that use internet to communicate (for instance, to exchange pictures, videos,...). Each agent chooses between two internet connection speeds. The first connection (efficient) is fast, but its price per hour is high. The second one (risk-dominant) is slow but cheap. When two agents communicate, each of them pays for the price of her connection, but their benefits are constrained by the slower connection speed.

In this paper, we study this kind of coordination problems in the framework of the two-way flow (network formation) model with decay described in Bala and Goyal [1], BG hereafter. Our novelty is to consider that the decay is endogenous and idiosyncratic to the link. As in BG, we study a dynamic setting in which agents unilaterally form (costly) links in order to access the (non-rival) benefits generated by other agents. Benefits flow in both directions of a link (*i.e.*, links are two-way or non-directed), no matter who bears its cost. A link to another agent allows access to the benefits available to the latter via his own links. Therefore, individual links produce externalities whose value depends on the level of decay associated with indirect links. We depart from BG in the way we model the decay. Specifically, in the present paper, all the agents choose between two actions (communication technologies) for a coordination game: One efficient and the other risk-dominant. Whenever two agents get linked, their choices of actions determine the level of decay that each of them incurs in.[4] If an agent chooses the efficient action, she incurs in no decay (*i.e.*, she receives the other agents's benefits without frictions) if the other agent chooses the same action, whereas she incurs in full decay (*i.e.*, she receives no benefits from the link) if the other agent chooses the alternative action. On the contrary, if an agent chooses the risk-dominant action, she incurs in an intermediate and fixed level of decay (independent of the other agent's choice). Note that, since agents get benefits from indirect communication, the decay of a link not only affects the payoffs to the two involved agents, but also to all the agents that use the link in their indirect connections. We propose a stochastic adjustment process (agents revise actions and links), and focus our analysis on the study of the stochastically stable states, *i.e.*, those action profiles and network structures that are robust enough to be observed a significant fraction of time in the long run.

We first address the *coordination issue* (we characterize the action profiles associated to the stochastically stable states). We show that, in the long run, all players coordinate on the same action. The selected action depends both on the link cost and on the trade-off between efficiency and risk-dominance. When the link cost is low enough, the risk-dominant action is selected and, when it is high enough, both the

[4] A key feature of our model is that each agent chooses a single action (device or technology) that she uses in all the interactions with her neighbors.

efficient action and the risk-dominant one coexist. Moreover, provided that the risk-dominant action delivers a sufficiently low decay level as compared to the efficient one (lower than the inverse of the golden ratio), the efficient action is (uniquely) selected for intermediate link cost. We then turn to the *network issue* (network structures associated to the different stochastically stable states). We first show that, in those stochastically stable states in which agents coordinate on the efficient action, the networks are minimally connected. Moreover, such states maximize aggregate payoffs.[5] On the other hand, when coordination is on the risk-dominant action, the link cost determines the long-run network architectures: Low link costs result in the complete network whereas high link costs result in stars.

There is a large literature on the issue of equilibrium selection in social coordination games by means of stochastic stability techniques.[6] Within this literature, various papers consider that agents have the ability to choose their interaction partners by creating links. Goyal and Vega-Redondo [12] (GV hereafter) and Hojman and Szeidl [15] (HS hereafter) study one-sided models: Links are unilaterally formed. GV consider two-way links and focus on the case in which connections are costly and each agent interacts with her direct neighbors. They obtain that, for low link cost, all agents coordinate on the risk-dominant action whereas, for high link cost, all players coordinate on the efficient one. Moreover, the network is complete in all the stochastically stable states, regardless the action profile.[7] HS focus on the case of one-way (*i.e.*, directed) links and consider that agents receive benefits from indirect connections (without decay). They analyze two main cases: (i) The link costs are negligible but miscoordination is punished and (ii) agents are only allowed to form one link each.[8] In both cases, HS obtain that, in the stochastically stable states, all players coordinate on the same action and the network is a wheel. They prove that the efficient action is (uniquely) selected if the risk-dominant action delivers sufficiently low payoffs as compared to the efficient one; otherwise, the risk-dominant action is selected. In contrast to our case, both in GV and HS, different actions profiles never coexist in the set of stochastically stable states. However, the present paper integrates the two effects obtained by GV and HS (link cost and trade-off between efficiency and risk-dominance), provided the link cost is not too high. Other related papers in this literature are Jackson and Watts [16] and Meléndez-Jiménez [20], in which the formation of links requires the mutual agreement between the parties.[9] They both obtain

[5] This result allows us to compare efficiency and stability.

[6] This literature was pioneered by Kandori et al. [18], Young [23] and Ellison [7]. See also Bhaskar and Vega-Redondo [2], Dieckmann [6], Ely [8] and Mailath et al. [19].

[7] GV also show that their main result qualitatively extends to the case of indirect links (without frictions). In such a case, the long run networks are not complete but stars.

[8] They also consider a third case, in which links are costless and miscoordination is not punished. In such a case, all links are formed and risk-dominance considerations prevail.

[9] In Jackson and Watts [16] the link cost is equally shared between the parties and, in Meléndez-Jiménez [20], the cost shares result from bargaining. In Meléndez-Jiménez [20], the results on equilibrium selection are qualitatively similar to those of GV. Differently, Jackson and Watts [16]

that the complete network is uniquely observed in all the stochastically stable states. Thus, one interesting pattern missing from this literature that arises from the present model is that changes in parameters not only affect the action profile, but also the network architecture.

Also related to our paper are the models of Bloch and Dutta [3] and Deroian [5], who analyze network formation in the presence of endogenous decay. Instead of modelling the decay as a coordination game, they consider an allocation problem: Each player has an endowment that she allocates among different links and, the higher the total investment of the players in a link, the higher the link strength.[10] These decisions result in a weighted network. Bloch and Dutta [3] propose a non-directed communication model (two-way flow links) and obtain that the efficient and stable networks are stars. In contrast, Deroian [5] considers directed communication and obtains that the efficient and stable networks are wheels. This alternative approach to endogenize the decay is complementary to ours: Our framework represents situations where the benefits that an agent receives from all her links are affected (limited) by a single decision (like, *e.g.*, the choice of a communication technology), whereas their setting corresponds to situations where agents have flexibility to determine the strength of each link (for instance, investing more time in some relationships than in others).

The paper is organized as follows. In section 2, we describe the model. In Section 3, we present our results. In Section 4, we discuss the assumptions on our parameters. Section 5 concludes. In Appendix A we introduce the techniques that we use to study the stochastic stability and prove our results. In Appendix B we prove some technical results.

2 The Model

Networks. Let $N = \{1, \ldots, n\}$ be the set of players, where $n > 2$. Previously to the specification of the game we shall introduce some definitions. Let $G := \{g \subset N \times N : \text{for each } i, j \in N,\ (i,i) \notin g \text{ and } (i,j) \in g \iff (j,i) \in g\}$ be the set of *undirected networks*. We say (i,j) is a *link* of $g \in G$ if $(i,j) \in g$. Let $N_g = \{i \in N : \{(i,1), ...(i,n)\} \cap g \neq \emptyset\}$. Given $g \in G$, we say that $g' \subseteq g$ is a *sub-network* of g if, for each $i, j \in N_{g'}$, $(i,j) \in g \Longrightarrow (i,j) \in g'$.

We define a *path* of length $m \in \mathbb{N}$ in $g \in G$ from $i \in N$ to $j \in N\backslash\{i\}$ as a sequence of m consecutive links $\{(j_1, j_2), (j_2, j_3), \ldots, (j_m, j_{m+1})\} \subseteq g$ such that $j_1 = i$ and $j_{m+1} = j$.[11] Let $P^m_{i,j}(g)$ denote the set of all the paths of length m that exist from i to j in network g and let $P_{i,j}(g) = \cup_{m \in \mathbb{N}} P^m_{i,j}(g)$. We define the *distance* between players i and j in network g, $d_{i,j}(g)$, as the number of links in the shortest path between i and

find a parameter range in which both (homogeneous) action profiles coexist in the long run.

[10] Given their formulation, they use the terminology link strength instead of decay.

[11] Two links $(i,i'), (i'',i''') \in g$ are consecutive if $i' = i''$.

j in g, *i.e.*, $d_{i,j}(g) = \min_{p \in P_{i,j}(g)} |p|$. If $P_{i,j}(g) = \emptyset$, we set $d_{i,j}(g) = \infty$. We say that a sub-network g' of g is a *component* of g if, for each $i, j \in N_{g'}$ and each $m \in N \backslash N_{g'}$, $P_{i,j}(g) \neq \emptyset$ and $P_{i,m}(g) = \emptyset$. We say that $g \in G$ is *connected* if for each $i \in N$ and $j \in N \backslash \{i\}$, $P_{i,j}(g) \neq \emptyset$.

We say that $g \in G$ is *minimally connected* if it is connected and, for each $(i, j) \in g$, $g \backslash \{(i, j)\}$ is not connected. Let $G^m \subset G$ be the set of all minimally connected networks. The complete network is $g^{co} = (N \times N) \backslash \{(i, i)_{i \in N}\}$. A star with center $i \in N$ is a minimally connected network g^{st} such that, for all $j \in N \backslash \{i\}$, $(i, j) \in g^{st}$. Let $G^{st} \subset G^m$ be the set of all stars and, for each $g^{st} \in G^{st}$, let $\hat{\imath}(g^{st}) \in N$ denote the center of g^{st}.

Strategies, payoffs and efficiency. The strategy of each $i \in N$, $s_i = (L_i, a_i)$, consists on a subset of players $L_i \subseteq N \backslash \{i\}$ with whom to form links and an action $a_i \in \{\alpha, \beta\}$. Let $l_i = |L_i|$. For each $i \in N$, let $S_i = 2^{(N \backslash \{i\})} \times \{\alpha, \beta\}$ be the set of strategies of player i and let $S = \prod_{i \in N} S_i$.

Link formation is one-sided and non-cooperative, *i.e.*, the formation of a link only requires the consent of the player who initiates it. In this sense, $s \in S$ results in a network $g(s) \in G$ such that, for each $i, j \in N$, $(i, j) \in g(s)$ if and only if either $i \in L_j$ and/or $j \in L_i$. A strategy profile $s \in S$ is *essential* if, for each $i, j \in N$, $i \in L_j \Longrightarrow j \notin L_i$. Let $S^* \subset S$ be the set of essential strategy profiles.

Players obtain earnings from the network. We assume that each agent is endowed with one unit of non-rival good of value 1. In what follows, we shall refer to such non-rival good as information. Agents can access others' information through the network. In particular, we model a situation where the information flow is subjected to decay. The decay that the information suffers when it flows through a link $(i, j) \in g(s)$ is endogenous and depends on a_i and a_j. More precisely, the decay that player i incurs in when she receives the information from j, $\delta(a_i, a_j)$, is derived from the following 2×2 matrix,

	α	β
α	1	0
β	x	x

Table 1: The endogenous decay factor

where $1/2 < x < 1$. Table 1 is the payoff matrix of the bilateral stag-hunt game proposed by Carlsson and van Damme [4]. The safe action β yields a fixed decay factor $\delta(\beta, \alpha) = \delta(\beta, \beta) = x$. In contrast, action α yields the best decay factor $\delta(\alpha, \alpha) = 1$ if the other player also chooses α (*i.e.*, the information flows perfectly from j to i), but it yields the worst possible one $\delta(\alpha, \beta) = 0$ otherwise (*i.e.*, there is no information transmission from j to i).[12] Therefore, since $x \in (1/2, 1)$, (β, β) is the risk-dominant equilibrium of the bilateral stag-hunt game, as defined by Harsanyi and Selten [13], whereas (α, α) is the Pareto efficient equilibrium. This

[12]In Section 4 we discuss the implications of allowing for $x < 1/2$, $\delta(\alpha, \beta) > 0$ and $\delta(\alpha, \alpha) < 1$.

normalized game allows us to interpret our results in terms of a single parameter x that measures the degree of risk-dominance of action β.[13] Given $s \in S$, we assume that each $i \in N$ access the information from $j \in N^i(g) := \{j' : P_{i,j'}(g) \neq \emptyset\}$ using the path that allows her to receive it with the highest quality, *i.e.*, using $p_{i,j}(s) := \arg\max_{p \in P_{i,j}(g(s))} \prod_{(m,m') \in p} \delta(a_m, a_{m'})$. Thus, the total earnings that i obtains from $g(s)$ are $\sum_{j \in N^i(g(s))} \prod_{(m,m') \in p_{i,j}(s)} \delta(a_m, a_{m'})$.

On the other hand, links are costly. Each player pays a cost $c < 1$ for each link that she initiates.[14] Thus, the total cost that player i bears at period t is $l_i \cdot c$. Therefore, given $s \in S$, the payoff to agent i is

$$\Pi_i(s) = \sum_{j \in N^i(g(s))} \left(\prod_{(m,m') \in p_{i,j}(s)} \delta(a_m, a_{m'}) \right) - l_i \cdot c.$$

Regarding the notion of efficiency, we follow the convention in the literature of network formation and focus on the sum of payoffs of all players.[15] A state s is *efficient* if, for each $s' \in S$, $\sum_{i \in N} \Pi_i(s) \geq \sum_{i \in N} \Pi_i(s')$.

Dynamics. Time is considered discrete, $t = 0, 1, 2, ...$ At each period t, the state of the system is represented by a strategy profile $s(t) = \{s_i(t)\}_{i \in N} \in S$, where, for each $i \in N$, $s_i(t) = (L_i(t), a_i(t))$ as defined above. For simplicity, we will refer to the network associated to the state prevailing at period t as $g(t)$, *i.e.*, $g(t) = g(s(t))$. Let $s(0) \in S$ denote the initial state. At each period $t \geq 1$, one player is randomly selected to revise her strategy.[16] When a player receives a revision opportunity, she selects a myopic best response to the strategy profile of the previous period. There is also a small probability ε that the player trembles, and chooses a strategy that he did not intend to. Thus, with probability ε, there is a *mutation* and the strategy $s_i(t)$ is chosen at random (each $s_i \in S_i$ is chosen with positive probability) and, with probability $1 - \varepsilon$,

$$s_i(t) \in \arg\max_{s_i \in S_i} \Pi\left(s_i, s_{-i}(t-1)\right).$$

If there are several best responses, each of them is chosen with positive probability.

Stochastic stability. Let ΔS be the set of probability distributions over S. If we assume that $s(0)$ is chosen through a certain $\mu(0) \in \Delta S$, the dynamics described above defines a Markov chain on S. Let Q_ε be the $|S| \times |S|$ transition matrix, where $(Q_\varepsilon)_{s,s'} := \Pr\left(s(t) = s' \mid_{s(t-1)=s}\right)$ for each $s, s' \in S$. Then, the probability that the Markov process (S, Q_ε) leads to each state at period $t \geq 1$ is $\mu_\varepsilon(t) = \mu(0) \cdot (Q_\varepsilon)^t$.

[13] Given this normalization, we may also interpret x as the ratio between the decay factors associated to the efficient equilibrium and the risk-dominant one, *i.e.*, $x = \delta(\beta, \beta) / \delta(\alpha, \alpha)$.

[14] In Section 4 we discuss the implications of allowing for $c > 1$.

[15] This notion corresponds to the concept of strong efficiency in Jackson and Wolinsky [17].

[16] Note that, in each period, we only permit one player to revise her strategy. One interpretation is that strategy revisions are governed by a Poisson process, so that only one revision takes place during any sufficiently brief interval of time.

Given a Markov process (S, Q_ε), $\mu \in \Delta S$ is an invariant probability distribution if $\mu \cdot Q_\varepsilon = \mu$. For each $\varepsilon > 0$, (S, Q_ε) defines a Markov chain that is aperiodic and irreducible and, therefore, has a *unique* invariant probability distribution, namely μ_ε, and $\hat{\mu} = \lim_{\varepsilon \to 0} \mu_\varepsilon$ is well defined.[17] A state $s \in S$ is *stochastically stable* when $\hat{\mu}(s) > 0$. Let $\hat{S} := \{s \in S : \hat{\mu}(s) > 0\}$.

3 Results

In this section, after characterizing the efficient states, we analyze the properties of the stochastic stable states: Action profiles and network structures. For each $s \in S$, let the set of $\alpha - players$ of state s be $K_\alpha(s) := \{i \in N : a_i(s) = \alpha\}$ and let $k_\alpha(s) := |K_\alpha(s)|$. With some abuse of notation, when there is no confusion, we will denote by $K_\alpha(t)$ the set of $\alpha - players$ of the state prevailing at period t, *i.e.*, $K_\alpha(t) = K_\alpha(s(t))$ and, by $k_\alpha(t)$, its cardinality. The next proposition characterizes the efficient states.

Proposition 1 *A state s is efficient if and only if $s \in S^*$, $k_\alpha(s) = n$ and $g(s) \in G^m$.*

A formal proof is omitted since, once we show that an efficient state requires all players to choose action α, the fact that the strategy must be essential and the resulting network minimally connected directly follows from Proposition 4.3 of Bala and Goyal [1]. We provide the intuition. First, note that any state with all (or some) agents coordinated on action β is dominated by a state that prescribes the same link decisions and all agents coordinated on action α. When all players are coordinated on α, there are no differences in payoffs between direct and indirect links. Thus, a minimally connected network allows to connect all the players efficiently, since it uses the minimum possible number of links, *i.e.*, $n - 1$. On the other hand, an efficient strategy must be essential, since, otherwise, there is at least one link such that the cost is redundantly paid (both players pay c).

We shall now analyze the dynamic model and study the properties of the stochastically stable states in terms of our parameters (x and c). We first characterize the action profiles included in stochastically stable states (Theorem 1). Then, we study the networks that are associated to the different stochastically stable states (Propositions 2 and 3) and compare stability and efficiency (Corollary 1). All the proofs are in Appendix A.

The next theorem characterizes the action profiles associated to the stochastically stable states. Prior to the statement, we define $\bar{c}(x) := \max\{2 - \frac{1}{x}, 1 + x - \frac{x^3}{2x-1}\}$ and $\hat{c}(x) := \max\{x, 1 - x^2 - \frac{x}{n-2}\}$. In the statement of the theorem, ϕ represents the golden ratio.[18]

[17] See, for instance, Freidlin and Wentzell [11].

[18] The golden ratio, also known as the divine proportion, golden section, or golden mean, is a number often encountered when taking the ratios of distances in simple geometric figures. It is the only positive number satisfying $\phi = 1 + 1/\phi$. Specifically, $\phi = (\sqrt{5} + 1)/2$ and $1/\phi = (\sqrt{5} - 1)/2$.

Theorem 1 *Let $s \in \hat{S}$. For n large enough,*
I) If $x < 1/\phi$, (i) if $c < \bar{c}(x)$, $k_\alpha(s) = 0$, (ii) if $\bar{c}(x) < c < \hat{c}(x)$, $k_\alpha(s) = n$ and, (iii) if $c > \hat{c}(x)$, $k_\alpha(s) \in \{0, n\}$ and there is $s' \in \hat{S}$ such that $k_\alpha(s) \neq k_\alpha(s')$.
II) If $x > 1/\phi$, (i) if $c < \hat{c}(x)$, $k_\alpha(s) = 0$ and, (ii) if $c > \hat{c}(x)$, $k_\alpha(s) \in \{0, n\}$ and there is $s' \in \hat{S}$ such that $k_\alpha(s) \neq k_\alpha(s')$.

Since $x > 1/2$, our model presents a tension between efficiency and risk-dominance. Theorem 1 shows that, when c is higher than a certain threshold ($c > \hat{c}(x)$), both homogeneous action profiles ($k_\alpha = 0$ and $k_\alpha = n$) are significantly played in the long run. In contrast, when c is lower ($c < \hat{c}(x)$), only one of them is significantly observed. In this case, the golden ratio conjugate ($1/\phi$) defines a threshold for x.[19] If $x > 1/\phi$, in all the stochastically stable states, $k_\alpha = 0$, *i.e.*, risk-dominance consideration prevail. On the other hand, if $x < 1/\phi$, the value of c determines which action profile, $k_\alpha = 0$ or $k_\alpha = n$, prevails: For low cost ($c < \bar{c}(x)$), in all the stochastically stable states, $k_\alpha = 0$, whereas, for intermediate cost ($\bar{c}(x) < c < \hat{c}(x)$), only the efficient action profile is played in the stochastically stable states.

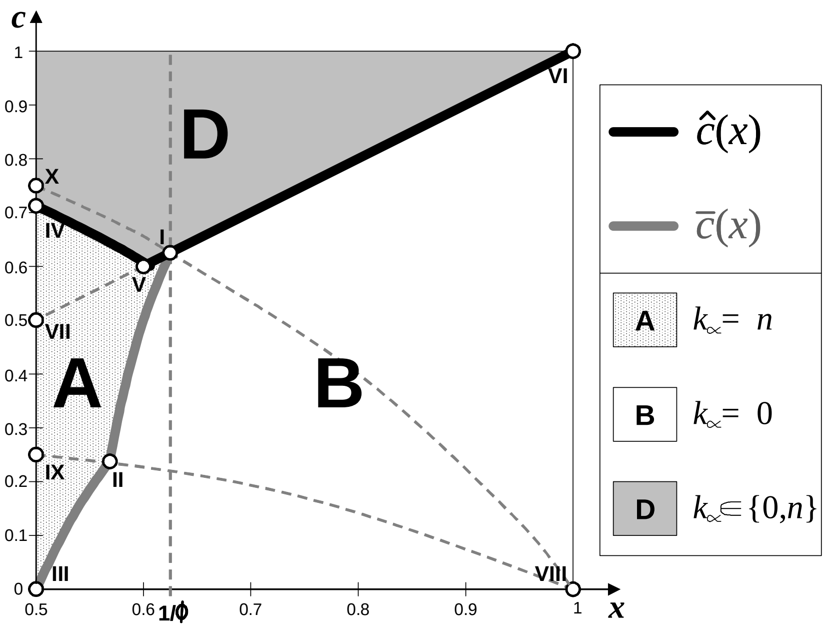

Figure 1: Action profiles in the stochastically stable states. The arc I-II is given by $c = 1 + x - \frac{x^3}{2x-1}$, II-III by $c = 2 - \frac{1}{x}$, VI-VII by $c = x$, VIII-IX by $c = x - x^2$, VIII-X by $c = 1 - x^2$ and IV-V by $c = 1 - x^2 - \frac{x}{n-2}$.[20]

In Figure 1, we provide a graphic illustration of Theorem 1. The thick lines represent $\hat{c}(x)$ and $\bar{c}(x)$ (black and dark grey, respectively), that intersect in point I

[19] Note that x may be interpreted as the ratio between the decay factors associated to the risk-dominant equilibrium and to the efficient one (*cf.* Footnote 13).

[20] Note that only arc IV-V depends on n. When n increases this arc moves up. In the limit, when $n \to \infty$, we get $c = 1 - x^2$ and, therefore, it coincides with arc I-X.

($x = 1/\phi$). For each $s \in \hat{S}$, in area A, $k_\alpha(s) = n$, in area B, $k_\alpha(s) = 0$ and, in area D, $k_\alpha(s) \in \{0, n\}$.

We now summarize the main arguments of the proof of Theorem 1. The proof is based on the notion of recurrent sets. A *recurrent set* is a collection of absorbing sets of states with the property that it is impossible for a single mutation, followed by the unperturbed dynamics, to lead to an absorbing set not contained in it.[21] We show that all the states with heterogenous action profiles (*i.e.*, those where there are players choosing different actions) are transient, in the sense that there are no recurrent sets containing them. We prove that there is a region where there is only one recurrent set (area above VII-I-VIII of Figure 1), and a region where there are two recurrent sets (area below VII-I-VIII). When there is just one recurrent set, all the states that are contained in it are stochastically stable (*cf.* Proposition 4 in Appendix A). Hence, in such a case, we characterize the action profiles played in all the states contained in the (unique) recurrent set. In contrast, when there are two recurrent sets, we analyze which of them is more robust to perturbations since, in this case, the states contained in it conform the set of stochastically stable states. Then, we characterize the action profiles played in the more robust recurrent set.

We first focus on the case where there is one recurrent set, *i.e.*, the area above VII-I-VIII. Intuitively, the higher the link cost, the easier it is, in terms of number of mutations, to leave states with all the players coordinated on some action and transit to a state with all the players coordinated in the alternative one. That is why, when c is high enough, there is only one recurrent set. For instance, in Figure 1, in the area above arc VI-VII (arc VIII-X), one mutation followed by the unperturbed dynamics allows for transitions from states with conformity on action β (α) to states with conformity on α (β). Therefore, there are states with $k_\alpha = n$ ($k_\alpha = 0$) contained in the unique recurrent set. For this reason, in the region above VI-I-X, both homogeneous action profiles are significantly observed in the long run. In fact, arcs VI-VII and VIII-X intersect in point I ($x = 1/\phi$), that separates the two cases of Theorem 1. As expected, since x is the risk-dominance parameter that measures the advantage of action β with respect to α, arc VI-VII (arc VIII-X) is increasing (decreasing) in x. In area I-VI-VIII, two or more mutations are needed to transit from states with $k_\alpha = 0$ to states with $k_\alpha = n$ and, therefore, in all the states contained in the unique recurrent set, $k_\alpha = 0$. In area I-VII-X, there are two regions: In region IV-V-VII, in all the states contained in the unique recurrent set, $k_\alpha = n$. In contrast, in region I-V-VI-X, one mutation is enough to transit from states with $k_\alpha = n$ to states with $k_\alpha = 0$ and, therefore, both action profiles are played in the unique recurrent set. This last region vanishes as n increases (*cf.* footnote 20) and, therefore, in the limit,

[21] The unperturbed dynamics corresponds to the case without mutations, *i.e.*, $\varepsilon = 0$. An absorbing set is a collection of states such that: (i) there is zero probability that the unperturbed dynamics can cause the system to exit it, and (ii) there is a positive probability of moving from one state in the set to any other state in a finite number of periods. See Appendix A and Samuelson [21] (chapter 7) for technical details.

only $k_\alpha = n$ would be significantly observed in area I-VII-X.

We now focus on the case where there are two recurrent sets, *i.e.*, the area below VII-I-VIII. We show that one recurrent set only contains states with $k_\alpha = n$ and the other one only contain states with $k_\alpha = 0$. When x is low ($x < 1/\phi$), if the cost is higher than a certain threshold ($c > \bar{c}(x)$) the recurrent set where $k_\alpha = n$ dominates whereas, if the cost is lower, risk-dominance considerations prevail. As expected, the threshold is increasing in x. In contrast, when x is high ($x > 1/\phi$), the recurrent set where $k_\alpha = 0$ prevails.

One important difference between our results and those of GV (and HS) is the presence of area D (where both homogeneous action profiles coexist). The main reason why GV do not obtain such a case is that, since they do not consider externalities from indirect links (in the main body of the paper), when c exceeds the payoffs from the risk-dominant action (but is lower than the payoff from the efficient one), no state where agents coordinate on β belongs to absorbing sets.[22] In this cost range, GV obtain that there is just one recurrent set, which only contains states with $k_\alpha = n$.

Once we have addressed the issue of coordination, a natural question is to study the network structures that are robust enough to be significantly observed in the long run. In the next proposition, we characterize the networks associated to the stochastically stable states where the efficient action profile is played.

Proposition 2 *For n large enough, if there is $s \in \hat{S}$ such that $k_\alpha(s) = n$, then $s \in S^*$, $g(s) \in G^m$ and, for each $g' \in G^m$ and each $s' \in S^*$ such that $g(s') = g'$ and $k_\alpha(s') = n$, $s' \in \hat{S}$.*

In Proposition 2 we show that, in areas A and D of Figure 1, in each stochastically stable state where $k_\alpha = n$, the strategy profile must be essential and the associated network minimally connected. The next corollary allows us to compare efficiency and stability in our framework.

Corollary 1 *For n large enough, the efficient states are contained in $\hat{S}$ if and only if there exists $s \in \hat{S}$ such that $k_\alpha(s) = n$. If there is no $s' \in \hat{S}$ such that $k_\alpha(s') = 0$, then $\hat{S}$ coincides with the set of efficient states.*

The result directly follows from Theorem 1 and Propositions 1 and 2. In area A of Figure 1 the set of efficient states and the set of stochastically stable states coincide. In area D, the set of efficient states is a subset of the set of stochastically stable states. Hence, in this region, efficient states are significantly observed in the long run, but also inefficient ones. In area B there is a conflict between efficiency and stability: Due to risk-dominance considerations, in this broad area of our parameter

[22] Moreover, in GV's extension (where agents obtain benefits from indirect connections), for this cost range, the fact that there are no frictions ($\delta = 1$) produces miscoordination problems (regarding the choices of links) when all players choose β. Therefore, also in this case no absorbing set contains states where $k_\alpha = 0$.

space no stochastically stable state is efficient.[23] Finally, in the following proposition, we study the network structures associated to the stochastically stable states where all players coordinate on the risk-dominant action.

Proposition 3 *For n large enough, if there is $s \in \hat{S}$ such that $k_\alpha(s) = 0$, then $s \in S^*$ and*

I) If $c < x - x^2$, then $g(s) = g^{co}$ and, for each $s' \in S^*$ such that $g(s') = g^{co}$ and $k_\alpha(s') = 0$, $s' \in \hat{S}$.

II) If $x - x^2 < c < x - x^3$, then, for each $s' \in S^*$ such that $g(s') \in G^{st}$ and $k_\alpha(s') = 0$, $s' \in \hat{S}$.

III) If $x - x^3 < c < x$, then $g(s) \in G^{st}$ and for each $s' \in S^*$ such that $g(s') \in G^{st}$ and $k_\alpha(s') = 0$, $s' \in \hat{S}$.

IV) If $c > x$, then, for each $s' \in S^*$ such that $k_\alpha(s') = 0$, $g(s') \in G^{st}$ and $L_{\hat{\imath}(g(s'))} = \emptyset$, $s' \in \hat{S}$.[24]

In Proposition 3 we analyze which networks arise in the long run in area B of Figure 1 and which networks are associated to the stochastically stable states with $k_\alpha = 0$ of area D. In both cases, the strategy profile must be essential. Area B has two differentiated regions regarding the network structure: When the cost is low enough, *i.e.*, $c < x - x^2$ (region II-III-VIII), the complete network is the unique stochastically stable network. When c is intermediate, *i.e.*, $x - x^2 < c < x$ (region I-II-VIII-VI) all the stars are associated to stochastically stable states. Moreover, in the wider part of this region ($x - x^3 < c < x$), we show that only the stars are significantly observed in the long run. In area D (high cost), we find that all the periphery-sponsored stars (*cf.* footnote 24) are associated to stochastically stable states with $k_\alpha = 0$, although there might be additional network structures related to them.

Interestingly, in region III-VIII-XI of Figure 1 (low cost), the network structures associated to the stochastically stable states with $k_\alpha = n$ and the network associated to those ones with $k_\alpha = 0$ are polar cases:[25] While the former ones are minimally connected, the later one is complete. In contrast, in region VI-VII-IX-VIII (intermediate cost), there are networks of the very same nature associated to stochastically stable states with $k_\alpha = 0$ and with $k_\alpha = n$ (for instance, the stars). Hence, for low cost, the completeness of the network associated to the stochastically stable states with $k_\alpha = 0$ makes the recurrent set containing them relatively more robust to perturbations than the efficient one (with respect to the case of intermediate cost). This issue is clear in Figure 1, since in region III-VIII-IX, area B relatively grows at the expense of area A with respect to region VI-VII-IX-VIII.

[23] In a pioneering paper, Jackson and Wolinsky [17] show that efficiency and stability in networks do not always coincide.

[24] Note that $s' \in S^*$, $g(s') \in G^{st}$ and $L_{\hat{\imath}(g(s'))} = \emptyset$ imply that each peripheral player meets the cost of her link. This is known in the literature as a periphery-sponsored star (*cf.* Bala and Goyal [1]).

[25] They are polar with respect to the total number of links within the set of connected networks.

4 Discussion

In this section, we discuss our assumptions on the parameters of the model.

So far, we have assumed $x > 1/2$ and have shown (Theorem 1) that, in the long run, the tension between efficiency and risk-dominance is differently solved in three regions of our space of parameters (areas A, B and D of Figure 1). If we were to consider $x \leq 1/2$, then (α, α) would be both the efficient and the risk-dominant equilibrium of the bilateral game presented in Table 1. Hence, we might expect the region where the efficient action profile ($k_\alpha = n$) is played in stochastically stable states to increase significantly. In the proofs of our main results (Appendix A and B), we basically use the assumption $x > 1/2$ to show that, when $c > x$, those states where an heterogenous action profile is played are transient.[26] Thus, when $x \leq 1/2$, if $c > x$, we can not assert that in the stochastically stable states $k_\alpha \in \{0, n\}$. Hence, there might be additional stochastically stable states as compared to Theorem 1. Abstracting from these considerations, our intuition is that, when $x \leq 1/2$, (i) all the efficient states will be stochastically stable (*i.e.*, there will be no region equivalent to area B of Figure 1), and (ii) when the cost is high enough, there will be inefficient stochastically stable states (those essential states where $k_\alpha = 0$ and a periphery-sponsored star forms).[27]

In our model, we have considered $\delta(\alpha, \beta) = 0$. We shall now discuss the possible effects of allowing for $\delta(\alpha, \beta) > 0$. If $\delta(\alpha, \beta)$ were close to zero, our main results should hold, since no $\alpha - player$ would be willing to form links to $\beta - players$. In contrast, if $\delta(\alpha, \beta)$ were higher, then $\alpha - players$ may form links to $\beta - players$. In this case, the compatibility of action α would increase and, therefore, it would become more attractive. Thus, intuitively, it should be easier (in terms of number of mutations) to reach recurrent sets where α is played (and more difficult to leave them). Thus, we conjecture that the region of parameters where the efficient states are stochastically stable increases with respect to Figure 1. If $2x - 1 < \delta(\alpha, \beta) < 1$, then, as in the previous point ($x \leq 1/2$), (α, α) would be both the efficient and the risk-dominant equilibrium of the bilateral game. Thus, both forces (efficiency and risk-dominance) would move the system in the same direction, *i.e.*, coordination on action α.

We have also assumed $\delta(\alpha, \alpha) = 1$. This assumption implies not only that action α is more efficient than β, but also that there is an additional asymmetry between actions (there is decay when choosing one action and not when choosing the other). We now discuss the possible effects of allowing for $\delta(\alpha, \alpha) = y$, with $x < y < 1$. In such a case, when $c < y - y^2$, those states contained in recurrent sets such that $k_\alpha = n$ would be associated to the complete network. In contrast, when $c > y - y^2$, these states could be associated to (a subset of) minimally connected networks (including stars) and, maybe, also to other kinds of networks. The fact that recurrent sets admit different interaction structures would affect the robustness to perturbations

[26] See Lemma 11.

[27] This should occur when $c > 1 - x^2 - \frac{x}{n-2}$, *i.e.*, $c > \hat{c}(x)$, analogously to the case $x > 1/2$.

and the subsequent stochastic selection. Nevertheless, we believe that the results of Theorem 1 should qualitatively hold for the following two reasons: (i) When c is close to zero, those states contained in recurrent sets will be characterized by the fact that all players coordinate on the same action (either α or β) and the complete network forms. In such a case, the model becomes very similar to GV. Thus, we can invoke their result that states that, when the link cost is sufficiently low, those states with all players coordinated on the risk-dominant action will be selected in the long run. (ii) When $c > y - y^2$, it is possible to show that, among the set of states with all agents coordinated on α, only one recurrent set can exist, and it must contain those essential states such that the associated network is a star. In fact, the proofs of our results suggest that the robustness of the recurrent set of states with $k_\alpha = n$ depends on the robustness of states where the network is a star.[28] Therefore, it is quite likely that, for high link cost, the long run selection will produce similar results to those obtained in Theorem 1.

Last, we have restricted our analysis to the case $c < 1$, which, in our view, provides the most interesting results. However, if we were to consider $c > 1$ (but not too high), in the stochastically stable states, either the empty network (no links) would arise and/or the action profile played would be $k_\alpha = 0$.[29] If c were high enough, only the empty network would be significantly observed in the long run. The fact that, when $c > 1$, no efficient state is stochastically stable is deeply affected by the assumption $\delta(\alpha, \alpha) = 1$. When $c > 1$ and $\delta(\alpha, \alpha) = 1$, it is possible to show that, starting from any state where $k_\alpha = n$, the unperturbed dynamic leads the system into a state characterized by the empty network. This fact is not true if we assume $\delta(\alpha, \alpha) < 1$.

5 Conclusion

We have analyzed, in a stylized form, an evolving social network with endogenous decay. Our framework integrates the coordination problem of GV and HS and the network formation problem in the presence of decay of BG. Indeed, in our model, the link cost and the trade-off between efficiency and risk-dominance affect both the action in which agents coordinate in the long run (as in GV and HS) and the network architecture (as in BG). A novelty of the present paper is to show that, depending on parameters, different stochastically stable sets can admit different network structures.

Further developments can be made in different directions. For instance, it would be interesting to consider endogenous decay in a two-sided link formation model or to endogenize the decay using different social games, which may better represent other real world situations. These extensions are left for future research.

[28] See Lemmas 3, 4 and 6 in Appendix A.

[29] The interested reader is referred to Feri [10] for a derivation of these results.

References

[1] V. Bala, S. Goyal, A non cooperative model of network formation, Econometrica 68 (2000), 1181-1231

[2] V. Bhaskar, F. Vega-Redondo, Migration and the evolution of conventions, Journal of Economic Behavior and Organization 55 (2004), 397-418

[3] F. Bloch, B. Dutta, Communication networks with endogenous link strength, Games Econ. Behav. 66 (2009), 39-56

[4] H. Carlsson, E. van Damme, Equilibrium selection in stag hunt games. In: K.G. Binmore, A. Kirkman, P. Tani (eds.) Frontiers of Game Theory, MIT Press, Cambridge, MA, (1993)

[5] F. Deroian, Endogenous link strength in directed communication networks, Mathematical Social Sciences 57 (2009), 110-116

[6] T. Dieckmann, The evolution of conventions with mobile players, Journal of Economic Behavior and Organization 38 (1998), 93-111

[7] G. Ellison, Learning, local interaction and coordination. Econometrica 61 (1993), 1047-1071

[8] J. Ely, Local Conventions, Advances in Theoretical Economics 2 (2002)

[9] F. Feri , Stochastic stability in networks with decay, J. Econ. Theory 135 (2007), 442-457

[10] F. Feri, Network formation with endogenous decay, Working Papers in Economics and Statistics 2007-14, University of Innsbruck (2007)

[11] M. Freidlin, A. Wentzell, Random Perturbations of Dynamical Systems. Springer Verlag, New York, (1984)

[12] S. Goyal, F. Vega-Redondo, Network formation and social coordination, Games Econ. Behav. 50 (2005), 178-207

[13] J. Harsanyi, R. Selten, A General Theory of Equilibrium Selection. MIT Press, Cambridge, MA, (1988)

[14] D. Hojman, A. Szeidel, Core and periphery in endogenous networks, J. Econ. Theory 139 (2008), 295-309

[15] D. Hojman, A. Szeidl, Endogenous networks, social games, and evolution, Games Econ. Behav. 55 (2006), 112-130

[16] M.O. Jackson, A. Watts, On the formation of interaction networks in social coordination games, Games Econ. Behav. 41 (2002), 265-291

[17] M.O. Jackson, A. Wolinsky, A strategic model of social and economic networks, J. Econ. Theory 71 (1996), 44-74

[18] M. Kandori, G.J. Mailath, R. Rob, Learning, mutation and long run equilibria in games, Econometrica 61 (1993), 29-56

[19] G. Mailath, L. Samuelson, A. Shaked, Endogenous interactions. In: A. Nicita, U. Pagano (ed.) The Evolution of Economic Diversity. Routledge, New York (2001)

[20] M.A. Meléndez-Jiménez, A bargaining approach to coordination in networks, Int. J. Game Theory 37 (2008), 439-456

[21] L. Samuelson, Evolutionary Games and Equilibrium Selection, MIT Press, Cambridge, MA, (1997)

[22] A. Watts, A dynamic model of network formation, Games Econ. Behav. 34 (2001), 331-341

[23] H.P. Young, The evolution of conventions, Econometrica 61 (1993), 57-84

Appendix A

We shall prove Theorem 1 and Propositions 2 and 3. To this aim, in order to identify the stochastically stable states, we use the techniques introduced by Kandori, Mailath and Rob [18] and Young [23], that are summarized as follows.

We call *unperturbed dynamics* to the extreme case in which $\varepsilon = 0$ (*i.e.*, players do not make mistakes when they revise their strategies). A set $\widetilde{S} \subset S$ is an *absorbing set* if the unperturbed Markov process (S, Q_0) is such that, (i) for each $s' \in \widetilde{S}$, $\sum_{s \in \widetilde{S}} (Q_0)_{s',s} = 1$ and, (ii) for each $s', s'' \in \widetilde{S}$, there exists $t \geq 1$ such that $(Q_0)^t_{s',s''} > 0$. Let $\mathcal{S}$ be the set of absorbing sets of (S, Q_0). A $\widetilde{S} - tree$ is a directed graph on $\mathcal{S}$ whose root is $\widetilde{S}$ and such that there is a unique (directed) path in the graph from each $\widetilde{S}' \in \mathcal{S} \backslash \{\widetilde{S}\}$ to $\widetilde{S}$. Given a $\widetilde{S} - tree$, we define a *cost for each arrow* $\widetilde{S}' \to \widetilde{S}''$ as the minimum number of mutations that are required for the transition from $\widetilde{S}'$ to $\widetilde{S}''$ to be feasible through the unperturbed dynamics. The *cost of the* $\widetilde{S} - tree$ is obtained by adding up the costs associated with all its arrows. The *stochastic potential* of $\widetilde{S}$ is defined as the minimum cost across all $\widetilde{S} - trees$. Then an absorbing set $\widetilde{S}$ is *stochastically stable* if it has the minimum stochastic potential across $\mathcal{S}$.

Definition 1 *(Samuelson [21], Definition 7.4.) A recurrent set $R \subseteq \mathcal{S}$ is a collection of absorbing sets with the following two properties:*
(I) Starting from $\widetilde{S} \in R$, by means of a single perturbation followed by the unperturbed dynamics, it is impossible to end up in $\widetilde{S}' \notin R$;
(II) Given a pair $\widetilde{S}', \widetilde{S}'' \in R$, there is a sequence $\{\widetilde{S}_1, \ldots, \widetilde{S}_V\} \subseteq R$ such that $\widetilde{S}_1 = \widetilde{S}'$, $\widetilde{S}_V = \widetilde{S}''$ and, for each $v \in [2, V]$, by a single mutation followed by the unperturbed dynamics, the transition from $\widetilde{S}_{v-1}$ to $\widetilde{S}_v$ is feasible.

In what follows, we shall refer the kind of sequence described in Property (II) of Definition 1 as a *path of one step mutations*. Let $\mathcal{R} \subset 2^{\mathcal{S}}$ be the set of recurrent sets and, for each $R \in \mathcal{R}$, let $E(R) = \bigcup_{\widetilde{S} \in R} \widetilde{S}$. We shall use the following two results.

Lemma 1 *(Samuelson [21], Lemma 7.1) At least one recurrent set exists. Recurrent sets are disjoint.*

Proposition 4 *(Samuelson [21], Proposition 7.7) $\hat{S} \subseteq \bigcup_{R \in \mathcal{R}} E(R)$ and, for each $R \in \mathcal{R}$, $\hat{S} \cap E(R) \in \{\emptyset, E(R)\}$.*[30]

Hence, as a first step to prove Theorem 1 and Propositions 2 and 3, we begin by stating Lemmas 2 to 6, that are proven in Appendix B. In Lemmas 2, 3 and 4, we identify $\mathcal{R}$ when $c < x - x^2$, $x - x^2 < c < x$ and $c > x$, respectively.[31] These three lemmas show that $|\mathcal{R}| \in \{1, 2\}$. By Proposition 4, when $|\mathcal{R}| = 1$, *i.e.*, $\mathcal{R} = \{R\}$, $\hat{S} = E(R)$. In contrast, if $|\mathcal{R}| = 2$, *i.e.*, $\mathcal{R} = \{R_h, R_{h'}\}$, in order to characterize $\hat{S}$, we need to compute the stochastic potential of the absorbing sets belonging to R_h and $R_{h'}$. We denote by $\omega_{hh'}$ the *minimum (mutation) cost* across all paths connecting one absorbing set of R_h to one absorbing set of $R_{h'}$. If $\omega_{hh'} > \omega_{h'h}$, then the absorbing sets in R_h have the minimum stochastic potential across $\mathcal{S}$ and, therefore, $\hat{S} = E(R_h)$. On the other hand, if $\omega_{hh'} = \omega_{h'h}$, then all the absorbing sets in $R_h \cup R_{h'}$ have the minimum stochastic potential across $\mathcal{S}$ and, therefore, $\hat{S} = E(R_h) \cup E(R_{h'})$. Hence, in Lemmas 5 and 6, we obtain the minimum (mutation) costs for the ranges of c such that, given Lemmas 2 to 4, $|\mathcal{R}| = 2$, *i.e.*, $c < x - x^2$ and $x - x^2 < c < \min\{x, 1 - x^2\}$. For each $z \in \mathbb{R}$, we denote by $\lceil z \rceil$ the least (strictly) positive integer not less than z.

Lemma 2 *Let $c < x - x^2$. For n large enough, $\mathcal{R} = \{R_\alpha, R_\beta\}$, where $E(R_\alpha) = \{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\}$ and $E(R_\beta) = \{s \in S^* : k_\alpha(s) = 0$ and $g(s) = g^{co}\}$.*

Lemma 3 *Let $x - x^2 < c < x$. For n large enough,*
I) If $c < 1 - x^2$, then $\mathcal{R} = \{R_\alpha, R_{\beta'}\}$, where $E(R_\alpha) = \{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\}$ and, for each $s' \in E(R_{\beta'})$, $k_\alpha(s') = 0$.
II) If $c > 1 - x^2$, then $\mathcal{R} = \{R_{\beta''}\}$ and, for each $s \in E(R_{\beta''})$, $k_\alpha(s) = 0$.
III) For each $s \in S^*$ such that $k_\alpha(s) = 0$ and $g(s) \in G^{st}$, $s \in \bigcup_{R \in \mathcal{R}} E(R)$.

[30] We have adapted the statement of Proposition 7.7 of Samuelson [21] to our notation.
[31] Note that, in Figure 1, this partition is provided by arcs VI-VII and VIII-IX.

Lemma 4 *Let $c > x$. For n large enough,*
I) If $c < 1 - x^2 - \frac{x}{n-2}$, then $\mathcal{R} = \{R_\alpha\}$ where $E(R_\alpha) = \{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\}$.
II) If $c > 1 - x^2 - \frac{x}{n-2}$, then $\mathcal{R} = \{R_1\}$, where R_1 is such that
(i) $E(R_1) \supseteq \{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\}$,
(ii) $E(R_1) \supseteq \{s \in S^* : k_\alpha(s) = 0$, $g(s) \in G^{st}$ and $l_{\hat{\imath}(g(s))} = 0\}$ and
(iii) for each $s \in E(R_1)$, $k_\alpha(s) \in \{0, n\}$ and, if $k_\alpha(s) = n$, then $s \in \{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\}$.

Lemma 5 *If $c < x - x^2$, then $\omega_{\alpha\beta} = \lceil (n-1)(1-x) \rceil$ and $\omega_{\beta\alpha} = \left\lceil \frac{(n-1)(x-c)}{1-c} \right\rceil$.*

Lemma 6 *If $x - x^2 < c < \min\{x, 1 - x^2\}$, then $\omega_{\alpha\beta'} = \left\lceil \frac{(n-1)(1-c-x^2)}{1-c-x^2+x} \right\rceil$ and $\omega_{\beta'\alpha} = \left\lceil \frac{(n-1)(x-c)}{1-c} \right\rceil$.*

Proof of Theorem 1

First, we claim that, if $c < \min\{2 - \frac{1}{x}, x - x^2\}$, for each $\hat{s} \in \hat{S}$, $k_\alpha(\hat{s}) = 0$ whereas, if $2 - \frac{1}{x} < c < x - x^2$, for each $\hat{s} \in \hat{S}$, $k_\alpha(\hat{s}) = n$. Let $c < x - x^2$. By Lemma 2, for n large enough, $\mathcal{R} = \{R_\alpha, R_\beta\}$. Thus, by Proposition 4, either $\hat{S} = E(R_\alpha)$, $\hat{S} = E(R_\beta)$, or $\hat{S} = E(R_\alpha) \cup E(R_\beta)$. Given Lemma 5, if $c > 2 - \frac{1}{x}$, then $\omega_{\alpha\beta} > \omega_{\beta\alpha}$ and, therefore, $\hat{S} = E(R_\alpha)$. In contrast, if $c < 2 - \frac{1}{x}$, then $\omega_{\beta\alpha} > \omega_{\alpha\beta}$ and, therefore, $\hat{S} = E(R_\beta)$. Hence, by Lemma 2, the claim follows.

Second, we claim that, if $x - x^2 < c < \min\{x, 1 + x - \frac{x^3}{2x-1}\}$, for each $\hat{s} \in \hat{S}$, $k_\alpha(\hat{s}) = 0$ whereas, if $\max\{x - x^2, 1 + x - \frac{x^3}{2x-1}\} < c < x$, for each $\hat{s} \in \hat{S}$, $k_\alpha(\hat{s}) = n$. Let $x - x^2 < c < x$. By Lemma 3, for n large enough, if $c > 1 - x^2$, then $\mathcal{R} = \{R_{\beta''}\}$ and, if $c < 1 - x^2$, then $\mathcal{R} = \{R_\alpha, R_{\beta'}\}$. Hence, by Proposition 4, if $c > 1 - x^2$, then $\hat{S} = E(R_{\beta''})$ whereas, if $c < 1 - x^2$, then either $\hat{S} = E(R_\alpha)$, $\hat{S} = E(R_{\beta'})$, or $\hat{S} = E(R_\alpha) \cup E(R_{\beta'})$. In the latter case, by Lemma 6, if $c > 1 + x - \frac{x^3}{2x-1}$, then $\omega_{\alpha\beta'} > \omega_{\beta'\alpha}$ and, therefore, $\hat{S} = E(R_\alpha)$. In contrast, if $c < 1 + x - \frac{x^3}{2x-1}$, then $\omega_{\beta'\alpha} > \omega_{\alpha\beta'}$ and, therefore, $\hat{S} = E(R_{\beta\prime})$. Hence, by Lemma 3, the claim follows.[32]

Third, we claim that if $x < c < 1 - x^2 - \frac{x}{n-2}$, for each $\hat{s} \in \hat{S}$, $k_\alpha(\hat{s}) = n$ whereas, if $c > \max\{x, 1 - x^2 - \frac{x}{n-2}\}$, for each $\hat{s} \in \hat{S}$, $k_\alpha(\hat{s}) \in \{0, n\}$ and there exist $\hat{s}', \hat{s}'' \in \hat{S}$ such that $k_\alpha(\hat{s}') = n$ and $k_\alpha(\hat{s}'') = 0$. Let $c > x$. By Lemma 4, for n large enough, if $c < 1 - x^2 - \frac{x}{n-2}$, then $\mathcal{R} = \{R_\alpha\}$ and, if $c > 1 - x^2 - \frac{x}{n-2}$, then $\mathcal{R} = \{R_1\}$. Hence, by Proposition 4, if $c < 1 - x^2 - \frac{x}{n-2}$, then $\hat{S} = E(R_\alpha)$ and, if $c > 1 - x^2 - \frac{x}{n-2}$, then $\hat{S} = E(R_1)$. Hence, by Lemma 4, the claim follows.

The third claim proves parts I.(iii) and II.(ii). Hence, in what follows, we assume that $c < \hat{c}(x)$. Since $\bar{c}(x) = \max\{2 - \frac{1}{x}, 1 + x - \frac{x^3}{2x-1}\}$ and $\hat{c}(x) = \max\{x, 1 - x^2 - \frac{x}{n-2}\}$,

[32] Note that, since $c > x$, $c > 1 - x^2$ implies $c < 1 + x - \frac{x^3}{2x-1}$.

$\bar{c}(x) < \hat{c}(x)$ if and only if $x < \frac{\sqrt{5}-1}{2}$ (*i.e.*, $x < 1/\phi$). It is directly verifiable that, if $c < x - x^2$, then $\bar{c}(x) = 2 - \frac{1}{x}$ and that, if $c > x - x^2$, then $\bar{c}(x) = 1 + x - \frac{x^3}{2x-1}$.

Let $x < 1/\phi$. We distinguish the cases $c < \bar{c}(x)$ and $c > \bar{c}(x)$. (i) $c < \bar{c}(x)$ if and only if either $c < \min\{2 - \frac{1}{x}, x - x^2\}$ or $x - x^2 < c < \min\{x, 1 + x - \frac{x^3}{2x-1}\}$. Hence, by the first two claims, if $c < \bar{c}(x)$, then for each $\hat{s} \in \hat{S}$, $k_\alpha(\hat{s}) = 0$. (ii) $\bar{c}(x) < c < \hat{c}(x)$ if and only if either $2 - \frac{1}{x} < c < x - x^2$, $\max\{x - x^2, 1 + x - \frac{x^3}{2x-1}\} < c < x$, or $x < c < 1 - x^2 - \frac{x}{n-2}$. Hence, by the three claims, if $\bar{c}(x) < c < \hat{c}(x)$, then for each $\hat{s} \in \hat{S}$, $k_\alpha(\hat{s}) = n$. This proves part I.

Let $x > 1/\phi$. Then, $c < \hat{c}(x)$ if and only if either $c < \min\{2 - \frac{1}{x}, x - x^2\}$ or $x - x^2 < c < \min\{x, 1 + x - \frac{x^3}{2x-1}\}$.[33] Hence, by the first two claims, if $c < \hat{c}(x)$, then for each $\hat{s} \in \hat{S}$, $k_\alpha(\hat{s}) = 0$. This proves part II. □

Proof of Proposition 2

Assume that there is $s \in \hat{S}$ such that $k_\alpha(s) = n$. By Proposition 4, there is a recurrent set, namely R', such that $s \in E(R')$ and $E(R') \subseteq \hat{S}$. By Lemmas 2 to 4, $s \in S^*$, $g(s) \in G^m$ and $E(R') \supseteq \{s' \in S : k_\alpha(s') = n \text{ and } g(s') \in G^m\}$. □

Finally, prior to the proof of Proposition 3, we shall state the following Lemma, that is proven in Appendix B.

Lemma 7 *Let $x - x^3 < c < x$. From any state in $\{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$, for n large enough, one mutation followed by the unperturbed dynamics leads the system with probability one to a state in $\{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$.*

Proof of Proposition 3

Assume there is $s \in \hat{S}$ such that $k_\alpha(s) = 0$.

First, let $c < x - x^2$. By Lemma 2, $\mathcal{R} = \{R_\alpha, R_\beta\}$. By Proposition 4, either $\hat{S} = E(R_\alpha)$, $\hat{S} = E(R_\beta)$, or $\hat{S} = E(R_\alpha) \cup E(R_\beta)$. Since $k_\alpha(s) = 0$, $s \in E(R_\beta)$. This proves I.

Second, let $x - x^2 < c < x$. By Proposition 4, there is a recurrent set, namely R', such that $s \in E(R')$ and $E(R') \subseteq \hat{S}$. By Lemma 3, if $c < 1 - x^2$, $R' = R_{\beta'}$ and, if $c > 1 - x^2$, $R' = R_{\beta''}$. Moreover, By Lemma 3, $E(R') \supseteq \{s' \in S^* : k_\alpha(s') = 0$ and $g(s') \in G^{st}\}$. This proves II. If $x - x^3 < c < x$, by Lemma 7, $\{s' \in S^* : k_\alpha(s') = 0$ and $g(s') \in G^{st}\}$ satisfies property (I) of Definition 1. Hence, in such a case, $E(R') = \{s' \in S^* : k_\alpha(s') = 0$ and $g(s') \in G^{st}\}$ and, by Proposition 4, either $\hat{S} = E(R_\alpha)$, $\hat{S} = E(R')$, or $\hat{S} = E(R_\alpha) \cup E(R')$. This proves III.

Finally, let $c > x$. By Proposition 4, there is a recurrent set, namely R'', such that $s \in E(R'')$ and $E(R'') \subseteq \hat{S}$. Since $k_\alpha(s) = 0$, by Lemma 4, $R'' = R_1$, $\mathcal{R} = \{R_1\}$ and $E(R_1) \supseteq \{s' \in S^* : k_\alpha(s') = 0$, $g(s') \in G^{st}$ and $l_{\hat{\imath}(g(s'))} = 0\}$. □

[33] If $x > 1/\phi$, then it is directly verifiable that $\hat{c}(x) = x$, $\min\{2 - \frac{1}{x}, x - x^2\} = x - x^2$ and $\min\{x, 1 + x - \frac{x^3}{2x-1}\} = x$.

Appendix B

This Appendix is devoted to prove Lemmas 2 to 7. To this aim, as a first step, we begin by stating and proving Lemmas 8 to 12. In Lemmas 8 and 9, we study the convergence of the unperturbed dynamics when the initial state has conformity in the choice of actions ($k_\alpha \in \{0, n\}$). The main lessons from these two lemmas are: (i) there are states with $k_\alpha = n$ and states with $k_\alpha = 0$ in absorbing sets, (ii) the states with $k_\alpha = n$ that belong to absorbing sets must have associated a minimally connected network and (iii) the states with $k_\alpha = 0$ that belong to absorbing sets must be in absorbing sets that do not include any state with $k_\alpha = n$. Lemma 10 is instrumental for the proof of Lemma 11, that identifies, for different parameter ranges, the states that belong to absorbing sets and that, therefore, are candidates to belong to recurrent sets. We show that a state in an absorbing set must be essential and have $k_\alpha \in \{0, n\}$. In Lemma 12, we show that if there is a recurrent set that contains a state with $k_\alpha = n$, then all the essential states with $k_\alpha = n$ that result in a minimally connected network must be in such a recurrent set. Once we have formally stated and proved these Lemmas, we use them to prove Lemmas 2 to 7.

Prior to the statement of Lemmas 8 to 12, we shall introduce some notation. For each $s \in S$ and $M_\alpha(s) \subseteq K_\alpha(s)$, let $g_{M_\alpha(s)}$ be the sub-network of $g(s)$ such that $N_{g_{M_\alpha(s)}} = M_\alpha(s)$. We say that $M_\alpha(s)$ is an $\alpha - group$ of s if $g_{M_\alpha(s)}$ is a component of $g_{K_\alpha(s)}$. Let $\mathcal{M}_\alpha(s)$ be the set of $\alpha - groups$ of state s. We say that $M_\alpha(s) \in \mathcal{M}_\alpha(s)$ is minimally connected if, for each $(i,j) \in g_{M_\alpha(s)}$, $M_\alpha(s)$ is split into two distinct $\alpha{-}groups$ in $g(s) \backslash \{(i,j)\}$. For each period $t > 0$, we denote by $i_t \in N$ the player that is (randomly) chosen to revise her strategy at t. Moreover, $a_{i_t} \rightsquigarrow \alpha$ denotes i_t chooses action α when she revises her strategy (similarly for $a_{i_t} \rightsquigarrow \beta$). We define $K_\beta(s)$ (*i.e.*, the set of $\beta - players$ of state s) and $k_\beta(s)$ analogously to $K_\alpha(s)$ and $k_\alpha(s)$. Finally, for each $s = \{(L_i, a_i)_{i \in N}\} \in S$, $a \in \{\alpha, \beta\}$ and $j \in N$, let l_j^a be the number of links that j supports to players in $K_a(s)$.[34]

Lemma 8 *Let* $\varepsilon = 0$.

I) If, for some $t \geq 0$, $k_\alpha(t) = n$, *then there is* $T > t$ *such that, for each* $t' \geq T$, $s(t') \in S^*$, $k_\alpha(t') = n$ *and* $g(t') \in G^m$.

II) If $s \in S^*$, $k_\alpha(s) = n$ *and* $g(s) \in G^m$, *then* $s \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$.

Proof. We first prove I). Let $s(t) \in S$ be such that $k_\alpha(t) = n$. For each $L_{i_{t+1}}(t+1) \subseteq N \backslash \{i_{t+1}\}$, if $a_{i_{t+1}} \rightsquigarrow \alpha$, $\Pi_{i_{t+1}}(s(t+1))$ is strictly higher than if $a_{i_{t+1}} \rightsquigarrow \beta$.[35] Hence, $a_{i_{t+1}} \rightsquigarrow \alpha$. Analogously, for each $\tau > t+1$, $a_{i_\tau} \rightsquigarrow \alpha$. Let period T be such that all players have received a revision opportunity. Then, for each $\tau > t$ and $j \in N$ such that $i_\tau \in L_j(\tau - 1)$, $j \notin L_{i_\tau}(\tau)$. Thus, for each $t' \geq T$, $s(t') \in S^*$. We now

[34] Note that, for each $j \in N$, $l_j = l_j^\alpha + l_j^\beta$.

[35] Only if $i_{t+1} \notin N_{g(t+1)}$, i_{t+1} is indifferent between choosing α or β. However, in such a case, $L_{i_{t+1}}(t+1) = \emptyset$ is not a best response.

claim that at each $\tau > t$, $g(\tau)$ is connected. Assume not, *i.e.*, there is $j \in N$ such that $P_{i_\tau,j} = \emptyset$. Since $a_j(\tau) = a_i(\tau) = \alpha$ and $c < 1$, $j \notin L_{i_\tau}(\tau)$ contradicts that i_τ has chosen a best response. Finally, we claim that for each $\tau > t$ and $j \in N$ such that $j \in L_{i_\tau}(\tau)$, $g(\tau)\backslash\{(i_\tau, j)\}$ is not connected. Assume not. Then, since $k_\alpha(\tau) = n$, $L'_{i_\tau}(\tau) = L_{i_\tau}(\tau)\backslash\{j\}$ increases the payoff to i_τ at τ by $c > 0$, a contradiction with a best response. Thus, for all $t' > T$, $g(t') \in G^m$.

We now prove II). Let $s' \in S^*$, $k_\alpha(s') = n$ and $g(s') \in G^m$ and assume, for the sake of contradiction, that $s' \notin \bigcup_{\widetilde{S}\in\mathcal{S}} \widetilde{S}$. Then, if $s(t) = s'$ for some period $t \geq 0$, eventually at some $T \geq t+1$, the following two conditions must hold: (i) there is $\widetilde{S} \in \mathcal{S}$ such that $s(T) \in \widetilde{S}$ and (ii) for each $t' \in [t, T-1]$, $s(t') \notin \bigcup_{\widetilde{S}\in\mathcal{S}} \widetilde{S}$. Since $k_\alpha(t) = n$, $a_{i_{t+1}} \rightsquigarrow \alpha$. We claim that a best response implies $l_{i_{t+1}}(t+1) = l_{i_{t+1}}(t)$ and $g(t+1) \in G^m$. Assume not. If either $l_{i_{t+1}}(t+1) = l_{i_{t+1}}(t)$ and $g(t+1) \notin G^m$ or $l_{i_{t+1}}(t+1) < l_{i_{t+1}}(t)$, since $s(t) \in S^*$ and $g(t) \in G^m$, $g(t+1)$ is not connected. Hence, since $k_\alpha(t+1) = n$, i_{t+1} could increase her payoff by creating a new link to a player in $N\backslash(N^i(g(t+1)) \cup \{i\})$, which contradicts a best response. If $l_{i_{t+1}}(t+1) > l_{i_{t+1}}(t)$, the payoff to i_{t+1} at $t+1$ is at most $n - 1 - l_{i_{t+1}}(t+1) \cdot c$, which is lower than $\Pi_{i_{t+1}}(s(t)) = n - 1 - l_{i_{t+1}}(t) \cdot c$, a contradiction. Therefore, $\Pi_{i_{t+1}}(s(t+1)) = \Pi_{i_{t+1}}(s(t))$. The same reasoning holds for each $t' \in [t, T]$, *i.e.*, $k_\alpha(t') = n$, $g(t') \in G^m$ and, for each $j \in N$, $\Pi_j(s(t')) = \Pi_j(s(t))$. Finally, let $i_{T+1} = i_T$. Since $\Pi_{i_T}(s(T)) = \Pi_{i_T}(s(T-1))$, to choose $s_{i_{T+1}}(T+1) = s_{i_{T+1}}(T-1)$ is a best response. Hence, with positive probability, $s(T+1) = s(T-1)$. Since $s(T) \in \bigcup_{\widetilde{S}\in\mathcal{S}} \widetilde{S}$, this contradicts $s(T-1) \notin \bigcup_{\widetilde{S}\in\mathcal{S}} \widetilde{S}$. $\square$

Lemma 9 *Let $\varepsilon = 0$. If, for some $t \geq 0$, $k_\alpha(t) = 0$, then there is $T \geq t$ such that, for each $t' > T$, $s(t') \in S^*$ and*
I) If $c < x - x^2$, then $k_\alpha(t') = 0$ and $g(t') = g^{co}$.
II) If $x - x^2 < c < x$, then $k_\alpha(t') = 0$.
III) If $c > x$, then either $k_\alpha(t') = n$ and $g(t') \in G^m$, or $k_\alpha(t') = 0$.

Proof. Let $s(t) \in S$ be such that $k_\alpha(t) = 0$ and let period T be such that all players have revised.

First consider I). For each $\tau \geq 0$ such that $k_\alpha(\tau) = 0$, if $a_{i_{\tau+1}} \rightsquigarrow \alpha$, the payoff to $i_{\tau+1}$ at $\tau + 1$ is at most 0. In contrast, if $a_{i_{t+1}} \rightsquigarrow \beta$, by choosing $\hat{L}_{i_{\tau+1}} = \{j \in N : i_{\tau+1} \notin L_j(\tau)\}$, her payoff is at least $(n-1)(x-c) > 0$. Hence, $a_{i_{t+1}} \rightsquigarrow \beta$. Moreover, since $c < x - x^2$, $L_{i_{\tau+1}}(\tau+1) = \hat{L}_{i_{\tau+1}}$.

Now consider II). By the same arguments used in part I), for each $\tau > t$, $a_{i_\tau} \rightsquigarrow \beta$. It is immediate that, for each $\tau > t$ and each $j \in N$ such that $i_\tau \in L_j(\tau - 1)$, $j \notin L_{i_\tau}(\tau)$. Thus, for each $t' \geq T$, $s(t') \in S^*$.

Finally, consider III). For each $s \in S$, order the players in N and let them revise in consecutive periods, *i.e.*, at each $\tau, \tau' \in [t+1, t+n]$, $i_\tau \neq i_{\tau'}$. At $t+n$ choose a new order afresh and let all the players revise again. Repeat the process, exploring all possible orders, until some $\hat{t} > t$ such that $a_{i_{\hat{t}}} \rightsquigarrow \alpha$ whereas, for each $\tau \in (t, \hat{t})$, $a_{i_\tau} \rightsquigarrow \beta$. If such $\hat{t}$ does not exist, then the proof follows. Otherwise, since, for each

$j \in N\backslash\{i_{\hat{t}}\}$, $a_j(\hat{t}) = \beta$, $i_{\hat{t}} \notin N_{g(\hat{t})}$ and $\Pi_{i_{\hat{t}}}(s(\hat{t})) = 0$. Hence, no link in $g(\hat{t})$ is profitable. Then, if all players in $N\backslash\{i_{\hat{t}}\}$ revise consecutively, for each $\hat{\tau} \in [\hat{t}+1, \hat{t}+n-1]$, either $a_{i_{\hat{\tau}}} \rightsquigarrow \alpha$, $L_{i_{\hat{\tau}}}(\hat{\tau}) \subseteq K_\alpha(\hat{\tau}-1)$ and $l_{i_{\hat{\tau}}}(\hat{\tau}) = 1$ or $a_{i_{\hat{\tau}}} \rightsquigarrow \alpha$, $L_{i_{\hat{\tau}}}(\hat{\tau}) = \emptyset$ and $i_{\hat{\tau}} \in N_{g(\hat{\tau})}$. Thus, for each $t' \geq \hat{t}+n-1$, $s(t') \in S^*$, $k_\alpha(t') = n$ and $g(t') \in G^m$. $\square$

Lemma 10 *Let $\varepsilon = 0$. From each $s \in S$, with positive probability the dynamics leads the system either (i) to a state s' such that $k_\alpha(s') = 0$ or (ii) to a state s'' such that $K_\alpha(s'')$ is a minimally connected $\alpha-group$, $k_\alpha(s'') \geq 2$ and, for each $i \in K_\alpha(s'')$, $L_i(s'') \subset K_\alpha(s'')$.*[36]

Proof. Let $t_1 \geq 0$ and $s(t_1) = s$. If $k_\alpha(t_1) = 0$, (i) holds. If $k_\alpha(t_1) \geq 1$, let all players in $K_\alpha(t_1)$ revise consecutively until $t_2 = t_1 + k_\alpha(t_1)$. If $k_\alpha(t_1) = 1$, then, if $a_{i_{t_1+1}} \rightsquigarrow \alpha$, the payoff to i_{t_1+1} is at most 0, whereas, if $a_{i_{t_1+1}} \rightsquigarrow \beta$ and $L_{i_{t_1+1}}(t_1+1) = \emptyset$, the payoff to i_{t_1+1} is at least 0. Thus, with positive probability, $a_{i_{t_1+1}} \rightsquigarrow \beta$ and (i) holds. Therefore, let $k_\alpha(t_1) \geq 2$. For each $\tau \in [t_1+1, t_2]$, if $a_{i_\tau} \rightsquigarrow \alpha$, then $L_{i_\tau}(\tau)$ must be such that $|\mathcal{M}_\alpha(s(\tau))| = 1$ and that $j \in L_{i_\tau}(\tau)$ if and only if both $j \in K_\alpha(\tau-1)$ and $L_{i_\tau}(\tau)\backslash\{j\}$ implies $|\mathcal{M}_\alpha(s(\tau))| > 1$. Given the revision structure, each $M \in \mathcal{M}_\alpha(s(t_2))$ is minimally connected.[37] If $a_{i_{t_2}} \rightsquigarrow \alpha$, (ii) holds. Otherwise, let all players in $K_\alpha(t_2)$ revise consecutively until $t_3 = t_2 + k_\alpha(t_2)$. If for each $\tau \in [t_2+1, t_3]$, $a_{i_\tau} \rightsquigarrow \beta$, (i) holds. Otherwise, (ii) holds at period $\hat{\tau} \in [t_2+1, t_3]$ such that $a_{i_{\hat{\tau}}} \rightsquigarrow \alpha$. $\square$

Lemma 11 *Let $\varepsilon = 0$. For n large enough,*
I) If $c < x - x^2$, then $s \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$ if and only if $s \in S^*$ and, either $k_\alpha(s) = n$ and $g(s) \in G^m$, or $k_\alpha(s) = 0$ and $g(s) = g^{co}$.
II) If $c > x - x^2$ and $s \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$, then $s \in S^*$ and, either $k_\alpha(s) = n$ and $g(s) \in G^m$, or $k_\alpha(s) = 0$.

Proof. We first claim that, if there is $\widetilde{S}' \in \mathcal{S}$ such that for some $s' \in \widetilde{S}'$, $k_\alpha(s') = n$, then for each $s \in \widetilde{S}'$, $k_\alpha(s) = n$, $s \in S^*$ and $g(s) \in G^m$. Assume not, *i.e.*, there is $s'' \in \widetilde{S}'$ such that $s'' \notin \{s \in S^* : k_\alpha(s) = n, \text{and } g(s) \in G^m\}$.[38] By Lemma 8, for each $t \geq 1$, $(Q_0)^t_{s',s''} = 0$, a contradiction to $s', s'' \in \widetilde{S}'$.

We now claim that, if there is $\widetilde{S}' \in \mathcal{S}$ such that for some $s' \in \widetilde{S}'$, $k_\alpha(s') = 0$, then for each $s \in \widetilde{S}'$, $k_\alpha(s) = 0$, $s \in S^*$ and, if $c < x - x^2$, $g(s) = g^{co}$. Assume not. First, consider the case where there is $s'' \in \widetilde{S}'$ such that $k_\alpha(s'') > 0$. If $c < x$, by Lemma 9, for each $t \geq 1$, $(Q_0)^t_{s',s''} = 0$, a contradiction to $s', s'' \in \widetilde{S}'$. For each $t \geq 1$, if $c > x$, by Lemma 9, $(Q_0)^t_{s',s''} > 0$ only if $k_\alpha(s'') = n$. But, then by the former claim, this contradicts $s' \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$. Second, consider the case where there is $s'' \in \widetilde{S}'$ such that $s'' \notin S^*$. By Lemma 9, for each $t \geq 1$, $(Q_0)^t_{s',s''} = 0$, a contradiction to $s', s'' \in \widetilde{S}'$.

[36] Note that the fact that $K_\alpha(s'')$ is an $\alpha-group$ implies $\mathcal{M}(s'') = \{K_\alpha(s'')\}$.
[37] Note that $|\mathcal{M}_\alpha(s(t_2))| > 1$ only if $a_{i_{t_2}} \rightsquigarrow \beta$.
[38] Note that this formulation allows for the case $s'' = s'$.

Finally, consider the case where $c < x - x^2$ and there is $s'' \in \widetilde{S}'$ such that $g(s'') \neq g^{co}$. By Lemma 9, for each $t \geq 1$, $(Q_0)^t_{s',s''} = 0$, a contradiction to $s', s'' \in \widetilde{S}'$.

Given these two results, in order to prove part II) and the necessary condition of part I) of the lemma, we just need to show that, from each $s(0) \in S$, there is a sequence of events with positive probability that leads the system to a state in $\{s \in S : k_\alpha(s) \in \{0, n\}\}$. If $k_\alpha(0) \in \{0, n\}$, the result is immediate. Thus, let $1 \leq k_\alpha(0) \leq n - 1$. By Lemma 10, with positive probability, at some $t_1 > 0$ the system is either in a state s' such that $k_\alpha(s') = 0$ or in a state $s'' = \{(L''_j, a''_j)\}_{j \in N}$ such that $K_\alpha(s'')$ is a minimally connected $\alpha - group$, $k_\alpha(s'') \geq 2$ and, for each $i \in K_\alpha(s'')$, $L''_i \subset K_\alpha(s'')$. In the first case, the corresponding result follows. Thus, consider the second case, *i.e.*, $s(t_1) = s''$.

We first prove I). Let $c < x - x^2$. We first show the necessary condition ($\Rightarrow$). Let the players in $K_\beta(t_1)$ revise consecutively until $t_2 = t_1 + k_\beta(t_1)$. If $k_\alpha(t_2) > k_\alpha(t_1)$ repeat the process, letting the players in $K_\beta(t_2)$ revise consecutively until, at some t_r either $k_\alpha(t_r) = n$ or $k_\alpha(t_r) = k_\alpha(t_{r-1}) < n$. In the first case, the result follows. In the second case, for each $j, j' \in K_\beta(t_r)$, $|L_j(t_r) \cap K_\alpha(t_r)| = 1$ and $(j, j') \in g(t_r)$. Moreover, at t_r, $K_\alpha(t_r)$ is a minimally connected $\alpha - group$ and, for each $i \in K_\alpha(t_r)$, $L_i(t_r) \subset K_\alpha(t_r)$. Consider the event (with strictly positive probability) that, for each $j \in K_\beta(t_r)$, $L_j(t_r) \cap K_\alpha(t_r) = \{m\}$, where $m \in \{j' \in K_\alpha(t_r) : \left|g_{K_\alpha(t_r)} \cap \{(j', 1), ..., (j', n)\}\right| = 1\}$, *i.e.*, all players who form a link to the $\alpha - group$ get linked to the same player, m, who is linked to only one $\alpha - player$.[39] Hence, at t_r no player in $K_\beta(t_r)$ has incentives to switch to α. Let $j \in K_\beta(t_r)$ and $\widetilde{s}_j = (\alpha, \{m\}) \in S_j$. Then, since $\Pi_j(s(t_r)) \geq \Pi_j(\widetilde{s}_j, s_{-j}(t_r))$,

$$k_\alpha(t_r) \leq (n - 1) \cdot x - (l_j(t_r) - 1) \cdot c. \tag{1}$$

Let $i_{t_r+1} = m$. Then $a_{i_{t_r+1}} \rightsquigarrow \alpha$ only if

$$k_\alpha(t_r) \geq (n - 1) \cdot x + 1, \tag{2}$$

which, for each $l_j(t_r) \geq 1$, is not compatible with (1). Then $a_m \rightsquigarrow \beta$ and $L_m(t_r + 1) = L_m(t_r)$. Note that $K_\alpha(t_r + 1)$ is a minimally connected $\alpha - group$. Then, let the players in $K_\beta(t_r + 1)$ revise consecutively until $T = t_r + 1 + k_\beta(t_r + 1)$. Since $k_\alpha(t_r + 1) < k_\alpha(t_r)$, for each $t' \in [t_r + 2, T]$, $a_{i_{t'}} \rightsquigarrow \beta$, $L_{i_{t'}}(t') \cap K_\beta(t_r + 1) = L_{i_{t'}}(t_r + 1) \cap K_\beta(t_r + 1)$ and $|L_{i_{t'}}(t') \cap K_\alpha(t_r + 1)| = 1$. Consider the event (with strictly positive probability) that, for each $t' \in [t_r + 2, T]$, $L_{i_{t'}}(t') \cap K_\alpha(t_r + 1) = \{m'\}$, where $m' \in \{j' \in K_\alpha(t_r + 1) : \left|g_{K_\alpha(t_r+1)} \cap \{(j', 1), ..., (j', n)\}\right| = 1\}$. Then, by the same arguments used above (equations (1)-(2)), if we let $i_{T+1} = m'$, $a_{m'} \rightsquigarrow \beta$ and $L_{m'}(T + 1) = L_{m'}(T)$. Therefore, continuing recursively this process, at some finite T', $k_\alpha(T') = 0$ and the result follows. This proves the necessary condition.

[39]Note that, since the $\alpha - group$ is minimally connected, there are at least two players that are linked to only one member of the $\alpha - group$.

We prove the sufficient condition ($\Leftarrow$). First, let $s' \in S^*$, such that $k_\alpha(s') = 0$ and $g(s') = g^{co}$ and let $t \geq 0$ such that $s(t) = s'$. Since $c < x - x^2$, for each $t' > t$, $s(t') = s'$. Hence, $s' \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$. On the other hand, for each $s \in S^*$, such that $k_\alpha(s) = n$ and $g(s) \in G^m$, by Lemma 9, $s \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$. This proves I).

We now prove II). Let $c > x - x^2$. Let $\bar{d}$ be the maximum element of $\{d \in \mathbb{Z} : 2x^d \geq 2x - c\}$, let $\bar{n}$ be the minimum element of $\{n \in \mathbb{Z} : (n-1)x^{\bar{d}+1} > c/(x - x^2)\}$ and assume that $n > \bar{n}$. Let the players in $K_\beta(t_1)$ revise consecutively from $t_1 + 1$ to $t_2 = t_1 + k_\beta(t_1)$ (recall that $s(t_1) = s''$). Then, for each $t' \in [t_1 + 1, t_2]$, there is a unique and minimally connected $\alpha - group$ and $|L_{i_{t'}}(t') \cap K_\alpha(t')| \leq 1$. If $k_\alpha(t_2) = n$, by Lemma 8, the result follows. If $k_\alpha(t_2) < n$, consider the event (with strictly positive probability) that, for each $j \in K_\beta(t_2)$, $L_j(t_2) \cap K_\alpha(t_2) \in \{\emptyset, \{m\}\}$, where $m \in \{j' \in K_\alpha(t_2) : \left|g_{K_\alpha(t_2)} \cap \{(j',1), ..., (j',n)\}\right| = 1\}$. For each t, let $\bar{K}_\beta(t) := \{j \in K_\beta(t) : d_{j,m}(g(t)) > \bar{d}\}$ and let $i_{t_2+1} \in \bar{K}_\beta(t_2)$. Since, for each $d > \bar{d}$, $2x^d < 2x - c$, $L_{i_{t_2+1}}(t_2+1)$ is such that $d_{i_{t_2+1},m}(g(t_2+1)) \leq \bar{d}$.[40] Then, $\bar{K}_\beta(t_2+1) \subset \bar{K}_\beta(t_2)$. Repeat the process, letting a player in $\bar{K}_\beta(t_2+1)$ revise in t_2+2 and so on, until some period T such that $\bar{K}_\beta(T) = \emptyset$ and, for each $j \in K_\beta(T)$, $L_{i_T}(T) \cap K_\alpha(T) \in \{\emptyset, \{m\}\}$. Consider the event (with strictly positive probability) that $\left|g_{K_\alpha(T)} \cap \{(m,1), ..., (m,n)\}\right| = 1$. Let $i_{T+1} = m$. There are two cases: (i) $k_\alpha(T) < 1 + (n-1)x^{\bar{d}+1}$ and (ii) $k_\alpha(T) \geq 1 + (n-1)x^{\bar{d}+1}$. Former to the analysis of the two cases, for each period $t > T$, let us denote by $\widetilde{L}_{i_t}(t) \in 2^{(N \setminus \{m\})}$ be the set of links that maximize the payoff to i_t at t conditional on $a_{i_t} \rightsquigarrow \alpha$, and let $\widetilde{l}_{i_t}(t) = |\widetilde{L}_{i_t}(t)|$.

Consider case (i), *i.e.*, $k_\alpha(T) < 1 + (n-1)x^{\bar{d}+1}$. In this case, $\widetilde{l}_m(T+1) \in \{0,1\}$.[41] Assume first that $k_\alpha(T) = 2$. If $a_m \rightsquigarrow \alpha$, she gets a maximum payoff of $1 - \widetilde{l}_m(T+1) \cdot c$ whereas, if $a_m \rightsquigarrow \beta$, she gets a payoff greater than $(n-2)\, x^{\bar{d}} - \widetilde{l}_m(T+1) \cdot c$. Hence, for n large enough, $a_m \rightsquigarrow \beta$ and $k_\alpha(T+1) = 1$. Then, let $i_{T+2} \in K_\alpha(T+1)$. If $a_{i_{T+2}} \rightsquigarrow \alpha$, the payoff i_{T+2} obtains is at most 0, whereas, if $a_{i_{T+2}} \rightsquigarrow \beta$ and $L_{i_{T+2}}(T+2) = \emptyset$, the payoff to i_{T+2} is at least 0. Thus, with positive probability, $a_{i_{T+2}} \rightsquigarrow \beta$. Hence, $k_\alpha(T+2) = 0$ and the result follows. Assume now that $k_\alpha(T) > 2$. Then, if $a_m \rightsquigarrow \alpha$, m gets a maximum payoff of $k_\alpha(T) - 1 - \widetilde{l}_m(T+1) \cdot c$ whereas, if $a_m \rightsquigarrow \beta$, since $k_\alpha(T) > 2$, she gets a payoff greater than $(n-1)\, x^{\bar{d}} - \widetilde{l}_m(T+1) \cdot c$. Then, since $k_\alpha(T) < 1 + (n-1)x^{\bar{d}+1}$ and $x < 1$, $a_m \rightsquigarrow \beta$.

If $k_\alpha(T+1) = 2$, we are in the previous situation and the result follows. If $k_\alpha(T+1) > 2$, let $i_{T+2} \in \{j' \in K_\alpha(T+1) : \left|g_{K_\alpha(T+1)} \cap \{(j',1), ..., (j',n)\}\right| = 1\}$. If $a_{i_{T+2}} \rightsquigarrow \alpha$ she gets at most $k_\alpha(T+1) - 1 - \widetilde{l}_{i_{T+2}}(T+2) \cdot c$. If $a_{i_{T+2}} \rightsquigarrow \beta$, she can form a link to m (so that she observes all the players at distance at most $\bar{d}+1$)

[40] Note that since $k_\alpha(t_2) \geq 2$, either $a_{i_{t_2+1}} \rightsquigarrow \alpha$ and she forms a direct link to the $\alpha - group$ or $a_{i_{t_2+1}} \rightsquigarrow \beta$ and she creates links such that she is at distance at most $\bar{d}$ from the $\alpha - group$. In the former case, we assume that the link is to a player different from m and, in the later case, if i_{t_2+1} forms a (direct) link to the $\alpha - group$, we assume that the link is to m.

[41] Note that $\widetilde{l}_m(T+1) = 1$ if and only if, for each $j' \in K_\alpha(T)$, $m \notin L_{j'}(T)$.

and, therefore, the payoff to i_{T+2} is at least $(n-1)\,x^{\bar{d}+1} - (\widetilde{l}_{i_{T+2}}(T+2)+1)\cdot c$. Since $k_\alpha(T+1) = k_\alpha(T) - 1$ and $k_\alpha(T) < 1 + (n-1)x^{\bar{d}+1}$, $a_{i_{T+2}} \rightsquigarrow \beta$.[42] Then, either $k_\alpha(T+2) = 2$, or we repeat recursively the process until some $\hat{T}$, such that $k_\alpha(\hat{T}) = 2$. Then, we are in the previous situation and the result follows.

Consider case (ii), *i.e.*, $k_\alpha(T) \geq 1 + (n-1)x^{\bar{d}+1}$. Let all those player in $K_\beta(T)$ that are not directly linked to m revise consecutively, say until period T'. Since $n > \bar{n}$ and $k_\alpha(T) \geq 1 + (n-1)x^{\bar{d}+1}$, $k_\alpha(T) > c/(x - x^2)$. Hence, for each $t \in [T+1, T']$, $|L_{i_t}(t) \cap K_\alpha(t)| = 1$. Consider the event (which has positive probability) that, for each $j \in K_\beta(T')$, $L_j(T') \cap K_\alpha(T') = \{m\}$ and $\left|g_{K_\alpha(T')} \cap \{(m,1),...,(m,n)\}\right| = 1$. Thus, $k_\alpha(T') \geq k_\alpha(T)$. Then, let the players in $K_\beta(T')$ revise consecutively until a period $T_1 \in [T'+1, T'+k_\beta(T)]$ such that either $a_{i_{T_1}} \rightsquigarrow \beta$ and, for each $t \in [T+1, T_1-1]$, $a_{i_t} \rightsquigarrow \alpha$ or $T_1 = T' + k_\beta(T')$ and, for each $t \in [T+1, T_1]$, $a_{i_t} \rightsquigarrow \alpha$. In the second case, the result follows. Thus, assume the first case. Since $k_\alpha(T_1 - 1) \geq k_\alpha(T)$, $k_\alpha(T_1-1) \geq 1 + (n-1)x^{\bar{d}+1}$. Thus i_{T_1} gets directly linked to the $\alpha - group$. As in the previous cases, assume $L_{i_{T_1}}(T_1) \cap K_\alpha(T_1) = \{m\}$ and $\left|g_{K_\alpha(T_1)} \cap \{(m,1),...,(m,n)\}\right| = 1$.

Since $a_{i_{T_1}} \rightsquigarrow \beta$, the largest payoff to i_{T_1} by choosing β, *i.e.*, $(n-1)\,x - \widetilde{l}_{i_{T_1}}(T_1)\cdot c$, must be higher than the payoff that she gets choosing α, *i.e.*, $k_\alpha(T_1 - 1) - \widetilde{l}_{i_{T_1}}(T_1)\cdot c$. Hence, $k_\alpha(T_1 - 1) \leq (n-1)\cdot x$. Let $i_{T_1+1} = m$. Since $k_\alpha(T_1) = k_\alpha(T_1 - 1)$, $k_\alpha(T_1) \leq (n-1)\cdot x$ and, therefore, $a_{i_m} \rightsquigarrow \beta$ and $L_m(T_1+1) = L_m(T_1)$.[43] Assume that from T_1+2 all players in $\{j \in K_\beta(T_1) : L_j(T_1) \cap K_\beta(T_1) \neq \emptyset\}$ revise consecutively, say until period T_2. Then, for each $t' \in [T_1+2, T_2]$, if $a_{i_{t'}} \rightsquigarrow \beta$, then $L_{i_{t'}}(t') \cap K_\beta(t') = \{m\}$ and $|L_{i_{t'}}(t') \cap K_\alpha(t')| = 1$.[44] Moreover, if there is $\tau \in [T_1+2, T_2-1]$ such that $a_{i_\tau} \rightsquigarrow \alpha$, then $a_{i_{T_2}} \rightsquigarrow \alpha$.[45] Hence, there are two possibilities. First, if $a_{i_{T_2}} \rightsquigarrow \alpha$, let the players in $k_\beta(T_2)$ revise consecutively. Then all of them switch to α and the result follows. Second, if $a_{i_{T_2}} \rightsquigarrow \beta$, then $k_\beta(T_2) = k_\beta(T_1+1)$. If $a_{i_{T_2}} \rightsquigarrow \alpha$, her payoff is $k_\alpha(T_2) - \widetilde{l}_{i_{T_2}}(T_2)\cdot c$. If $a_{i_{T_2}} \rightsquigarrow \beta$, her payoff is $x + (k_\beta(T_2) - 2)\cdot x^2 + k_\alpha(T_2)\cdot x - (\widetilde{l}_{i_{T_2}}(T_2)+1)\cdot c$. Then, since $a_{i_{T_2}} \rightsquigarrow \beta$ is a best response,

$$k_\alpha(T_2)\cdot(1-x) \leq x + (k_\beta(T_2) - 2)\cdot x^2 - c. \tag{3}$$

Then, let $i_{T_2+1} \in \{j' \in K_\alpha(T_2) : \left|g_{K_\alpha(T_2)} \cap \{(j',1),...,(j',n)\}\right| = 1\}$. If $a_{i_{T_2+1}} \rightsquigarrow \alpha$, her payoff is at most $k_\alpha(T_2) - 1 - \widetilde{l}_{i_{T_2+1}}(T_2+1)\cdot c$. If $a_{i_{T_2+1}} \rightsquigarrow \beta$, by creating a link to m, the payoff to i_{T_2+1} is at least $k_\alpha(T_2)x + (k_\beta(T_2) - 1)\,x^2 - (1 + \widetilde{l}_{i_{T_2+1}}(T_2+1))c$.

[42] Note that $a_{i_{T+2}} \rightsquigarrow \beta$ if $k_\alpha(T+1) - 1 < (n-1)x^{\bar{d}+1} - c$ that, since $k_\alpha(T+1) = k_\alpha(T) - 1$, can be rewritten as $k_\alpha(T) - (1-c) < (n-1)x^{\bar{d}+1} + 1$.

[43] Note that the payoff to m at T_1+1 if $a_m \rightsquigarrow \alpha$ exceeds her payoff if $a_m \rightsquigarrow \beta$ only if $k_\alpha(T_1) > (n-1)\,x + 1$.

[44] Note that, since $k_\alpha(T_1) \geq k_\alpha(T)$, and $a_{i_{T_1+1}} \rightsquigarrow \beta$, $k_\alpha(t') \geq k_\alpha(T) - 1$. Thus, since $k_\alpha(T) \geq 1 + (n-1)x^{\bar{d}+1}$, $k_\alpha(t') \geq (n-1)x^{\bar{d}+1}$. Moreover, since $n > \bar{n}$, $k_\alpha(t') > c/(x - x^2)$. Hence, $|L_{i_t}(t') \cap K_\alpha(t')| = 1$.

[45] Note that i_{T_2} does not receive any links from $\beta - players$ at period T_2.

Hence, $a_{i_{T_2+2}} \rightsquigarrow \alpha$ only if $k_\alpha(T_2) - 1 \geq k_\alpha(T_2)x + (k_\beta(T_2) - 1)\, x^2 - c$, which is incompatible with (3). Hence $a_{i_{T_2+1}} \rightsquigarrow \beta$. Then, repeat recursively the process until $T_3 = T_2 + 1 + k_\alpha(T_2 + 2)$, choosing for each $t \in [T_2 + 2, T_3]$, $i_t \in \{j' \in K_\alpha(t-1) : \left|g_{K_\alpha(t-1)} \cap \{(j',1), ..., (j',n)\}\right| = 1\}$. Since $k_\alpha(T_2+2) = k_\alpha(T_2+1) - 1$, by the former argument (using (3)), $a_{i_{T_2+2}} \rightsquigarrow \beta$. Following the same reasoning for the subsequent periods, $k_\alpha(T_3) = 0$ and the result follows. □

Lemma 12 *For each pair $\widetilde{S}', \widetilde{S}'' \in \mathcal{S}$ such that that $\widetilde{S}', \widetilde{S}'' \subseteq \{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\}$, there is a path of one step mutations in the set $\{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\}$ from $\widetilde{S}'$ to $\widetilde{S}''$.*

Proof. Let $s' = \{(L'_j, a'_j)\}_{j\in N} \in \widetilde{S}'$ and $s'' = \{(L''_j, a''_j)\}_{j\in N} \in \widetilde{S}''$. Assume that, for some $t \geq 0$, $s(t) = s'$. Consider that, for each $j \in N$, there is one period t_j where j mutates and, after each mutation, the players that have not mutated yet revise consecutively. We order the periods of mutation inversely to the player indices ($t_n < ... < t_1$) such that, after the mutation of player j, from t_j+1 to $t'_j = t_j + (j-1)$, players from 1 to $j-1$ revise (for instance, for each $j' \in \{1, ..., j-1\}$, $j' = i_{t_j+j'}$). Then, let $t_n = t+1$ and, for each $j < n$, $t_j = t'_{j+1} + 1$.[46] For each $j \in N$, let the mutation at t_j be such that $a_j \rightsquigarrow \alpha$ and $L_j\,(t_j) = L''_j$. Then, since $k_\alpha(t) = n$ and each player that mutates chooses α, for each $\tau \in [t_n, t'_1]$, $k_\alpha(\tau) = n$. Since after j's mutation at period t_j, each $j' \in \{1, ..., j-1\}$ revises consecutively until t'_j, the best responses of these players imply the choice of link strategies such that $s(t'_j) \in S^*$ and $g(t'_j) \in G^m$. Hence, for each $j \in N$, $s\left(t'_j\right) \in \{s \in S^* : k_\alpha(s) = n$ and $g(s) = G^m\}$. By construction, since at t'_1 there has been a mutation for each $j \in N$ in the direction of s'', $s\,(t'_1) = s''$. □

Proof of Lemma 2

We proceed in 3 steps. By part I of Lemma 11, $\bigcup_{R\in\mathcal{R}} E(R) \subseteq \{s \in S^* :$ either $k_\alpha(s) = 0$ and $g(s) = g^{co}$ or $k_\alpha(s) = n$ and $g(s) \in G^m\}$.

Step 1. Let $s' \in S^*$ such that $k_\alpha(s') = 0$ and $g(s') = g^{co}$. We claim that, if there is $R \in \mathcal{R}$ such that $s' \in E(R)$, then $\{s \in S^* : k_\alpha(s) = 0$ and $g(s) = g^{co}\} \subseteq E(R)$. To prove it, it suffices to show that, for each pair $s', s'' \in \{s \in S^* : k_\alpha(s) = 0$ and $g(s) = g^{co}\}$, there is a path of one-step mutations in $\{s \in S^* : k_\alpha(s) = 0$ and $g(s) = g^{co}\}$ from s' to s''. We construct it as follows. Let $s' = \{(L'_j, a'_j)\}_{j\in N}$ and $s'' = \{(L''_j, a''_j)\}_{j\in N}$. Assume that, for some $t \geq 0$, $s(t) = s'$. Consider that, for each $j \in N$, there is one period t_j where j mutates and, after each mutation, the players that have not mutated yet revise consecutively. We order the periods of mutation inversely to the player indices ($t_n < ... < t_1$) such that, after the mutation of player j, from $t_j + 1$ to $t'_j = t_j + (j-1)$, players from 1 to $j-1$ revise. Then, let $t_n = t+1$ and, for each $j < n$, $t_j = t'_{j+1} + 1$. For each $j \in N$, let the mutation at t_j be such that $a_j \rightsquigarrow \beta$ and $L_j\,(t_j) = L''_j$. Then, since $k_\alpha(t) = 0$ and $c < x - x^2$, for each $j \in N$,

[46] Player j mutates after $j+1$ has mutated and all the players in $\{1, ..., j\}$ have revised.

$s\left(t_j'\right) \in \{s \in S^* : k_\alpha(s) = 0 \text{ and } g(s) = g^{co}\}$. By construction, since at t_1' there has been a mutation for each $j \in N$ in the direction of s'', $s\left(t_1'\right) = s''$.

Step 2. We claim that, for n large enough, there is $R_\beta \in \mathcal{R}$ such that $R_\beta = \{s \in S^* : k_\alpha(s) = 0 \text{ and } g(s) = g^{co}\}$. Given Step 1, in order to prove the claim, it suffices to show that, if $s(t) \in \{s \in S^* : k_\alpha(s) = 0 \text{ and } g(s) = g^{co}\}$, after any possible single mutation at $t+1$ followed by the unperturbed dynamics, the system goes with probability one to a state in $\{s \in S^* : k_\alpha(s) = 0 \text{ and } g(s) = g^{co}\}$. If the mutation at $t+1$ is such that $a_{i_{t+1}} \rightsquigarrow \beta$, by Lemma 9, the claim follows. Thus, let the mutation be such that $a_{i_{t+1}} \rightsquigarrow \alpha$. Let $i_{t+2} \in N \backslash \{i_{t+1}\}$. If $a_{i_{t+2}} \rightsquigarrow \alpha$ the payoff to i_{t+2} is at most 1. If $a_{i_{t+2}} \rightsquigarrow \beta$ the payoff to i_{t+2} is at least $(n-1)(x-c)$. Hence, for n large enough, $a_{i_{t+2}} \rightsquigarrow \beta$. By the same reasoning, for each $t' > t+2$, $a_{i_{t'}} \rightsquigarrow \beta$. Eventually, at some $t'' > t+1$, with positive probability $i_{t''} = i_{t+1}$. Hence, $k_\alpha(t'') = 0$ and, by Lemma 9, the claim follows.

Step 3. We claim that, for n large enough, there is $R_\alpha \in \mathcal{R}$ such that $E(R_\alpha) = \{s \in S^* : k_\alpha(s) = n \text{ and } g(s) \in G^m\}$. By Lemma 8.II and Lemma 12, $\{s \in S^* : k_\alpha(s) = n \text{ and } g(s) \in G^m\}$ satisfies property II of Definition 1. Thus, we just need to prove that it also satisfies property I. To this aim, we now show that, if $s(t) \in \{s \in S^* : k_\alpha(s) = n \text{ and } g(s) \in G^m\}$ and n is large enough, then after any possible single mutation at $t+1$ followed by the unperturbed dynamics, the system goes with probability one to a state in $\{s \in S^* : k_\alpha(s) = n \text{ and } g(s) \in G^m\}$. First note that, if the mutation at $t+1$ is such that $a_{i_{t+1}} \rightsquigarrow \alpha$, by Lemma 8, the system goes to a state in $\{s \in S^* : k_\alpha(s) = n \text{ and } g(s) \in G^m\}$. Thus, let the mutation be such that $a_{i_{t+1}} \rightsquigarrow \beta$. Let $i_{t+2} \in N \backslash \{i_{t+1}\}$. If $a_{i_{t+2}} \rightsquigarrow \beta$ the payoff to i_{t+2} is at most $(n-1)x$. If $a_{i_{t+2}} \rightsquigarrow \alpha$ the payoff to i_{t+2} is at least $(n-2)(1-c)$. Then, if $(n-2)(1-x-c) > x$, $a_{i_{t+2}} \rightsquigarrow \alpha$. Hence, since $c < x - x^2 < 1 - x$, for n large enough, $a_{i_{t+2}} \rightsquigarrow \alpha$. By the same reasoning, for each $t' > t+2$, $a_{i_{t'}} \rightsquigarrow \alpha$. Eventually, at some $t'' > t+1$, with positive probability $i_{t''} = i_{t+1}$. Hence, $k_\alpha(t'') = n$ and, by Lemma 8, the system goes to a state in $\{s \in S^* : k_\alpha(s) = n \text{ and } g(s) \in G^m\}$.

Step 4. Since $E(R_\alpha) \cup E(R_\beta) = \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$, $\mathcal{R} = \{R_\alpha, R_\beta\}$. □

Proof of Lemma 3

Let $x - x^2 < c < x$. We proceed in 5 steps.

Step 1. We shall prove the following two claims. *Claim* 1: Let $s(t) \in S^*$ such that $k_\alpha(t) = n$ and $g(t) \in G^m$. For n large enough, if $c < 1 - x^2$, after any possible single mutation at $t+1$ followed by the unperturbed dynamics, the system goes with probability one to a state $s' \in S^*$ such that $k_\alpha(s') = n$ and $g(s') \in G^m$. *Claim* 2: Let $s(t') \in S^*$ such that $k_\alpha(t') = n$, $g(t') \in G^{st}$ and $l_{\hat{\imath}(g(t'))} = n-1$. If $c > 1 - x^2$, then there is one single mutation at $t'+1$ that, followed by the unperturbed dynamics, leads the system to a state s'' such that $k_\alpha(s'') = 0$.

By Lemma 8, a mutation at $t+1$ such that $a_{i_{t+1}} \rightsquigarrow \alpha$ satisfies the statement of Claim 1 and does not allow to prove Claim 2. Thus, consider a mutation such that $a_{i_{t+1}} \rightsquigarrow \beta$. Let $i_{t+2} \in N \backslash \{i_{t+1}\}$. Let $\bar{s}'_{i_{t+2}} = (\bar{L}'_{i_{t+2}}, \alpha) \in S_{i_{t+2}}$ be the strategy where

i_{t+2} chooses action α and the set of links $\bar{L}'_{i_{t+2}}$ that maximizes her payoff conditional on $a_{i_{t+2}} = \alpha$ and the strategies of the other players at $t+2$. Define analogously $\bar{s}''_{i_{t+2}} = (\bar{L}''_{i_{t+2}}, \beta) \in S_{i_{t+2}}$. Denote by $\mathcal{M}_1 \subseteq \mathcal{M}_\alpha(\bar{s}''_{i_{t+2}}, s_{-i_{t+2}}(t+2))$ the set of those $\alpha-$*groups* to which i_{t+2} gets directly linked if $a_{i_{t+2}} \rightsquigarrow \beta$. Denote by $\mathcal{M}_2 \subseteq \mathcal{M}_\alpha(\bar{s}''_{i_{t+2}}, s_{-i_{t+2}}(t+2)) \backslash \mathcal{M}_1$ the set of those $\alpha-$*groups* to which i_{t+2} gets indirectly linked at distance 2 via her link to i_{t+1} if $a_{i_{t+2}} \rightsquigarrow \beta$.[47] Let $m_1 = \left|\bigcup_{M \in \mathcal{M}_1} M\right|$ and $m_2 = \left|\bigcup_{M \in \mathcal{M}_2} M\right|$. Note that $m_1 + m_2 = k_\alpha(t+1) - 1$. Then $\Pi_{i_{t+2}}(\bar{s}''_{i_{t+2}}, s_{-i_{t+2}}(t+2)) = (1+m_1)x + m_2 x^2 - \bar{l}''_{i_{t+2}} c$. On the other hand, if $a_{i_{t+2}} \rightsquigarrow \alpha$, she optimally gets (directly) linked to every $\alpha-$*group* and $i_{t+1} \notin \bar{L}'_{i_{t+2}}$. Let $d = 0$ if $i_{t+2} \in L_{i_{t+1}}(t+2)$ and $d = 1$ otherwise. Then $\bar{l}'_{i_{t+2}} = (\bar{l}''_{i_{t+2}} - d) + |\mathcal{M}_2|$. Hence, $\Pi_{i_{t+2}}(\bar{s}'_{i_{t+2}}, s_{-i_{t+2}}(t+2)) = n - 2 - (\bar{l}''_{i_{t+2}} - d + |\mathcal{M}_2|)c$. Therefore, $\Pi_{i_{t+2}}(\bar{s}''_{i_{t+2}}, s_{-i_{t+2}}(t+2)) - \Pi_{i_{t+2}}(\bar{s}'_{i_{t+2}}, s_{-i_{t+2}}(t+2))$ equals

$$m_1 x + m_2 x^2 + |\mathcal{M}_2|\, c - dc + x - (n-2). \tag{4}$$

We now determine the state $s^{\max}$ that maximizes (4) in $S(t+1) := \{s(t+1) \in S : s(t) \in S^*,\ k_\alpha(t) = n,\ g(t) \in G^m \text{ and } a_{i_{t+1}} \rightsquigarrow \beta\}$. We analyze (4) by parts. First note that $x - (n-2)$ is common to all $s \in S(t+1)$. We claim that, in $s^{\max}$, $d = 0$, $|\mathcal{M}_2| = m_2$ and $m_2 = n-2$. The first two conditions are straightforward. Thus, we shall prove the third one. Let $d = 0$, $|\mathcal{M}_2| = m_2$ and assume for the sake of contradiction that, in $s^{\max}$, $m_2 < n-2$. Then $\mathcal{M}_1 \neq \emptyset$. Let $M \in \mathcal{M}_1$ and $j \in M$. Consider $s' \in S(t+1)$ that only differs from $s^{\max}$ in the fact that $\{j\}$ is a (separated) $\alpha-$*group* and $j \in L_{i_{t+1}}(t+1)$. Then, since $c > x - x^2$, in state s', $\{j\} \in \mathcal{M}'_2$. Thus, $|\mathcal{M}'_2| = |\mathcal{M}_2| + 1$, *i.e.*, $m'_2 = m_2 + 1$, and $m'_1 = m_1 - 1$. Since $x^2 + c > x$, expression (4) is higher for s' than for $s^{\max}$, a contradiction. Finally, we show that there exists $s^{\max} \in S(t+1)$ such that $d = 0$, $|\mathcal{M}_2| = m_2$ and $m_2 = n-2$. Let $s(t) \in S^*$ be such that $g(t) \in G^{st}$ and $L_{\hat{\imath}(g(t))}(t) = n-1$. Then, just consider the mutation where $i_{t+1} = \hat{\imath}(g(t))$, $a_{i_{t+1}} \rightsquigarrow \beta$ and $L_{i_{t+1}}(t+1) = L_{i_{t+1}}(t)$, and we obtain the desired $s^{\max}$. Substituting $d = 0$ and $|\mathcal{M}_2| = m_2 = n-2$ in (4) we obtain

$$x - (n-2)(1 - x^2 - c). \tag{5}$$

If $1 - x^2 - c > 0$, then, for n large enough, (5) is negative. Thus, no player in $N \backslash \{i_{t+1}\}$ switches to β when revising her action and when, eventually, at some finite $t' > t+1$, $i_{t'} = i_{t+1}$, $i_{t'} \rightsquigarrow \alpha$. Since $k_\alpha(t') = n$, by Lemma 8, Claim 1 follows. Now, let $1 - x^2 - c < 0$ and consider that, starting at $s(t+1) = s^{\max}$, from period $t+2$ to $T = t+n$, the players in $N \backslash \{i_{t+1}\}$ revise consecutively. Since (5) is positive $a_{i_{t+2}} \rightsquigarrow \beta$ and $L_{i_{t+2}}(t+2) = \emptyset$. This, in turn, increases the incentives to switch to β for the next player who revises. Hence, for each $t \in [t+2, T]$, $a_{i_t} \rightsquigarrow \beta$ and $L_{i_t}(t) = \emptyset$. Then $k_\alpha(T) = 0$ and Claim 2 follows.

Step 2. Let $s(t) \in S^*$ such that $k_\alpha(t) = 0$. We claim that, for n large enough, after any possible single mutation at $t+1$ followed by the unperturbed dynamics, the

[47]Note that, since $c < x$, if $i_{t+2} \notin L_{i_{t+1}}(t+1)$ then $i_{t+1} \in L''_{i_{t+2}}$.

system goes with probability one to a state $s'' \in S^*$, such that $k_\alpha(s'') = 0$. Note that, if the mutation is such that $a_{i_{t+1}} \rightsquigarrow \beta$, by Lemma 9, the claim follows. Thus, let the the mutation be such that $a_{i_{t+1}} \rightsquigarrow \alpha$. Let $i_{t+2} \in N \backslash \{i_{t+1}\}$. If $a_{i_{t+2}} \rightsquigarrow \alpha$ the payoff to i_{t+2} is at most 1. If $a_{i_{t+2}} \rightsquigarrow \beta$ the payoff to i_{t+2} is at least $(n-1)(x-c)$. Since $c < x$, for n large enough, $a_{i_{t+2}} \rightsquigarrow \beta$. By the same reasoning, for each $t' > t+2$, $a_{i_{t'}} \rightsquigarrow \beta$. Eventually, at some $t'' > t+1$, with positive probability $i_{t''} = i_{t+1}$. Hence, $k_\alpha(t'') = 0$ and, by Lemma 9, the claim follows.

Step 3. Let $s(t) \in S$ such that $k_\alpha(t) = 0$. We claim that, for each $s' \in \{s \in S^* : k_\alpha(s) = 0,\ g(s) \in G^{st}$ and $l_{\hat{\imath}(g(s))} = n-1\}$, there is a single mutation at $t+1$ that, followed by the unperturbed dynamics, leads the system to s'. Moreover, if $T \geq t+1$ is such that $s(T) = s'$, then, for each $t' > T$, $s(t') = s'$. Let $i_{t+1} = \hat{\imath}(s')$ and the mutation be such that $a_{i_{t+1}} \rightsquigarrow \beta$ and $L_{i_{t+1}}(t+1) = N \backslash \{i_{t+1}\}$. Then, consider that, from period $t+2$ to $T = t+n$, the players in $N \backslash \{i_{t+1}\}$ consecutively revise. Then, for each $L_{i_{t+2}} \in 2^{N \backslash \{i_{t+2}\}}$, if $a_{i_{t+2}} \rightsquigarrow \alpha$ then the payoff to i_{t+2} is $-l_{i_{t+2}} c$ and, if $a_{i_{t+2}} \rightsquigarrow \beta$, the payoff to i_{t+2} is $x + (n-1)x^2 - l_{i_{t'}} c$. Hence, $a_{i_{t'}} \rightsquigarrow \beta$ and $L_{i_{t'}}(t') = \emptyset$. The same reasoning holds for each $t' \in [t+3, T]$, *i.e.*, $a_{i_{t'}} \rightsquigarrow \beta$ and $L_{i_{t'}}(t') = \emptyset$. Therefore, $s(T) = s'$. It is immediate to see that, for each $t' > T$, $s(t') = s(T)$.

Step 4. We claim that for each $\bar{s}, \bar{s}' \in \{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$ there is a path of one step mutations in $\{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$ from $\bar{s}$ to $\bar{s}'$. The claim directly follows from Lemma 2 of Feri [9].[48]

Step 5. Finally, we complete the proof using the previous steps. By part II of Lemma 11, $\bigcup_{R \in \mathcal{R}} E(R) \subseteq \{s \in S^* :$ either $k_\alpha(s) = n$ and $g(s) \in G^m$ or $k_\alpha(s) = 0\}$. We first prove part I. Let $x - x^2 < c < \min\{x, 1-x^2\}$, by Lemma 12, R_α satisfies property (II) of Definition 1. By Claim 1 of Step 1, R_α also satisfies property (I). Hence $R_\alpha \in \mathcal{R}$. We claim that, if for some $\bar{R} \in \mathcal{R}$ there is $s' \in E(\bar{R})$ such that $k_\alpha(s') = 0$, then $E(\bar{R}) \supseteq \{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$. Assume not. Then, by Steps 3 and 4, $\bar{R}$ does not satisfy property (I) of Definition 1, a contradiction. Note that the claim implies that $E(\bar{R}) \supseteq \{s \in S : k_\alpha(s) = 0\} \cap (\bigcup_{R \in \mathcal{R}} E(R))$. We now claim that there is $R_{\beta'} \in \mathcal{R}$ such that, for each $s' \in E(R_{\beta'})$, $k_\alpha(s') = 0$. Let $R_{\beta'} \subseteq \mathcal{S}$ that satisfies the following two conditions. First, $E(R_{\beta'}) \supseteq \{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$. Second, $E(R_{\beta'})$ is formed by all the states in $\{s \in S^* : k_\alpha(s) = 0\}$ such that, for each $s', s'' \in E(R_{\beta'})$, a path of one step mutations allows for a transition from s' to s''. By Step 4, the second condition is satisfied by the states in $\{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$, which guarantees the existence of $R_{\beta'}$. Hence, $R_{\beta'}$ satisfies property (II) of Definition 1. By Step 2, it is not possible to move from $R_{\beta'}$ to R_α using paths of one step mutations. Hence, $R_{\beta'}$ also satisfies property (I), which proves the claim. We now prove part II. Let $1 - x^2 < c < x$. By Claim 2 of Step 1, R_α does not satisfy

[48] In Feri [9], players' strategies only have the links dimension and all the links that form are subjected to an exogenous decay factor $\delta \in (0,1)$. If we consider paths of one step mutations where players only change links (and, therefore, all of them remain using action β), we can identify x to δ and use Feri's [9] result. Note that, our set $\{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$ corresponds to G^s in Feri [9].

property (I) of Definition 1. By an analogous reasoning to that used in part I, there exists $\bar{R}'$ such that $\bar{R}' \in \mathcal{R}$, $E(\bar{R}') \supseteq \{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$ and, for each $s \in E(\bar{R}')$, $k_\alpha(s) = 0$. Hence, let $R_{\beta''} = \bar{R}'$. Finally, part III is already shown in the proof of parts I and II. $\square$

Proof of Lemma 4

Let $c > x$. By part II of Lemma 11, if $s \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$, then $s \in S^*$ and either $k_\alpha(s) = n$ and $g(s) \in G^m$ or $k_\alpha(s) = 0$. By part II of Lemma 8, $\{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\} \subseteq \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$. We proceed in 4 steps.

Step 1. We claim that $\mathcal{R} = \{R\}$ and $\{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\} \subseteq E(R)$, which follows from the following two results. We first claim that from $s(t) \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S}$ such that $k_\alpha(t) = 0$, a single mutation at $t+1$ followed by the unperturbed dynamics, leads the system to a state in $\{s \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S} : k_\alpha(s) = n\}$. Consider a mutation such that $a_{i_{t+1}} \rightsquigarrow \beta$ and $L_{i_{t+1}}(t+1) = N \backslash \{i_{t+1}\}$. Then, let the players in $N \backslash \{i_{t+1}\}$ revise consecutively until $t_1 = t + n$. For each $t' \in [t+2, ..., t_1]$, $a_{i_{t'}} \rightsquigarrow \beta$ and $L_{i_{t'}}(t') = \emptyset$. Let $i_{t_1+1} = i_{t+1}$. Then, $L_{i_{t_1+1}}(t_1+1) = \emptyset$ and, with positive probability, $a_{i_{t_1+1}} \rightsquigarrow \alpha$. Thus, $g(t_1+1) = \emptyset$. Then, let the players in $N \backslash \{i_{t_1+1}\}$ revise consecutively until $t_2 = t_1 + n$. For each $t' \in [t_1+2, ..., t_2]$, a best response implies $a_{i_{t'}} \rightsquigarrow \alpha$, $l_{i_{t'}}(t') = 1$ and $L_{i_{t'}}(t') \subseteq K_\alpha(t'-1)$. Then $k_\alpha(t_2) = n$. By Lemma 8, the claim follows. Second, we claim that there is no $R \in \mathcal{R}$ such that $E(R) \cap \{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\} = \emptyset$. Assume not. Then by our first claim, R does not satisfy property (I) of Definition 1, a contradiction. Hence, by Lemma 12, for each $R' \in \mathcal{R}$, $\{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\} \subseteq E(R')$, which implies that $|\mathcal{R}| = 1$.

Step 2. Let $s(t) \in \{s \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S} : k_\alpha(s) = n\}$. We claim that, for n large enough, if $c < 1 - x^2 - \frac{x}{n-2}$, after any possible single mutation at $t+1$ followed by the unperturbed dynamics, the system goes with probability one to a state in $\{s \in \bigcup_{\widetilde{S} \in \mathcal{S}} \widetilde{S} : k_\alpha(s) = n\}$. By Lemma 8, a mutation at $t+1$ such that $a_{i_{t+1}} \rightsquigarrow \alpha$ satisfies the statement of the claim. Thus, consider a mutation such that $a_{i_{t+1}} \rightsquigarrow \beta$. Let $i_{t+2} \in N \backslash \{i_{t+1}\}$ and let $\bar{s}'_{i_{t+2}} = (\bar{L}'_{i_{t+2}}, \alpha)$, $\bar{s}''_{i_{t+2}} = (\bar{L}''_{i_{t+2}}, \beta)$, $\mathcal{M}_1$, $\mathcal{M}_2$, m_1, m_2 and d be defined exactly as in Step 1 of the proof of Lemma 3. Let $\mathcal{M}_\infty \subseteq \mathcal{M}_\alpha(\bar{s}''_{i_{t+2}}, s_{-i_{t+2}}(t+2))$ be the set of those $\alpha-$*groups* that i_{t+2} does not observe (neither directly nor indirectly) if she chooses $\bar{s}''_{i_{t+2}}$, and $m_\infty = \left|\bigcup_{M \in \mathcal{M}_\infty} M\right|$.[49] Note that $m_1 + m_2 + m_\infty = k_\alpha(t+1) - 1$. In this case, $\bar{l}'_{i_{t+2}} = (\bar{l}''_{i_{t+2}} - d) + |\mathcal{M}_2| + m_\infty$. Hence, $\Pi_{i_{t+2}}(\bar{s}''_{i_{t+2}}, s_{-i_{t+2}}(t+2)) = (1+m_1) \cdot x + m_2 \cdot x^2 - \bar{l}''_{i_{t+2}} \cdot c$ and $\Pi_{i_{t+2}}(\bar{s}'_{i_{t+2}}, s_{-i_{t+2}}(t+2)) = n - 2 - (\bar{l}''_{i_{t+2}} - d + |\mathcal{M}_2| + m_\infty)c$. Therefore, $\Pi_{i_{t+2}}(\bar{s}''_{i_{t+2}}, s_{-i_{t+2}}(t+2)) - \Pi_{i_{t+2}}(\bar{s}'_{i_{t+2}}, s_{-i_{t+2}}(t+2))$ equals

$$m_1 x + m_2 x^2 + |\mathcal{M}_2| \, c + m_\infty c - dc + x - (n-2). \tag{6}$$

By analogous arguments to those used in Step 1 of the proof of Lemma 3, (6) is maximized when $d = 0$, $|\mathcal{M}_2| = m_2$ and $m_2 = n - 2$. Introducing these values in (6)

[49] Each $M \in \mathcal{M}_\infty$ must be a singleton since, if $|M| \geq 2$, i_{t+2} would form a link to one player in M when $a_{i_{t+2}} \rightsquigarrow \beta$.

we obtain

$$x-(n-2)(1-x^2-c). \tag{7}$$

Since $c<1-x^2-\frac{x}{n-2}$, (7) is negative. Thus, no player in $N\backslash\{i_{t+1}\}$ switches to β when revising her action and when, eventually, at some finite $t'>t+1$, $i_{t'}=i_{t+1}$, $i_{t'}\rightsquigarrow\alpha$. Since $k_\alpha(t')=n$, by Lemma 8, the claim follows.

Step 3. Let $c>1-x^2-\frac{x}{n-2}$. We claim that, for n large enough, for each $j\in N$, a single mutation at $t+1$ followed by the unperturbed dynamics, allows for a transition from $s\in\bigcup_{\widetilde{S}\in\mathcal{S}}\widetilde{S}$ such that $k_\alpha(s)=n$, $g(s)\in G^{st}$, $\hat{\imath}(g(s))=j$ and $l_j=0$ to $s'\in S^*$ such that $k_\alpha(s')=0$, $g(s')\in G^{st}$, $\hat{\imath}(g(s'))=j$, and $l'_j=0$. Let $s(t)=s$. Let $i_{t+1}=j$ and consider the mutation at $t+1$ such that $a_j\rightsquigarrow\beta$ and $l_j(t+1)=3$. Let the players in $L_j(t+1)$ revise consecutively until $t+4$. If $a_{i_{t+2}}\rightsquigarrow\alpha$, the payoff to i_{t+2} at $t+2$ is $(n-2)(1-c)$ whereas, if $a_{i_{t+2}}\rightsquigarrow\beta$, her payoff is $x+x^2(n-2)$. Since, $c>1-x^2-\frac{x}{n-2}$, $a_{i_{t+2}}\rightsquigarrow\beta$. By the same reasoning, $a_{i_{t+3}}\rightsquigarrow\beta$ and $a_{i_{t+4}}\rightsquigarrow\beta$. Let the players in $N\backslash\{L_j(t+1)\cup\{j\}\}$ revise consecutively until $t+n$. For each $\tau\in\{5,...,n\}$, assuming that $a_{i_{t+\tau'}}\rightsquigarrow\beta$ for each $\tau'\in\{4,...,\tau-1\}$, if $a_{i_{t+\tau}}\rightsquigarrow\alpha$, then the payoff to $i_{t+\tau}$ at $t+\tau$ is $(n-\tau)(1-c)$ whereas, if $a_{i_{t+\tau}}\rightsquigarrow\beta$, then her payoff is $x-c+x^2(n-2)$. Hence, in such a case, $a_{i_{t+\tau}}\rightsquigarrow\beta$ if

$$x-c+x^2(n-2)\geq(n-\tau)(1-c). \tag{8}$$

The LHS of (8) does not depend on τ, whereas the RHS is decreasing in τ. Hence, since $a_{i_{t+4}}\rightsquigarrow\beta$, if (8) holds for $\tau=5$, then, for each $\hat{\tau}\in\{5,...,n\}$, $a_{i_{t+\hat{\tau}}}\rightsquigarrow\beta$. Hence, let $\tau=5$. We can rewrite (8) as $c\geq\frac{(n-5)-x^2(n-2)-x}{(n-6)}$. Since $c>1-x^2-\frac{x}{n-2}$, in order to show that $c>\frac{(n-5)-(n-2)x^2-x}{n-6}$ it suffices to show that $1-x^2-\frac{x}{n-2}>\frac{(n-5)-(n-2)x^2-x}{n-6}$, *i.e.*, $\frac{(n-2)(4x^2-1)+4x}{(n-6)(n-2)}>0$. Since $x>\frac{1}{2}$, for $n>7$, the inequality holds. Hence, $k_\alpha(t+n)=0$, $g(t+n)\in G^{st}$, $\hat{\imath}(g(t+n))=j$ and $l_j(t+n)=3$. Then, let $i_{t+n+1}=j$. If $a_j\rightsquigarrow\alpha$, her payoff is at most 0 whereas, if $a_j\rightsquigarrow\beta$, her payoff is strictly positive. Hence $a_j\rightsquigarrow\beta$ and $l_j(t+n+1)=0$. Let the players in $L_j(t+1)$ revise consecutively until $t+n+4$. If $a_{i_{t+n+2}}\rightsquigarrow\alpha$, her payoff is at most 0 whereas, if $a_{i_{t+n+2}}\rightsquigarrow\beta$ and $L_{i_{t+n+2}}(t+n+2)=\{j\}$, her payoff is $x+x^2(n-2)-c$. Hence, for n large enough, $a_{i_{t+n+2}}\rightsquigarrow\beta$ and $L_{i_{t+n+2}}(t+n+2)=\{j\}$. By the same reasoning, $a_{i_{t+n+3}}\rightsquigarrow\beta$, $a_{i_{t+n+4}}\rightsquigarrow\beta$ and $L_{i_{t+n+3}}(t+n+3)=L_{i_{t+n+4}}(t+n+4)=\{j\}$. Then, $s(t+n+4)\in S^*$, $k_\alpha(t+n+4)=0$, $g(t+n+4)\in G^{st}$, $\hat{\imath}(g(t+n+4))=j$ and $l_j=0$.

Step 4. Finally, we complete the proof using the previous steps. By Step 1, there is a unique recurrent set, namely R, such that $E(R)\supseteq\{s\in S^*:k_\alpha(s)=n$ and $g(s)\in G^m\}$. Moreover, by Lemma 11, $E(R)\subseteq\{s\in S^*:k_\alpha(s)=n$ and $g(s)\in G^m\}\cup\{s\in S^*:k_\alpha(s)=0\}$. We first prove part I). Since $c<1-x^2-\frac{x}{n-2}$, by Step 2, $\{s\in S^*:k_\alpha(s)=n$ and $g(s)\in G^m\}$ satisfies property (I) of Definition 1. By Lemma 12, $\{s\in S^*:k_\alpha(s)=n$ and $g(s)\in G^m\}$ satisfies property (II) of Definition 1. Hence, $R=R_\alpha$. We now prove part II). By Step 3, $E(R)\supset\{s\in S^*:k_\alpha(s)=0$, $g(s)\in G^{st}$ and $l_{\hat{\imath}(s)}=0\}$, since otherwise R does not satisfy property (I) of Definition 1. Hence, $R_1=R$. $\square$

Proof of Lemma 5

If $c < x - x^2$, by Lemma 2, $\mathcal{R} = \{R_\alpha, R_\beta\}$, where $E(R_\alpha) = \{s \in S^* : k_\alpha(s) = n$ and $g(s) \in G^m\}$ and $E(R_\beta) = \{s \in S^* : k_\alpha(s) = 0$ and $g(s) = g^{co}\}$. We proceed in two steps.

Step 1. We claim that $\omega_{\alpha\beta} = \lceil (n-1)(1-x) \rceil$. Consider the transition from $s' \in E(R_\alpha)$ to $s'' \in E(R_\beta)$. Let $s(t) = s'$ for some $t \geq 0$. Consider a sequence of ω consecutive mutations such that, for each $\tau \in [t+1, t+\omega]$, $a_{i_\tau} \rightsquigarrow \beta$. Let $t_1 = t + \omega + 1$ and $i_{t_1} \in N \backslash \{i_{t+1}, ..., i_{t+\omega}\}$. Let $\bar{s}'_{i_{t_1}} = (\bar{L}'_{i_{t_1}}, \alpha) \in S_{i_{t_1}}$ be the strategy where i_{t_1} chooses action α and the set of links $\bar{L}'_{i_{t_1}}$ that maximizes her payoff conditional on $a_{i_{t_1}} = \alpha$. Define analogously $\bar{s}''_{i_{t_1}} = (\bar{L}''_{i_{t_1}}, \beta) \in S_{i_{t_1}}$. Denote by $\mathcal{M}_1 \subseteq \mathcal{M}_\alpha(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1))$ the set of those $\alpha-$*groups* to which i_{t_1} gets directly linked by supporting one link if $a_{i_{t_1}} \rightsquigarrow \beta$. Then $\Pi_{i_{t_1}}(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1)) = (n-1)x - (|\mathcal{M}_1| + l''^{\beta}_{i_{t_1}})c$.[50] If $\bar{s}''_{i_{t_1}}$ is a best response then, for each $j \in \{i_{t+1}, ..., i_{t+\omega}\}$ such that $i_{t_1} \notin L_j(t_1)$, $j \in L_{i_{t_1}}(t_1)$ and $M \in \mathcal{M}_1$ if and only if, for each $j' \in M$, $i_{t_1} \notin L_j(t_1)$. On the other hand, $\Pi_{i_{t_1}}(\bar{s}'_{i_{t_1}}, s_{-i_{t_1}}(t_1)) = (n-1-\omega) - |\mathcal{M}_1|\, c$.[51] Player i_{t_1} prefers action β only if $\Pi_{i_{t_1}}(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1)) \geq \Pi_{i_{t_1}}(\bar{s}'_{i_{t_1}}, s_{-i_{t_1}}(t_1))$, *i.e.*,

$$\omega \geq l''^{\beta}_{i_{t_1}} c + (n-1)(1-x). \tag{9}$$

The RHS of (9) is minimized when $l''^{\beta}_{i_{t_1}} = 0$. Let

$$\bar{\omega} = \lceil (n-1)(1-x) \rceil. \tag{10}$$

Hence, $\omega \geq \bar{\omega}$ is a necessary condition for a transition from s' to s''. We now claim that there exist $\widetilde{s}' \in E(R_\alpha)$ and $\widetilde{s}'' \in E(R_\beta)$ such that $\omega \geq \bar{\omega}$ is also sufficient for a transition from $\widetilde{s}'$ to $\widetilde{s}''$. Let $\widetilde{s}' \in E(R_\alpha)$ and $\omega = \bar{\omega}$, and consider that, for each $\tau \in [t+1, t+\omega]$, $a_{i_\tau} \rightsquigarrow \beta$ and $L_{i_\tau}(\tau) = N \backslash \{i_\tau\}$. Then, let the players in $N \backslash \{i_{t+1}, ..., i_{t+\omega}\}$ revise consecutively from $t_1 = t + \omega + 1$ to $t_2 = t + n$. Then, for each $\tau \in [t_1, t_2]$, $a_{i_\tau} \rightsquigarrow \beta$. Hence $k_\alpha(t_2) = 0$ and, by Lemma 9, the claim follows. Hence, $\omega_{\alpha\beta} = \bar{\omega}$.

Step 2. We claim that $\omega_{\beta\alpha} = \lceil (n-1)(x-c)/(1-c) \rceil$. Consider the transition from $s'' \in E(R_\beta)$ to $s' \in E(R_\alpha)$. Let $s(t) = s''$ for some $t \geq 0$. Consider a sequence of ω consecutive mutations such that, for each $\tau \in [t+1, t+\omega]$, $a_{i_\tau} \rightsquigarrow \alpha$. Let $t_1 = t + \omega + 1$ and $i_{t_1} \in N \backslash \{i_{t+1}, ..., i_{t+\omega}\}$. Let $\bar{s}'_{i_{t_1}} = (\bar{L}'_{i_{t_1}}, \alpha) \in S_{i_{t_1}}$ and $\bar{s}''_{i_{t_1}} = (\bar{L}''_{i_{t_1}}, \beta) \in S_{i_{t_1}}$ and $\mathcal{M}_1$ be as defined in Step 1. Then, $\Pi_{i_{t_1}}(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1)) = (n-1)x - (|\mathcal{M}_1| + l''^{\beta}_{i_{t_1}})c$ and $\Pi_{i_{t_1}}(\bar{s}'_{i_{t_1}}, s_{-i_{t_1}}(t_1)) = \omega - |\mathcal{M}_1| \cdot c$. Player i prefers action α only if $\Pi_{i_{t_1}}(\bar{s}'_{i_{t_1}}, s_{-i_{t_1}}(t_1)) \geq$

[50] Recall that $l''^{\beta}_{i_{t_1}}$ is the number of links that i_{t_1} supports to players in $K_\beta(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1))$.

[51] If $a_{i_{t_1}} \rightsquigarrow \alpha$, i_{t_1} only receives a strictly positive payoff from agents in $K_\alpha(t_1 - 1)$ choosing α. Hence, her best response is to support a link to one player of each $\alpha - group$ such that none of the members of the $\alpha - group$ were supporting a link to i_{t_1} at $t_1 - 1$.

$\Pi_{i_{t_1}}(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1))$, *i.e.*,

$$\omega \geq (n-1)x - l''^{\beta}_{i_{t_1}} c. \tag{11}$$

The RHS of (11) is minimized when $l''^{\beta}_{i_{t_1}} = n-\omega-1$. If we substitute $l''^{\beta}_{i_{t_1}}$ by $n-\omega-1$ in (11), we obtain $\omega \geq (n-1)(x-c)/(1-c)$. Let

$$\bar{\omega}' = \lceil (n-1)(x-c)/(1-c) \rceil. \tag{12}$$

Hence, $\omega \geq \bar{\omega}'$ is a necessary condition for a transition from s'' to s'. We now claim that there exist $\widetilde{s}'' \in E(R_\beta)$ and $\widetilde{s}' \in E(R_\alpha)$ such that $\omega \geq \bar{\omega}'$ is also sufficient for a transition from $\widetilde{s}''$ to $\widetilde{s}'$. Let $\omega = \bar{\omega}'$ and $\widetilde{s}'' \in E(R_\beta)$ be such that, for each $j \in N$, $\widetilde{L}''_j = \{1, 2, ..., j-1\}$. For each $\tau \in \{1, ..., \omega\}$, consider that, at period $t+\tau$, $i_{t+\tau} = \tau$ and the mutation $a_{i_{t+\tau}} \rightsquigarrow \alpha$ occurs. Then, consider that, for each $\tau \in \{\omega+1, ..., n\}$, $i_{t+\tau} = \tau$. It is directly verifiable that condition (11) is satisfied for each $\tau \geq \bar{\omega}'+1$. Hence, for each $\tau \in \{\omega+1, ..., n\}$, $a_{i_{t+\tau}} \rightsquigarrow \alpha$. Thus $k_\alpha(t+n) = n$ and, by Lemma 8, the claim follows. Hence, $\omega_{\beta\alpha} = \bar{\omega}'$. □

Proof of Lemma 6

Let $x - x^2 < c < \min\{x, 1-x^2\}$. By Lemma 3, $\mathcal{R} = \{R_\alpha, R_{\beta'}\}$, where $E(R_\alpha) = \{s \in S^* : k_\alpha(s) = n \text{ and } g(s) \in G^m\}$, $E(R_{\beta'}) \supseteq \{s \in S^* : k_\alpha(s) = 0 \text{ and } g(s) \in G^{st}\}$ and, for each $s'' \in E(R_{\beta'})$, $k_\alpha(s'') = 0$. We proceed in 2 steps.

Step 1. We claim that $\omega_{\alpha\beta'} = \lceil (n-1)(1-c-x^2)/(1-c-x^2+x) \rceil$. Consider the transition from $s' \in E(R_\alpha)$ to $s'' \in E(R_{\beta'})$. Let $s(t) = s'$ for some $t \geq 0$. Consider a sequence of ω consecutive mutations such that, for each $\tau \in [t+1, t+\omega]$, $a_{i_\tau} \rightsquigarrow \beta$. Let $t_1 = t+\omega+1$ and $i_{t_1} \in N \backslash \{i_{t+1}, ..., i_{t+\omega}\}$. Let $\bar{s}'_{i_{t_1}} = (\bar{L}'_{i_{t_1}}, \alpha) \in S_{i_{t_1}}$ be the strategy where i_{t_1} chooses action α and the set of links $\bar{L}'_{i_{t_1}}$ that maximizes her payoff conditional on $a_{i_{t_1}} = \alpha$. Define analogously $\bar{s}''_{i_{t_1}} = (\bar{L}''_{i_{t_1}}, \beta) \in S_{i_{t_1}}$. For each $d \in \{1, ..., \omega+1\}$, denote by $\mathcal{M}_d \subseteq \mathcal{M}_\alpha(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1))$ the set of those α*−groups* that, using paths of $\beta - players$, are at distance d from i_{t_1} if $a_{i_{t_1}} \rightsquigarrow \beta$, and let $m_d = \left|\bigcup_{M \in \mathcal{M}_d} M\right|$. Note that $\sum_{d=1}^{\omega+1} m_d = n-\omega-1$. Additionally, for each $d \in \{1, ..., \omega\}$, denote by $\hat{m}_d$ the number of $\beta - players$ that, using paths of $\beta - players$, are at distance d from i_{t_1} if $a_{i_{t_1}} \rightsquigarrow \beta$. Then, $\Pi_{i_{t_1}}(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1)) = \sum_{d=1}^{\omega+1} m_d x^d + \sum_{d=1}^{\omega} \hat{m}_d x^d - \bar{l}''_{i_{t_1}} c$. On the other hand, since $\bar{l}'_{i_{t_1}} = \bar{l}''_{i_{t_1}} - \bar{l}''^{\beta}_{i_{t_1}} + \sum_{d=2}^{\omega+1} |\mathcal{M}_d|$ (by construction), $\Pi_{i_{t_1}}(\bar{s}'_{i_{t_1}}, s_{-i_{t_1}}(t_1)) = n-\omega-1-(\bar{l}''_{i_{t_1}} - \bar{l}''^{\beta}_{i_{t_1}} + \sum_{d=2}^{\omega+1} |\mathcal{M}_d|)c$. Player i_{t_1} prefers action β if and only if $\Pi_{i_{t_1}}(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1)) \geq \Pi_{i_{t_1}}(\bar{s}'_{i_{t_1}}, s_{-i_{t_1}}(t_1))$, *i.e.*,

$$\omega \geq n-1+c \cdot (\bar{l}''^{\beta}_{i_{t_1}} - \textstyle\sum_{d=2}^{\omega+1} |\mathcal{M}_d|) - \sum_{d=1}^{\omega+1} m_d \cdot x^d - \sum_{d=1}^{\omega} \hat{m}_d \cdot x^d. \tag{13}$$

We shall now explore the situation in which the RHS of (13) is minimized. The RHS of (13) can be rewritten as

$$n-1+c \cdot \bar{l}''^{\beta}_{i_{t_1}} - \textstyle\sum_{d=1}^{\omega} \hat{m}_d \cdot x^d - \sum_{d=2}^{\omega+1} |\mathcal{M}_d| \cdot c - \sum_{d=2}^{\omega+1} m_d \cdot x^d - m_1 \cdot x. \tag{14}$$

We claim that (14) is minimized when (i) $\bar{l}_{i_{t_1}}^{\prime\prime\beta} = 0$, (ii) $\sum_{d=1}^{\omega} \hat{m}_d x^d = \omega x$, (iii) for each $d \in \{2, ..., \omega+1\}$, $|\mathcal{M}_d| = m_d$ and (iv) $m_2 = n - \omega - 1$. Conditions (i)-(iii) are straightforward. Thus, we shall prove (iv). Let $\bar{l}_{i_{t_1}}^{\prime\prime\beta} = 0$, $\sum_{d=1}^{\omega} \hat{m}_d x^d = \omega x$ and, for each $d \in \{2, ..., \omega+1\}$, $|\mathcal{M}_d| = m_d$. Then, we want to minimize $-\sum_{d=2}^{\omega+1}(c + x^d)m_d - m_1 x$. Since $\sum_{d=1}^{\omega+1} m_d = n-\omega-1$, for each $d \geq 3$, $m_d = 0$. Thus, we want to minimize $-(c + x^2)m_2 - m_1 x$. Since, $c + x^2 > x$, $m_1 = 0$. Hence, $m_2 = n - \omega - 1$. This proves the claim. Thus, if we substitute the minimizing values in the RHS of (13), we obtain $\omega \geq (n-1)(1-c-x^2) / (1-c-x^2+x)$. Let

$$\hat{\omega} = \left\lceil (n-1)\left(1-c-x^2\right) / \left(1-c-x^2+x\right) \right\rceil. \tag{15}$$

Hence, $\omega \geq \hat{\omega}$ is a necessary condition for a transition from s' to s''.

We now claim that there exist $\widetilde{s}' \in E(R_\alpha)$ and $\widetilde{s}'' \in E(R_{\beta'})$ such that $\omega \geq \hat{\omega}$ is also sufficient for a transition from $\widetilde{s}'$ to $\widetilde{s}''$. Let $\widetilde{s}' \in \{s \in E(R_\alpha) : g(\widetilde{s}') \in G^{st}$ and $l_{\hat{\imath}(g(\widetilde{s}'))} = n-1\}$ and $\omega = \hat{\omega}$, and consider that, for each $\tau \in [t+1, t+\omega]$, $a_{i_\tau} \rightsquigarrow \beta$, $L_{i_\tau}(\tau) = N\backslash\{i_\tau\}$ and $i_{t+1} = \hat{\imath}(g(\widetilde{s}'))$. Then, let the players in $N\backslash\{i_{t+1}, ..., i_{t+\omega}\}$ revise consecutively from period $t_1 = t+\omega+1$ to $t_2 = t+n$. Then it is directly verifiable that, for i_{t_1}, conditions (i)-(iv) are satisfied. Since these conditions minimize (14), $a_{i_{t_1}} \rightsquigarrow \beta$ and $L_{i_{t_1}}(t_1) = \emptyset$. Hence, it is immediate that, for each $\tau \in [t_1+1, t_2]$, the incentives for i_τ to switch to action β increase in τ and, therefore, $a_{i_\tau} \rightsquigarrow \beta$ and $L_{i_\tau}(\tau) = \emptyset$. Then, let the players in $N\backslash\{\hat{\imath}(g(\widetilde{s}'))\}$ revise consecutively from period t_2+1 to $t_3 = t_2+n-1$. Then, for each $\tau \in [t_2+1, t_3]$, $a_{i_\tau} \rightsquigarrow \beta$ and $L_{i_{t_1}}(t_1) = \emptyset$. Since, $L_{\hat{\imath}(g(\widetilde{s}'))}(t_3) = L_{i_{t+1}}(t+1) = N\backslash\{\hat{\imath}(g(\widetilde{s}'))\}$, $s(t_3) \in \{s \in S^* : k_\alpha(s) = 0, g(s) \in G^{st}$ and $l_{\hat{\imath}(g(s))} = n-1\}$. Hence, by Lemma 3, $s(t_3) \in E(R_{\beta'})$. Therefore, the claim follows and $\omega_{\alpha\beta'} = \hat{\omega}$.

Step 2. We claim that $\omega_{\beta'\alpha} = \lceil (n-1)(x-c)/(1-c) \rceil$. Consider the transition from $s'' \in E(R'_\beta)$ to $s' \in E(R_\alpha)$. Let $s(t) = s''$ for some $t \geq 0$. Consider a sequence of ω consecutive mutations such that, for each $\tau \in [t+1, t+\omega]$, $a_{i_\tau} \rightsquigarrow \alpha$. Let $t_1 = t+\omega+1$ and $i_{t_1} \in N\backslash\{i_{t+1}, ..., i_{t+\omega}\}$. Let $\bar{s}'_{i_{t_1}} = (\bar{L}'_{i_{t_1}}, \alpha) \in S_{i_{t_1}}$ and $\bar{s}''_{i_{t_1}} = (\bar{L}''_{i_{t_1}}, \beta) \in S_{i_{t_1}}$ be as defined in Step 1. For each $d \in \{1, ..., n-\omega\}$, define $\mathcal{M}_d$ and m_d as in Step 1. Moreover, for each $d \in \{1, ..., n-\omega-1\}$, define $\hat{m}_d$ as in Step 1. Note that, in this case, $\sum_{d=1}^{n-\omega} m_d = \omega$. Then, $\Pi_{i_{t_1}}(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1)) = \sum_{d=1}^{n-\omega} m_d x^d + \sum_{d=1}^{n-\omega-1} \hat{m}_d x^d - \bar{l}''_{i_{t_1}} c$. On the other hand, since $\bar{l}'_{i_{t_1}} = \bar{l}''_{i_{t_1}} - \bar{l}_{i_{t_1}}^{\prime\prime\beta} + \sum_{d=2}^{n-\omega} |\mathcal{M}_d|$ (by construction), $\Pi_{i_{t_1}}(\bar{s}'_{i_{t_1}}, s_{-i_{t_1}}(t_1)) = \omega - (\bar{l}''_{i_{t_1}} - \bar{l}_{i_{t_1}}^{\prime\prime\beta} + \sum_{d=2}^{n-\omega} |\mathcal{M}_d|)c$. Player i_{t_1} prefers action α if and only if $\Pi_{i_{t_1}}(\bar{s}'_{i_{t_1}}, s_{-i_{t_1}}(t_1)) \geq \Pi_{i_{t_1}}(\bar{s}''_{i_{t_1}}, s_{-i_{t_1}}(t_1))$, *i.e.*,

$$\omega \geq c \cdot (-\bar{l}_{i_{t_1}}^{\prime\prime\beta} + \textstyle\sum_{d=2}^{n-\omega} |\mathcal{M}_d|) + \sum_{d=1}^{n-\omega} m_d \cdot x^d + \sum_{d=1}^{n-\omega-1} \hat{m}_d \cdot x^d. \tag{16}$$

We shall now explore the situation in which the RHS of (16) is minimized. The RHS of (16) can be rewritten as:

$$\textstyle\sum_{d=2}^{n-\omega-1} \hat{m}_d \cdot x^d + (\hat{m}_1 \cdot x - c \cdot \bar{l}_{i_{t_1}}^{\prime\prime\beta}) + \sum_{d=2}^{n-\omega} |\mathcal{M}_d| \cdot c + \sum_{d=2}^{n-\omega} m_d \cdot x^d + m_1 \cdot x. \tag{17}$$

We claim that (17) is minimized when the following two conditions hold: (i) $\overline{l}_{i_{t_1}}^{\prime\prime\beta} = n - \omega - 1$ and (ii) $m_1 = \omega$.

We prove condition (i). Consider the first part of (17), *i.e.*, $\sum_{d=2}^{n-\omega-1} \hat{m}_d \cdot x^d + (\hat{m}_1 \cdot x - c \cdot \overline{l}_{i_{t_1}}^{\prime\prime\beta})$. We claim that if, for some $d \geq 2$, $\hat{m}_d > 0$, then, we are not choosing the situation at t_1 that minimizes (17).[52] Since $\overline{s}_{i_{t_1}}^{\prime\prime}$ is a best response conditional on $a_{i_{t_1}} \rightsquigarrow \beta$, $\hat{m}_d > 0$ implies $x^d > x - c$. But, then if we aim to minimize (17), it is better a situation where i_{t_1} observes the $\beta - player$ at distance one by supporting a link to her. This proves the claim. Then, in order to minimize (17), for each $d \geq 2$, $\hat{m}_d = 0$. Hence, $\hat{m}_1 = n - \omega - 1$. Then, in order to minimize $(n - \omega - 1) \cdot x - c \cdot \overline{l}_{i_{t_1}}^{\prime\prime\beta}$, $\overline{l}_{i_{t_1}}^{\prime\prime\beta} = n - \omega - 1$.

We now prove condition (ii). Consider the second part of (17), *i.e.*, $\sum_{d=2}^{n-\omega} |\mathcal{M}_d| \cdot c + \sum_{d=2}^{n-\omega} m_d \cdot x^d + m_1 \cdot x$. We claim that if, for some $d \geq 2$, $m_d > 0$, then, we are not choosing the situation at t_1 that minimizes (17). Since $\overline{s}_{i_{t_1}}^{\prime\prime}$ is a best response conditional on $a_{i_{t_1}} \rightsquigarrow \beta$, $m_d > 0$ implies $m_d \cdot x^d > m_d \cdot x - |\mathcal{M}_d| \cdot c$. Hence, $m_d \cdot x^d + |\mathcal{M}_d| \cdot c > m_d \cdot x$. Then, if we aim to minimize (17), it is better a situation where i_{t_1} observes all the $\alpha-groups$ at distance one. Therefore, in order to minimize (17), $m_1 = \omega$.

Note that, since $k_\alpha(t_1) = \omega$, condition (i) implies $\sum_{d=2}^{n-\omega-1} \hat{m}_d \cdot x^d + (\hat{m}_1 \cdot x - c \cdot \overline{l}_{i_{t_1}}^{\prime\prime\beta}) = (n-\omega-1)(x-c)$ and condition (ii) implies $\sum_{d=2}^{n-\omega} |\mathcal{M}_d| \cdot c + \sum_{d=2}^{n-\omega} m_d \cdot x^d + m_1 \cdot x = \omega x$. Hence, substituting these values in the RHS of (13), we obtain $\omega \geq (n-1)(x-c)/(1-c)$. Let

$$\hat{\omega}' = \lceil (n-1)(x-c)/(1-c) \rceil. \tag{18}$$

Hence, $\omega \geq \hat{\omega}'$ is a necessary condition for a transition from s'' to s'.

We now claim that there exist $\widetilde{s}'' \in E(R'_\beta)$ and $\widetilde{s}' \in E(R_\alpha)$ such that $\omega \geq \hat{\omega}'$ is also sufficient for a transition from $\widetilde{s}''$ to $\widetilde{s}'$. Let $\widetilde{s}'' \in \{s \in E(R'_\beta) : g(\widetilde{s}'') \in G^{st}$ and $l_{\hat{\imath}(g(\widetilde{s}''))} = n - 1\}$ and $\omega = \hat{\omega}'$, and consider that, for each $\tau \in [t+1, t+\omega]$, $a_{i_\tau} \rightsquigarrow \alpha$, $L_{i_\tau}(\tau) = N \backslash \{i_\tau\}$ and $i_{t+1} = \hat{\imath}(g(\widetilde{s}'))$. Then, let the players in $N \backslash \{i_{t+1}, ..., i_{t+\omega}\}$ revise consecutively from period $t_1 = t + \omega + 1$ to $t_2 = t + n$. It is directly verifiable that, for i_{t_1}, conditions (i)-(ii) are satisfied. Since these conditions minimize (17), $a_{i_{t_1}} \rightsquigarrow \alpha$ and $L_{i_{t_1}}(t_1) = \emptyset$. Hence, it is immediate that, for each $\tau \in [t_1 + 1, t_2]$, the incentives for i_τ to switch to action α increase in τ and, therefore, $a_{i_\tau} \rightsquigarrow \alpha$ and $L_{i_\tau}(\tau) = \emptyset$. Hence $k_\alpha(t_2) = n$ and, by Lemma 8, the claim follows. Hence $\omega_{\beta'\alpha} = \hat{\omega}'$. □

Proof of Lemma 7

Let $x - x^3 < c < x$. Let $s(t) \in S^*$ such that $k_\alpha(t) = 0$ and $g(t) \in G^{st}$. First, consider a mutation at $t + 1$ such that $a_{i_{t+1}} \rightsquigarrow \beta$. Note that, in this case, since $k_\alpha(t+1) = 0$ and $c < x$, the choice of a best response implies that, for each $\tau \geq t+2$,

[52] Recall that we are conditioning our analysis on the choice $\overline{s}_{i_{t_1}}^{\prime\prime} = (\bar{L}_{i_{t_1}}^{\prime\prime}, \beta) \in S_{i_{t_1}}$ at t_1, that is a best response conditional on $a_{i_{t_1}} \rightsquigarrow \beta$. Then, we minimize (17) choosing among all situations where $s(t) \in E(R'_\beta)$ and there are ω mutations such that, for each $\tau \in [t+1, t+\omega]$, $a_{i_\tau} \rightsquigarrow \alpha$.

$a_{i_\tau} \rightsquigarrow \beta$ and, therefore, $k_\alpha(\tau) = 0$. Hence, convergence to a state in $\{s \in S^* : k_\alpha(s) = 0$ and $g(s) \in G^{st}\}$ directly follows from Lemma 3 in Feri [9] (*cf.* footnote 48).

Hence, consider a mutation at $t+1$ such that $a_{i_{t+1}} \rightsquigarrow \alpha$. Note that, in this case, since $k_\alpha(t+1) = 1$ and $c < x$, the choice of a best response implies that, for n large enough, for each $\tau \geq t+2$, $a_{i_\tau} \rightsquigarrow \beta$ and, therefore, $k_\alpha(\tau) \leq 1$.[53] We distinguish three cases: (i) $i_{t+1} = \hat{\imath}(g(t))$ and (ii) $i_{t+1} \in N\backslash\{\hat{\imath}(g(t))\}$ and $\{(i_{t+1}, 1), ..., (i_{t+1}, n)\} \cap g(t+1) = \emptyset$ and (iii) $i_{t+1} \in N\backslash\{\hat{\imath}(g(t))\}$ and $\{(i_{t+1}, 1), ..., (i_{t+1}, n)\} \cap g(t+1) \neq \emptyset$. If $i_{t+2} = i_{t+1}$, clearly $s(t+2) = s(t)$ and the result follows. Hence, let $i_{t+2} \in N\backslash\{i_{t+1}\}$.

In case (i), for each $L_{i_{t+1}}(t+1) \in 2^{N\backslash\{i_{t+1}\}}$ and each $j \in N\backslash\{i_{t+1}\}$ the payoff to j at $t+1$ is either x or $x-c$. Since $c < x$, if $i_{t+2} \notin L_{i_{t+1}}(t+1)$, $L_{i_{t+2}}(t+2) = N\backslash\{i_{t+2}\}$ and, otherwise, $L_{i_{t+2}}(t+2) = N\backslash\{i_{t+1}, i_{t+2}\}$. When a player $j \in N\backslash\{i_{t+1}, i_{t+2}\}$ revises her strategy at period $\tau > t+2$, $L_j(\tau) = \emptyset$, since j is optimally linked with all players via her link to i_{t+2}. When, at some period period $\tau' \geq t+2$, $i_{\tau'} = i_{t+1}$, $a_{i_{\tau'}} \rightsquigarrow \beta$ and $L_{i_{\tau'}}(\tau') \in \{\emptyset, \{i_{t+2}\}\}$.[54] Hence, at some period, say T, such that all players have revised their strategies, $s(T) \in S^*$, $k_\alpha(T) = 0$ and $g(t) \in G^{st}$.

Consider case (ii). All players in $N\backslash\{i_{t+1}\}$ are arranged in a star with center $\hat{\imath}(g(t))$. Since $c < x$, $L_{i_{t+2}}(t+2) = L_{i_{t+2}}(t) \cup \{i_{t+1}\}$. There are two possibilities. First, if $i_{t+2} = \hat{\imath}(g(t))$, $g(t+2) \in G^{st}$. Hence, when a player $j \in N\backslash\{i_{t+1}, i_{t+2}\}$ revises her strategy at period $\tau > t+2$, $L_j(\tau) = L_j(t)$. When, at some period period $\tau' > t+2$, $i_{\tau'} = i_{t+1}$, $a_{i_{\tau'}} \rightsquigarrow \beta$ and $L_{i_{t+1}}(\tau') = \emptyset$. Hence, at some period, say T, such that all players have revised their strategies, $s(T) \in S^*$, $k_\alpha(T) = 0$ and $g(t) \in G^{st}$. Second, if $i_{t+2} \in N\backslash\{\hat{\imath}(g(t))\}$, for each $j, j' \in N$, $d_{j,j'}(g(t+2)) \leq 3$. Since $c > x - x^3$, as long as i_{t+1} has not received a revision opportunity, when a player $j \in N\backslash\{i_{t+1}\}$ revises her strategy at period $\tau > t+2$, $L_j(\tau) = L_j(t)$. When, at some period period $\tau' > t+2$, $i_{\tau'} = i_{t+1}$, $a_{i_{\tau'}} \rightsquigarrow \beta$ and, if n is sufficiently large, $L_{i_{t+1}}(\tau') = \{\hat{\imath}(g(t))\}$. Hence, for each $\hat{\tau} > \tau'$, if $i_{\hat{\tau}} \neq i_{t+2}$, $L_{i_{\hat{\tau}}}(\hat{\tau}) = L_{i_{\hat{\tau}}}(\tau')$, whereas, if $i_{\hat{\tau}} = i_{t+2}$, $L_{i_{t+2}}(\hat{\tau}) = L_{i_{t+2}}(t+2)\backslash\{i_{t+1}\}$. Hence, at some period, say $T > \tau'$, such that $i_T = i_{\hat{\tau}}$, $s(T) \in S^*$, $k_\alpha(T) = 0$ and $g(t) \in G^{st}$.

Finally, consider case (iii). For each $j, j' \in N\backslash\{i_{t+1}\}$, $d_{j,j'}(g(t+1)) \leq 2$, and $d_{j,i_{t+1}}(g(t+1)) \leq 3$. Moreover, since $a_{i_{t+1}} \rightsquigarrow \alpha$ and, for each $j \in N\backslash\{i_{t+1}\}$, $a_j(t+1) = \beta$, no player receives any indirect payoff from a link to i_{t+1}.[55] Hence, for each $\hat{\tau} > t+2$, $L_{i_{\hat{\tau}}}(\hat{\tau}) = L_{i_{\hat{\tau}}}(t)$. Hence, at some period, say $T > t+1$, such that $i_T = i_{t+1}$, $s(T) = s(t)$. $\square$

[53] Note that if a revising player chooses α, her payoff is at most 1 whereas, if she chooses β, her payoff is at least $(n-1)(x-c)$.

[54] Note that $L_{i_{\tau'}}(\tau') = \{i_{t+2}\}$ if and only if $i_{\tau'} \notin L_{i_{t+2}}(\tau')$.

[55] A link to i_{t+1} just provides a payoff of x.

University of Innsbruck – Working Papers in Economics and Statistics
Recent papers

2009-19 **Francesco Feri and Miguel A.Meléndez-Jiménez:** Coordination in Evolving Networks with Endogenous Decay

2009-18 **Harald Oberhofer:** Firm growth, European industry dynamics and domestic business cycles

2009-17 **Jesus Crespo Cuaresma and Martin Feldkircher:** Spatial Filtering, Model Uncertainty and the Speed of Income Convergence in Europe

2009-16 **Paul A. Raschky and Manijeh Schwindt:** On the Channel and Type of International Disaster Aid

2009-15 **Jianying Qiu:** Loss aversion and mental accounting: The favorite-longshot bias in parimutuel betting

2009-14 **Siegfried Berninghaus, Werner Güth, M. Vittoria Levati and Jianying Qiu:** Satisficing in sales competition: experimental evidence

2009-13 **Tobias Bruenner, Rene Levinský and Jianying Qiu:** Skewness preferences and asset selection: An experimental study

2009-12 **Jianying Qiu and Prashanth Mahagaonkar:** Testing the Modigliani-Miller theorem directly in the lab: a general equilibrium approach

2009-11 **Jianying Qiu and Eva-Maria Steiger:** Understanding Risk Attitudes in two Dimensions: An Experimental Analysis

2009-10 **Erwann Michel-Kerjan, Paul A. Raschky and Howard C. Kunreuther:** Corporate Demand for Insurance: An Empirical Analysis of the U.S. Market for Catastrophe and Non-Catastrophe Risks

2009-09 **Fredrik Carlsson, Peter Martinsson, Ping Qin and Matthias Sutter:** Household decision making and the influence of spouses' income, education, and communist party membership: A field experiment in rural China

2009-08 **Matthias Sutter, Peter Lindner and Daniela Platsch:** Social norms, third-party observation and third-party reward

2009-07 **Michael Pfaffermayr:** Spatial Convergence of Regions Revisited: A Spatial Maximum Likelihood Systems Approach

2009-06 **Reimund Schwarze and Gert G. Wagner:** Natural Hazards Insurance in Europe – Tailored Responses to Climate Change Needed

2009-05 **Robert Jiro Netzer and Matthias Sutter:** Intercultural trust. An experiment in Austria and Japan

2009-04 **Andrea M. Leiter, Arno Parolini and Hannes Winner:** Environmental Regulation and Investment: Evidence from European Industries

2009-03 **Uwe Dulleck, Rudolf Kerschbamer and Matthias Sutter:** The Economics of Credence Goods: On the Role of Liability, Verifiability, Reputation and Competition

2009-02 **Harald Oberhofer and Michael Pfaffermayr:** Fractional Response Models - A Replication Exercise of Papke and Wooldridge (1996)

2009-01 **Loukas Balafoutas:** How do third parties matter? Theory and evidence in a dynamic psychological game.

2008-27 **Matthias Sutter, Ronald Bosman, Martin Kocher and Frans van Winden:** Gender pairing and bargaining – Beware the same sex! *Revised version forthcoming in Experimental Economics.*

2008-26 **Jesus Crespo Cuaresma, Gernot Doppelhofer and Martin Feldkircher:** The Determinants of Economic Growth in European Regions.

2008-25 **Maria Fernanda Rivas and Matthias Sutter:** The dos and don'ts of leadership in sequential public goods experiments.

2008-24 **Jesus Crespo Cuaresma, Harald Oberhofer and Paul Raschky:** Oil and the duration of dictatorships.

2008-23 **Matthias Sutter:** Individual behavior and group membership: Comment. *Revised Version forthcoming in American Economic Review.*

2008-22 **Francesco Feri, Bernd Irlenbusch and Matthias Sutter:** Efficiency Gains from Team-Based Coordination – Large-Scale Experimental Evidence.

2008-21 **Francesco Feri, Miguel A. Meléndez-Jiménez, Giovanni Ponti and Fernando Vega Redondo:** Error Cascades in Observational Learning: An Experiment on the Chinos Game.

2008-20 **Matthias Sutter, Jürgen Huber and Michael Kirchler:** Bubbles and information: An experiment.

2008-19 **Michael Kirchler:** Curse of Mediocrity - On the Value of Asymmetric Fundamental Information in Asset Markets.

2008-18 **Jürgen Huber and Michael Kirchler:** Corporate Campaign Contributions as a Predictor for Abnormal Stock Returns after Presidential Elections.

2008-17 **Wolfgang Brunauer, Stefan Lang, Peter Wechselberger and Sven Bienert:** Additive Hedonic Regression Models with Spatial Scaling Factors: An Application for Rents in Vienna.

2008-16 **Harald Oberhofer, Tassilo Philippovich:** Distance Matters! Evidence from Professional Team Sports.

2008-15 **Maria Fernanda Rivas and Matthias Sutter:** Wage dispersion and workers' effort.

2008-14 **Stefan Borsky and Paul A. Raschky:** Estimating the Option Value of Exercising Risk-taking Behavior with the Hedonic Market Approach. *Revised version forthcoming in Kyklos.*

2008-13 **Sergio Currarini and Francesco Feri:** Information Sharing Networks in Oligopoly.

2008-12 **Andrea M. Leiter:** Age effects in monetary valuation of mortality risks - The relevance of individual risk exposure.

2008-11 **Andrea M. Leiter and Gerald J. Pruckner:** Dying in an Avalanche: Current Risks and their Valuation.

2008-10 **Harald Oberhofer and Michael Pfaffermayr:** Firm Growth in Multinational Corporate Groups.

2008-09 **Michael Pfaffermayr, Matthias Stöckl and Hannes Winner:** Capital Structure, Corporate Taxation and Firm Age.

2008-08 **Jesus Crespo Cuaresma and Andreas Breitenfellner:** Crude Oil Prices and the Euro-Dollar Exchange Rate: A Forecasting Exercise.

2008-07 **Matthias Sutter, Stefan Haigner and Martin Kocher:** Choosing the carrot or the stick? – Endogenous institutional choice in social dilemma situations.

2008-06 **Paul A. Raschky and Manijeh Schwindt:** Aid, Catastrophes and the Samaritan's Dilemma.

2008-05 **Marcela Ibanez, Simon Czermak and Matthias Sutter:** Searching for a better deal – On the influence of group decision making, time pressure and gender in a search experiment. *Revised version published in Journal of Economic Psychology, Vol. 30 (2009): 1-10.*

2008-04 **Martin G. Kocher, Ganna Pogrebna and Matthias Sutter:** The Determinants of Managerial Decisions Under Risk.

2008-03 **Jesus Crespo Cuaresma and Tomas Slacik:** On the determinants of currency crises: The role of model uncertainty. *Revised version accepted for publication in Journal of Macroeconomics)*

2008-02 **Francesco Feri:** Information, Social Mobility and the Demand for Redistribution.

2008-01 **Gerlinde Fellner and Matthias Sutter:** Causes, consequences, and cures of myopic loss aversion - An experimental investigation. *Revised version published in The Economic Journal, Vol. 119 (2009), 900-916.*

2007-31 **Andreas Exenberger and Simon Hartmann:** The Dark Side of Globalization. The Vicious Cycle of Exploitation from World Market Integration: Lesson from the Congo.

2007-30 **Andrea M. Leiter and Gerald J. Pruckner:** Proportionality of willingness to pay to small changes in risk - The impact of attitudinal factors in scope tests. *Revised version forthcoming in Environmental and Resource Economics.*

2007-29 **Paul Raschky and Hannelore Weck-Hannemann:** Who is going to save us now? Bureaucrats, Politicians and Risky Tasks.

2007-28 **Harald Oberhofer and Michael Pfaffermayr:** FDI versus Exports. Substitutes or Complements? A Three Nation Model and Empirical Evidence.

2007-27 **Peter Wechselberger, Stefan Lang and Winfried J. Steiner:** Additive models with random scaling factors: applications to modeling price response functions.

2007-26 **Matthias Sutter:** Deception through telling the truth?! Experimental evidence from individuals and teams*. Revised version published in The Economic Journal, Vol. 119 (2009), 47-60.*

2007-25 **Andrea M. Leiter, Harald Oberhofer and Paul A. Raschky:** Productive disasters? Evidence from European firm level data. *Revised version forthcoming in Environmental and Resource Economics.*

2007-24 **Jesus Crespo Cuaresma:** Forecasting euro exchange rates: How much does model averaging help?

2007-23 **Matthias Sutter, Martin Kocher and Sabine Strauß:** Individuals and teams in UMTS-license auctions. *Revised version with new title "Individuals and teams in auctions" published in Oxford Economic Papers, Vol. 61 (2009): 380-394***).**

2007-22 **Jesus Crespo Cuaresma, Adusei Jumah and Sohbet Karbuz:** Modelling and Forecasting Oil Prices: The Role of Asymmetric Cycles. *Revised version accepted for publication in The Energy Journal.*

2007-21 **Uwe Dulleck and Rudolf Kerschbamer:** Experts vs. discounters: Consumer free riding and experts withholding advice in markets for credence goods. *Revised version published in International Journal of Industrial Organization, Vol. 27, Issue 1 (2009): 15-23.*

2007-20 **Christiane Schwieren and Matthias Sutter:** Trust in cooperation or ability? An experimental study on gender differences*. Revised version published in Economics Letters, Vol. 99 (2008): 494-497.*

2007-19 **Matthias Sutter and Christina Strassmair:** Communication, cooperation and collusion in team tournaments – An experimental study. *Revised version published in: Games and Economic Behavior, Vol.66 (2009), 506-525.*

2007-18 **Michael Hanke, Jürgen Huber, Michael Kirchler and Matthias Sutter:** The economic consequences of a Tobin-tax – An experimental analysis.

2007-17 **Michael Pfaffermayr:** Conditional beta- and sigma-convergence in space: A maximum likelihood approach. *Revised version forthcoming in Regional Science and Urban Economics.*

2007-16 **Anita Gantner:** Bargaining, search, and outside options. *Published in: Games and Economic Behavior, Vol. 62 (2008), pp. 417-435.*

2007-15 **Sergio Currarini and Francesco Feri:** Bilateral information sharing in oligopoly.

2007-14 **Francesco Feri:** Network formation with endogenous decay.

2007-13 **James B. Davies, Martin Kocher and Matthias Sutter:** Economics research in Canada: A long-run assessment of journal publications. *Revised version published in: Canadian Journal of Economics, Vol. 41 (2008), 22-45.*

2007-12 **Wolfgang Luhan, Martin Kocher and Matthias Sutter:** Group polarization in the team dictator game reconsidered. *Revised version published in: Experimental Economics, Vol. 12 (2009), 26-41.*

2007-11 **Onno Hoffmeister and Reimund Schwarze:** The winding road to industrial safety. Evidence on the effects of environmental liability on accident prevention in Germany.

2007-10 **Jesus Crespo Cuaresma and Tomas Slacik:** An “almost-too-late” warning mechanism for currency crises. *(Revised version accepted for publication in Economics of Transition***)**

2007-09 **Jesus Crespo Cuaresma, Neil Foster and Johann Scharler:** Barriers to technology adoption, international R&D spillovers and growth.

2007-08 **Andreas Brezger and Stefan Lang:** Simultaneous probability statements for Bayesian P-splines.

2007-07 **Georg Meran and Reimund Schwarze:** Can minimum prices assure the quality of professional services?.

2007-06 **Michal Brzoza-Brzezina and Jesus Crespo Cuaresma:** Mr. Wicksell and the global economy: What drives real interest rates?.

2007-05 **Paul Raschky:** Estimating the effects of risk transfer mechanisms against floods in Europe and U.S.A.: A dynamic panel approach.

2007-04 **Paul Raschky and Hannelore Weck-Hannemann:** Charity hazard - A real hazard to natural disaster insurance. *Revised version forthcoming in: Environmental Hazards.*

2007-03 **Paul Raschky:** The overprotective parent - Bureaucratic agencies and natural hazard management.

2007-02 **Martin Kocher, Todd Cherry, Stephan Kroll, Robert J. Netzer and Matthias Sutter:** Conditional cooperation on three continents. *Revised version published in: Economics Letters, Vol. 101 (2008): 175-178.*

2007-01 **Martin Kocher, Matthias Sutter and Florian Wakolbinger:** The impact of naïve advice and observational learning in beauty-contest games.

University of Innsbruck

Working Papers in Economics and Statistics

2009-19

Francesco Feri and Miguel A.Meléndez-Jiménez

Coordination in Evolving Networks with Endogenous Decay

Abstract

This paper studies an evolutionary model of network formation with endogenous decay, in which agents benefit both from direct and indirect connections. In addition to forming (costly) links, agents choose actions for a coordination game that determines the level of decay of each link. We address the issues of coordination (long-run equilibrium selection) and network formation by means of stochastic stability techniques. We find that both the link cost and the trade-off between efficiency and risk-dominance play a crucial role in the long-run behavior of the system.

ISSN 1993-4378 (Print)
ISSN 1993-6885 (Online)