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The favorite-longshot bias in parimutuel betting

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Loss aversion and mental accounting: The favorite-longshot bias in parimutuel betting

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Abstract

In this paper it is shown that the combination of mental accounting and loss aversion can fundamentally changes people's way of evaluating risky alternatives. The observation is applied in a market setting: Parimutuel betting markets. In parimutuel betting markets it has been found that for horses with lowest odds (favorites), market estimates of winning probabilities are smaller than objective winning probabilities; for horses with highest odds (longshot) the opposite is observed (the favorite-longshot bias). I build a game theoretical model and show that the favorite-longshot bias is the equilibrium play of the players with loss aversion, and that the degree of the favorite-longshot bias depends on the mental accounting process the players use.

Keywords: loss aversion, mental accounting, parimutuel betting, the favorite-longshot bias.

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1 Introduction

The decision of buying a lottery involves complicated mental processes. Standard financial theory prescribes that one should integrate the lottery at consideration with the current portfolio and evaluate the net effect of this additional lottery according to expected utility. This integration process, however, often needs time and substantial cognitive effort. When time and cognitive capacity are limited, as often the case in reality, one builds a separate mental account for the lottery and evaluate this lottery alone, i.e., the so called segregation process (Thaler and Ziemba, 1988). Additionally, as shown in Kahneman and Tversky (1979) and Kahneman and Tversky (1992), people evaluate lotteries with respect a reference point and care much more about losses relative to their reference points than about gains. In this paper it is shown that the combination of mental accounting and loss aversion can fundamentally changes people's way of evaluating risky lotteries.

I apply this observation in a market setting: Parimutuel betting markets. In parimutuel betting markets it has been found that for horses with lowest odds (favorites), market estimates of winning probabilities are smaller than objective winning probabilities; for horses with highest odds (longshot) the opposite is observed (the favorite-longshot bias). I build a game theoretical model and show that the favorite-longshot bias is the equilibrium play of the players with loss aversion, and that the degree of the favorite-longshot bias depends on the mental accounting process the players use.

This paper shares some features of Thaler and Ziemba (1988). I also rely on mental accounting. But there is one critical difference. Thaler and Ziemba (1988) aim to show that when players take part in *serial betting markets*, all outcomes of these bets are evaluated in the same mental account; thus players show higher risk preference if there is a loss in prior bets due to the risk seeking in the loss domain. In contrast, I show in this paper that, *in a single betting*, the bias results from loss aversion, and depending on the specific process of mental accounting, the degree of the bias differs. Intuitively, since people are more sensitive to losses than to gains, they are less willing to bet than otherwise. When the market is not in equilibrium, this prevents full movement back of relative frequencies of bets to objective probabilities and results in the bias. Moreover, in Pari-mutuel betting markets bets are paid well before the outcomes are known and are thus essentially prior losses. As pointed out by Thaler (1985) and Thaler and Johnson (1990), prior losses can either be integrated with, or segregated from, currently available alternatives, and that prospect theory often predicts different choices depending on which mental accounting process is used.

The remainder of the paper is organized as follows. Section 2 demonstrates a behavioral observation and its application in the evaluation of lotteries. Section 3 introduces the basic model, derives the theoretical results for two mental accounting processes, and explores the effects of mental accounting and loss aversion on the favorite-longshot bias. Section 4 concludes.

2 A behavioral observation and its application

In standard decision theory, players integrate the economic outcome of a decision with all other decisions and form one mental account. They decide wether to take an action by evaluating the marginal impact of this decision to this mental account. Thaler (1985) and Thaler and Johnson (1990) pointed out, however, people often build different mental accounts for different economic activities, and they act as if the money in these mental accounts is not exchangeable.

This idea together with loss aversion implies that the way players build mental accounts affects their decision. This is best illustrated by the following example. Consider a player who evaluates any prospect P of winning x with probability p or y with probability $1-p$ as follows:

$$V(P) = \pi(p)v(x) + \pi(1-p)v(y).$$

Here $\pi(\cdot)$ is the probability weighting function and $v(\cdot)$ is the prospect value function. Think of a bettor's decision to buy the following risky alternative at price c:

Risky alternative R

Outcome	Prob.	Outcome	Prob.
x	p	0	$1-p$

When the bettor integrates the prior loss, price c, with the risky alternative, the following mental account is established:

New risky alternative R'

Outcome	Prob.	Outcome	Prob.
$x-c$	p	$-c$	$1-p$

Thus

$$V(R') = \pi(p)v(x-c) + \pi(1-p)v(-c),$$

whereas, if the bettor segregates the prior loss from the risky alternative, his mental account can be presented as:

A sure loss of c and

The risky alternative R

which can be denoted as a compound prospect C. Notice $v(0) = 0$, it then follows

$$V(C) = v(-c) + \pi(p)v(x).$$

In general, $V(R') > 0$ does not imply $V(C) > 0$. Thus, depending on the mental accounting process people apply, different decisions may be made.

Segregating prior losses from risky alternatives may seem less intuitive and thus deserves some further remarks. Notice that how well people can integrate different attributes depends critically on the cognitive distance of these attributes. The integration process can be easily applied if different attributes are cognitively similar, whereas it would be difficult to apply if different attributes had large cognitive distance. Placing bets is like using safe money to buy lotteries. These two things are cognitively different: one is safe money, the other is lottery. Hence it is not obvious that people can integrate prior losses with risky alternatives.

I analyze the implication of loss aversion and mental accounting for economic decision making in a market setting: Parimutuel betting markets. Pari-mutuel betting is a betting system in which all bets of a particular type are placed together in a pool; taxes and a house take are removed, and payoff odds are calculated by sharing the pool among all placed bets. Pari-mutuel betting markets are special kinds of financial markets, where participants take a financial position on the outcome of a horse race. Although these markets are a tiny feature of most economies, they present significant opportunities for economic analysis. This stems from the fact that Pari-mutuel betting markets are simplified financial markets in which the scope of pricing has been greatly reduced:

- Fundamental value of traded assets, namely bets, are both observable and exogenous;
- The time horizon of assets is well defined.

Hence Pari-mutuel betting markets can provide a clearer view of pricing issues which are more complicated elsewhere.

Empirical studies generally conclude that betting markets are surprisingly efficient. But it has also been found that for horses with lowest odds (favorites) market estimates of winning probabilities are smaller than objective winning probabilities; for horses with highest odds (longshot) the opposite is observed. This puzzling regularity, called the favorite-longshot bias (hereafter referred to as the bias), cannot be reconciled with the behavior of risk averse individuals who behave according to the expected utility theory (von Neumann and Morgenstern, 1944).

In follows I build a game theoretical model and show that the favorite-longshot bias is the equilibrium play of the players with loss aversion, and that the degree of the favorite-longshot bias depends on the mental accounting process the players use. I rely on the sequential betting model developed by Koessler et al. (2003). The exogenously determined sequence greatly simplifies the analysis but retains the basic dynamic feature of Pari-mutuel betting markets: players place bets based on the odds they have observed so far and the expectation of final odds. Moreover, this structure is theoretically appealing for our purpose: it involves sequential evaluations of risky alternatives which, due to the institutional features of Pari-mutuel markets, change with every new bet.

3 The model

The model is a multi-stage game. There are n players (called strategic bettors) who place their bets sequentially at a predefined stage, and in each stage only one player moves. Players behave according to prospect theory and thus maximize the decision weighted value of bets. The set of players is denoted by N. There are two horses called F (standing for favorite) and L (standing for longshot), with respective objective winning probabilities of p and $1-p$, where $p > 1/2$. Before the start of the game, there are initial bets placed by some unmodeled noisy bettors with k units of money on each of the horses.

When a player moves in her predefined stage, she can choose to bet *one* unit of money on either of two horses, F or L, or refrain from betting. More precisely, each player chooses an action $a_i \in A = \{F, L, D\}$ in stage i, where F (respectively L) means to bet 1 unit of money on horse F (respectively L), and D means to refrain from betting. Let $h^t = (a_1, a_2, \ldots, a_t)$ denote the history up to stage t, $h^0 = \emptyset$ represents the starting of the game, and $h^n = z$ represents one terminal history. At the beginning of stage i, player i knows history h^{i-1}. Let H^t denote the set of histories up to stage t. For any non empty history $h^t \in H^t$, I partition the players that moved in $\{1, 2, \ldots, t\}$ into three sets:

$$
\begin{aligned}
F(h^t) &= \{i \in \{1, 2, \ldots, t\} : \textit{such that } a_i = F\}, \\
L(h^t) &= \{i \in \{1, 2, \ldots, t\} : \textit{such that } a_i = L\}, \\
D(h^t) &= \{i \in \{1, 2, \ldots, t\} : \textit{such that } a_i = D\}.
\end{aligned}
$$

Hence, after history h^t, $F(h^t)$ (respectively $L(h^t)$) denotes the set of players who have bet on horse F (respectively horse L), and $D(h^t)$ denotes the set of bettors who have refrained from betting. After history h^t, let $n_F(h^t) = |F(h^t)|$ (respectively $n_L(h^t) = |L(h^t)|$) denote the number of players who have bet on horse F (respectively horse L) and $n_D(h^t) = |D(h^t)|$ denotes the number of bettors who have refrained from betting.

By the institutional features of parimutuel betting markets, the total money bet on all horses, net of the track take, is shared proportionally among those who bet on the winning horse. This implies that when players choose to bet by conditioning bets on the histories observed so far, they need to take into account the effect of their own bet and future bets on the payoff. It has been argued by Hurley and McDonough (1995) that track take is responsible for the favorite-longshot bias. Since here I am mainly concerned with the effects of loss aversion and mental accounting on the bias, I assume a zero track take ratio to avoid possible disturbances.

Players evaluate risky alternatives in line with prospect theory. In order to rules out the effect of nonlinear probability weighting on the bias, the weighting function, $\pi(\cdot)$, is defined as

$$\pi(p) = p.$$

This form provides a good approximation when the parameter p is not in the extreme range. The prospect theory value function, $v(\cdot)$, is defined for present purposes as a segmented power function with three parameters (Kahneman and Tversky, 1992):

$$v(x) = \begin{cases} x^{\alpha} & if\ x \geq 0 \\ -\lambda(-x)^{\beta} & if\ x < 0, \end{cases}$$

where α determines the gain domain concavity, β determines loss domain convexity of the value function, and λ relates to the extent of loss aversion. Though the exact values of α, β, and λ are hard to determine, experimental findings generally conclude $0 < \alpha < 1$, $0 < \beta < 1$, and $\lambda > 1$, which implies the value function is convex on the loss domain and concave on the gain domain, and is loss averse. Since players are allowed to place only one unit of bet, their loss is always one, which implies the value of β does not influence players' behavior in this model since $v(-1) = -\lambda(1)^{\beta} = -\lambda$. It is also clear that this construction is in fact a special form of expected utility expression.

In parimutuel betting markets bets are paid well before the outcomes are known and are thus essentially prior losses. As pointed out by Thaler (1985) and Thaler and Johnson (1990), prior losses can either be integrated with (the integration process), or segregated from (the segregation process), currently available alternatives, and that prospect theory often predicts different choices depending on which mental accounting process is used.

Let Z denote the set of terminal histories. Since a terminal history represents an entire sequence of a play, i.e., the outcome of a play, and recalls the k units of initial bets on each of horses, the decision weighted value of *one unit* of bet based on the integration process and the segregation process is then respectively,

- the integration process

$$V_i : Z \to \Re$$

$$V_i(z) = \begin{cases} p(\frac{n_F(z)+n_L(z)+2k}{n_F(z)+k} - 1)^\alpha + (1-p)(-\lambda) \; if \; i \in F(z) \\ (1-p)(\frac{n_F(z)+n_L(z)+2k}{n_L(z)+k} - 1)^\alpha + p(-\lambda) \; if \; i \in L(z) \\ 0 \; if \; i \in D(z), \end{cases}$$

and

- the segregation process

$$V_i : Z \to \Re$$

$$V_i(z) = \begin{cases} p(\frac{n_F(z)+n_L(z)+2k}{n_F(z)+k})^\alpha - \lambda \; if \; i \in F(z) \\ (1-p)(\frac{n_F(z)+n_L(z)+2k}{n_L(z)+k})^\alpha - \lambda \; if \; i \in L(z) \\ 0 \; if \; i \in D(z). \end{cases}$$

As a terminal history z is uniquely defined by a strategy profile s, players' decision weighted value of one unit of bet can be written more explicitly as $V_i(z(s))$.

Bettor i's behavioral strategy is denoted by

$$s_i : h^{i-1} \longrightarrow A = \{F, L, D\},$$

and a profile of behavioral strategies is denoted by $s = (s_1, s_2, \ldots, s_n)$. To break the tie, I assume a player refrains from betting should she expect a zero decision weighted value from risky alternatives. Let $z(s|h^t)$ be the final history reached according to the strategy profile s, given the history $h^t \in H^t$; thus $z(s)$ is simply the final history generated by strategy profile s. A strategy profile s^* is the subgame perfect equilibrium if for $\forall h^{i-1} \in H_i^{i-1}$, and for $\forall i \in N$

$$V_i(z(s_i^*, s_{-i}^*|h^{i-1})) \geq V_i(z'(s_i', s_{-i}^*|h^{i-1})) \; for \; \forall s_i' \in S_i.$$

3.1 General results

In this section I derive the equilibrium outcome, using the subgame perfect equilibrium as solution concept.

Lemma 1 *There is no equilibrium outcome in which some players bet on the longshot and some players bet on the favorite.*

Proof for lemma 1: Assume by way of contradiction that z is an equilibrium outcome, where exist some i and j such that $a_i = F$ and $a_j = L$. I first prove for the case, where players apply the integration process. The fact that z is an equilibrium outcome implies

$$V_i(z) = p(\frac{n_F(z) + n_L(z) + 2k}{n_F(z) + k} - 1)^\alpha + (1-p)(-\lambda) > 0$$

and

$$V_j(z) = (1-p)(\frac{n_F(z) + n_L(z) + 2k}{n_L(z) + k} - 1)^\alpha + p(-\lambda) > 0,$$

which is equivalent to

$$\frac{n_L(z) + k}{n_F(z) + k} > \frac{(1-p)^{1/\alpha}\lambda^{1/\alpha}}{p^{1/\alpha}}$$

and

$$\frac{n_L(z) + k}{n_F(z) + k} < \frac{(1-p)^{1/\alpha}}{p^{1/\alpha}\lambda^{1/\alpha}}.$$

Since $\lambda > 1$, this yields a contradiction.

Similarly, when players apply the segregation process, the fact that z is an equilibrium outcome implies

$$V_i(z) = -\lambda + p(\frac{n_F(z) + n_L(z) + 2k}{n_F(z) + k})^\alpha > 0$$

and

$$V_j(z) = -\lambda + (1-p)(\frac{n_F(z) + n_L(z) + 2k}{n_L(z) + k})^\alpha > 0,$$

after some algebraic manipulation, we get

$$\frac{n_F(z) + k}{n_F(z) + n_L(z) + 2k} < (\frac{p}{\lambda})^{1/\alpha}$$

and

$$\frac{n_L(z)+k}{n_F(z)+n_L(z)+2k} < (\frac{1-p}{\lambda})^{1/\alpha},$$

which implies

$$(\frac{p}{\lambda})^{1/\alpha} + (\frac{1-p}{\lambda})^{1/\alpha} > 1,$$

which is a a contradiction to $\lambda > 1$. Thus, in equilibrium, players should bet only on the favorite or on the longshot.

Lemma 2 *Let s^* be a subgame perfect equilibrium, then in any subgame perfect equilibrium outcome $z(s^*)$, there is no bet on the longshot.*

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Proof for lemma 2: Assume by way of contradiction that $n_L(z(s^*)) > 0$, then by lemma 1, we must have $n_F(z(s^*)) = 0$. I first consider the integration process. $n_L(z(s^*)) > 0$ and $n_F(z(s^*)) = 0$ implies that for players $i \in L(z(s^*))$

$$V_i(z(s^*)) = (1-p)(\frac{n_L(z(s^*))+2k}{n_L(z(s^*))+k} - 1)^\alpha + p(-\lambda) > 0$$

which implies that

$$p < \frac{\left(\frac{k}{n_L(z(s^*))+k}\right)^\alpha}{\left(\frac{k}{n_L(z(s^*))+k}\right)^\alpha + \lambda} < \frac{1}{1+\lambda} < \frac{1}{2},$$

which is in contradiction to the assumption of $p > 1/2$.

Let B denote the aggregate betting volume, with $B = B^I$ if players apply the integration process and $B = B^S$ if players apply the segregation process. By lemma 1 and 2, it can be shown that $B^I(> 0)$ is the integer defined by

$$p\left(\frac{B^I+2k}{B^I+k} - 1\right)^\alpha - (1-p)\,\lambda > 0 \tag{1}$$

and

$$p\left(\frac{B^I+2k+1}{B^I+k+1} - 1\right)^\alpha - (1-p)\,\lambda \leq 0; \tag{2}$$

and $B^S(>0)$ is the integer defined by

$$-\lambda + p\left(\frac{B^S+2k}{B^S+k}\right)^\alpha > 0 \qquad (3)$$

and

$$-\lambda + p\left(\frac{B^S+2k+1}{B^S+k+1}\right)^\alpha \leq 0. \qquad (4)$$

Lemma 3 *In any Nash equilibrium s^*,*
(i) if $B \leq 0$, then $n_F(z(s^*)) = 0$;
(ii) if $B > 0$ and $n \leq B$, then $n_F(z(s^*)) = n$;
(iii)if $n > B > 0$, then $n_F(z(s^*)) = B$.

Proof for lemma 3: By lemmas 1 and 2, we have $n_L(z(s^*)) = 0$. In the following, we prove only for the integration process. The segregation process follows similarly.
(i) Notice that, from inequation (2), $B^I \leq 0$ implies $(\frac{p}{(1-p)\lambda})^{1/\alpha}k - k - 1 \leq 0$. Suppose $n_F(z(s^*)) > 0$, then it follows that for any player $i \in F(z(s^*))$

$$p\left(\frac{k}{n_F(z(s^*))+k}\right)^\alpha - (1-p)\,\lambda > 0$$

which yields

$$1 \leq n_F(z(s^*)) < (\frac{p}{(1-p)\lambda})^{1/\alpha}k - k.$$

which is in contradiction to $(\frac{p}{(1-p)\lambda})^{1/\alpha}k - k - 1 \leq 0$.

(ii) When $B^I > 0$ and $n \leq B^I$, it follows that

$$p\left(\frac{k}{n+k}\right)^\alpha - (1-p)\,\lambda \geq 0.$$

Since it is impossible to have $n_F(z(s^*)) > n$, suppose by way of contradiction that $n_F(z(s^*)) < n$. This implies $D(z(s^*))$ is not empty. Suppose player $i \in D(z(s^*))$ unilaterally deviates and chooses to bet on the favorite. Let z' denote the outcome after deviation, then this player i's decision weighted value is

$$V_i(z') = p\left(\frac{n_L(z')+k}{n_F(z')+k}\right)^\alpha - (1-p)\,\lambda$$

Notice that by $n_F(z') \leq n$ and $\frac{n_L(z')+k}{n_F(z')+k} \geq \frac{k}{n+k}$, combined with the above inequation, we have

$$V_i(z') \geq p\left(\frac{k}{n+k}\right)^{\alpha} - (1-p)\,\lambda > 0,$$

which contradicts s^* being a subgame perfect equilibrium.

(iii) By the construction of B^I, obviously $n_F(z(s^*)) > B^I$ cannot be supported as an equilibrium outcome. Suppose $n_F(z(s^*)) + 1 \leq B^I$, then by the construction of B^I, we get

$$p\left(\frac{k}{n_F(z(s^*)) + 1 + k}\right)^{\alpha} - (1-p)\,\lambda > 0$$

Since $n > B^I$ and $n_L(z(s^*)) = 0$, it follows that $D(z(s^*))$ is not empty. Consider the *last* player, player i, who refrains from betting. It is then clear that the $n-i$ players after player i all choose to bet on the favorite, and that there are $n_F(z(s^*)) - n + i$ bets before stage i.

Now suppose player i unilaterally deviates and chooses to bet on the favorite. This deviation does not affect the actions of players who move earlier than player i. Thus, at the end of stage i, there are $n_F(z(s^*)) - n + i + 1$ bets on the favorite and zero bets on the longshot. Let z' denote the terminal history after i's deviation, and let $n_L^{-i}(z')$ and $n_F^{-i}(z')$, respectively, denote the bets on the longshot and the favorite *after* stage i. Player i's decision weighted value then is

$$V_i(z') = p\left(\frac{n_L^{-i} + k}{n_F(z(s^*)) + i + 1 - n + n_F^{-i} + k}\right)^{\alpha} - (1-p)\,\lambda.$$

Notice that $n_F^{-i} \leq n - i$ and $n_L^{-i} \geq 0$, then

$$\frac{n_L^{-i} + k}{n_F(z(s^*)) + i + 1 - n + n_F^{-i} + k} > \frac{k}{n_F(z(s^*)) + 1 + k},$$

It follows that

$$V_i(z') > 0$$

which is in contradiction to s^* being a subgame perfect equilibrium.

Lemma 4 *If $n > B > 0$ and s^* is a subgame perfect equilibrium, then in the subgame following any h^t with $n_F(h^t) \leq B$ and $n_L(h^t) = 0$, bets realized according to s^* are such that zero bets are placed on the longshot and at most $B - n_F(h^t)$ bets on the favorite.*

Proof for lemma 4: analogous to lemma 3.

Proposition 1 *If $n > B > 0$, then in the outcome z^* supported by subgame perfect equilibrium s^*, the first B players bet on the favorite while the others refrain from betting.*

Proof: Let h^{*i} denote the history up to stage i and a_i the action of player i in z^*. Suppose by way of contradiction that in z^* there exists a player $j > B$ such that $a_j = F$. By Lemma 3, we know that in any Nash equilibrium outcome, there are B bets on the favorite and zero bets on the longshot. Thus $n_F(h^{*B}) < B$, and there exists at least one player $i \leq B$ such that $a_i = D$. Consider the unilateral deviation of player i choosing to bet on the favorite. This deviation does not affect the actions of players who move earlier than i. So after stage i, there are $n_F(h^{*i}) + 1 \leq B$ bets on the favorite and zero bets on the longshot. Since others are still playing the subgame perfect strategy, by Lemma 4, bets realized in the subsequent periods should be zero bets on the longshot and at most $B - n_F(h^{*i}) - 1$ bets on the favorite. Thus, in the outcome after deviation, z', there are at most B bets on the favorite and zero bets on the longshot, and player i receives positive decision weighted value. A contradiction to z^* is an equilibrium outcome.

3.2 The bias: integration vs. segregation

Solving the inequations defining the equilibrium betting volume under the integration process B^I and the segregation process B^S, (1), (2), (3), and (4), we have B^I as the integer defined by

$$Max\left\{0, Int^I\right\},$$

where Int^I is the integer in the interval of $\left[\frac{p^{1/\alpha}}{(1-p)^{1/\alpha}\lambda^{1/\alpha}}k - k - 1, \frac{p^{1/\alpha}}{(1-p)^{1/\alpha}\lambda^{1/\alpha}}k - k\right)$. B^S is the integer defined by

$$Max\left\{0, Int^B\right\},$$

where Int^B is the integer in the interval of $\left[\frac{p^{1/\alpha}}{\lambda^{1/\alpha}-p^{1/\alpha}}k - k - 1, \frac{p^{1/\alpha}}{\lambda^{1/\alpha}-p^{1/\alpha}}k - k\right)$.

As shown in Figure 4, if bettors apply the integration process, the prices they pay are combined with the risky alternatives. This limits the loss that individuals perceive and thus encourages betting. In contrast, in the segregation process, the price is segregated from the risky alternative and thus framed as prior loss. Due to a heightened sensitivity to losses as compared to equivalent gains, bettors are less willing to buy the risky alternative compared to the integration process.

Let ρ_i denote the ratio of aggregate bets on horse i ($i \in \{F, L\}$) divided by the total bets on two horses. With zero track take, rational players maximizing expected payoff evaluate one unit of bet on horse i with winning probability p_i, $i \in \{F, L\}$, as the following

$$p_i(\frac{1}{\rho_i} - 1) + (1 - p_i)(-1),$$

where $p_F = p$ and $p_L = 1 - p$. Rational decision making theory suggests that, in equilibrium, we should have

$$p_F(\frac{1}{\rho_F} - 1) + (1 - p_F)(-1) = p_L(\frac{1}{\rho_L} - 1) + (1 - p_L)(-1)$$

which implies

$$\frac{\rho_F}{p_F} = \frac{\rho_L}{p_L} = \kappa, \tag{5}$$

where κ is a constant. Since $\rho_F + \rho_L = p_F + p_L = 1$, we have $\kappa = 1$. It follows then that in markets where all players maximize expected payoff, we should, in equilibrium, have

$$\rho_i = p_i \text{ for } i \in \{F, L\}. \tag{6}$$

Hence ρ_i can be interpreted as the market estimate of horse i's objective winning probability. Obviously, when markets are efficient, odds should perfectly reveal horses' objective winning probabilities. I say there exists the bias if $p_F > p_L$ and

$$\frac{\rho_F}{p_F} < \frac{\rho_L}{p_L}.$$

This occurs when there are not sufficient bets on the favorite or too many bets on the longshot. In our model, the market estimate of the favorite's winning probability is $\rho_F =$

$\frac{B+k}{B+2k}$. Notice that ρ_F is increasing in B. Define ρ_F^I(respectively ρ_F^S) as the equilibrium market estimate of the favorite's winning probability when players apply the integration (segregation) process. Using the inequations defining B^I and B^S, when $B^I > 0$ and $B^S > 0$, we get

$$\frac{kp^{1/\alpha} - (1-p)^{1/\alpha}\lambda^{1/\alpha}}{kp^{1/\alpha} + (k-1)(1-p)^{1/\alpha}\lambda^{1/\alpha}} \leq \rho_F^I < \frac{p^{1/\alpha}}{p^{1/\alpha} + (1-p)^{1/\alpha}\lambda^{1/\alpha}} \tag{7}$$

and

$$\frac{kp^{1/\alpha} + p^{1/\alpha} - \lambda^{1/\alpha}}{kb^{1/\alpha} + p^{1/\alpha} - \lambda^{1/\alpha}} \leq \rho_F^S < \frac{p^{1/\alpha}}{\lambda^{1/\alpha}}. \tag{8}$$

I measure the degree of the bias by the ratio of market estimates divided by objective winning probabilities:

$$\tau_i = \frac{\rho_i}{p_i} \; i \in \{F, L\}.$$

Again, since $\rho_F + \rho_L = p_F + p_L = 1$, $\tau_F < 1$ implies $\tau_L > 1 > \tau_F$, which implies the existence of the bias. The smaller the τ_F, the worse are the markets in estimating horses' objective winning probability, and thus the more severe the bias. Let $\tau_F = \tau_F^I$ when players apply the integration process, and $\tau_F = \tau_F^S$ when players apply segregation process.

First consider the bias when the segregation process is used. Since $\lambda > 1$, from (8) we know that $\tau_F^S = \frac{\rho_F^S}{p} < \frac{p^{1/\alpha - 1}}{\lambda^{1/\alpha}} < 1$. Thus, if players apply the segregation process, the bias always emerges.

I now turn to the bias when players use the integration process. A closer examination of τ_F^I shows that, in principle, both $\tau_F^I > 1$ and $\tau_F^I < 1$ are possible. To understand this, notice that

$$\tau_F^I = \frac{\rho_F^I}{p} < \frac{p^{1/\alpha - 1}}{(1-p)^{1/\alpha}\lambda^{1/\alpha} + p^{1/\alpha}} = \frac{1}{(1/p - 1)^{1/\alpha}\lambda^{1/\alpha}p + p}.$$

Thus when

$$(1/p - 1)^{1/\alpha}\lambda^{1/\alpha}p + p > 1,$$

we have $\tau_F^I < 1$, which is equivalent to

$$(1/p - 1)^{1/\alpha}\lambda^{1/\alpha} > \frac{1}{p} - 1.$$

This inequation holds when both α and λ are sufficiently large, whereas when α and λ are sufficiently small, we may have $\tau_F^I > 1$, which implies the *inverse* bias.

The following numerical example illustrates the intuition:

$$p = 4/5, \lambda = 2, k = 40, a1 = 1/4, a2 = 1/2, a3 = 4/5,$$

where $a1$, $a2$, and $a3$ are values for the parameter α. It can easily be shown that, for different values of α, τ_F^I are, respectively,

$$a1 = 1/4, \tau_F^I = 1.18 \tag{9}$$

$$a2 = 1/2, \tau_F^I = 1 \tag{10}$$

$$a3 = 4/5, \tau_F^I = 0.88 \tag{11}$$

This can be more intuitively seen in Figure 4. Suppose the market is already at $\rho_F^I = 4/5$, and players are free to withdraw or increase bets. I want to find out whether $\rho_F^I = 4/5$ can be supported as an equilibrium outcome and, if not, how ρ_F^I will change in order to arrive at a new equilibrium.

As shown in Figure 4, the decision weighted value of one unit of bet on the favorite, depending on the value of α, is

$$\frac{4}{5}v(\frac{1}{\rho_F^I} - 1) + \frac{1}{5}v(-1) = \frac{4}{5}v(0.25) + \frac{1}{5}v(-1),$$

which can be represented by $V1$, $V2$, and $V3$ respectively. Thus when $\alpha = 4/5$, players refrain from betting well before $\rho_F = 4/5$ is reached, which results in the bias, whereas when $\alpha = 1/4$, after reaching $\rho_F = 4/5$, players still find betting on the favorite attractive and thus continue to bet on the favorite, which results in *the inverse bias.*

Intuitively, this inverse bias is due to the combined effects of loss aversion and concavity (convexity) of the value function. Though we have little evidence on the exact shape of individuals' value function, we tend to believe that rarely individuals simultaneously possess small α and small λ, which would imply strong risk aversion (risk seeking) in the gain (loss) domain and low loss aversion. Moreover, based on experimental evidence, Kahneman and Tversky (1992) suggest that the median value of α and λ are 0.88 and 2.25 respectively, which is unlikely to result in the inverse bias.

Since ρ increases with B and $B^I \geq B^S$, we always have $\tau_F^I \geq \tau_F^S$. Thus the bias is more severe in the segregation process. Moreover, when p approaches to one, τ_F^I converges to one:

$$\underset{p\to 1}{Lim}\ \tau_F^I \geq \underset{p\to 1}{Lim}\left(\frac{kp^{1/\alpha}-(1-p)^{1/\alpha}\lambda^{1/\alpha}}{\left(kp^{1/\alpha}+(k-1)(1-p)^{1/\alpha}\lambda^{1/\alpha}\right)p}\right)=1.$$

This is because when p increases, players put less weight on the loss, which encourages betting and decreases the degree of bias.

But, interestingly, τ_F^S seems rather insensitive to the increase of p:

$$\underset{p\to 1}{Lim}\ \tau_F^S \leq \underset{p\to 1}{Lim}\left(\frac{p^{1/\alpha}}{\lambda^{1/\alpha}}\right)\approx(\frac{1}{\lambda})^{1/\alpha}.$$

This can also be seen from players' decision weighted value function. Since the one unit bet is perceived as an ex ante loss, it is uncorrelated with p. Of course, it would be inappropriate to predict the existence of severe bias even when $p = 1$ since, as prospect theory suggests, people's perception is rather different when things are certain.

4 Conclusion

Behavioral assumptions motivated by empirical and experimental evidences, such as the nonlinear transformation of probability into decision weights, mental accounting, etc., have been adopted to shed new light on the bias. In this paper, I combine loss aversion and mental accounting and offer a new explanation for the bias. In the model, the bias exists in the absence of risk seeking preference, transaction costs, and a nonlinear decision weighting function. The degree of the bias is stronger if players apply the segregation process instead of the integration process.

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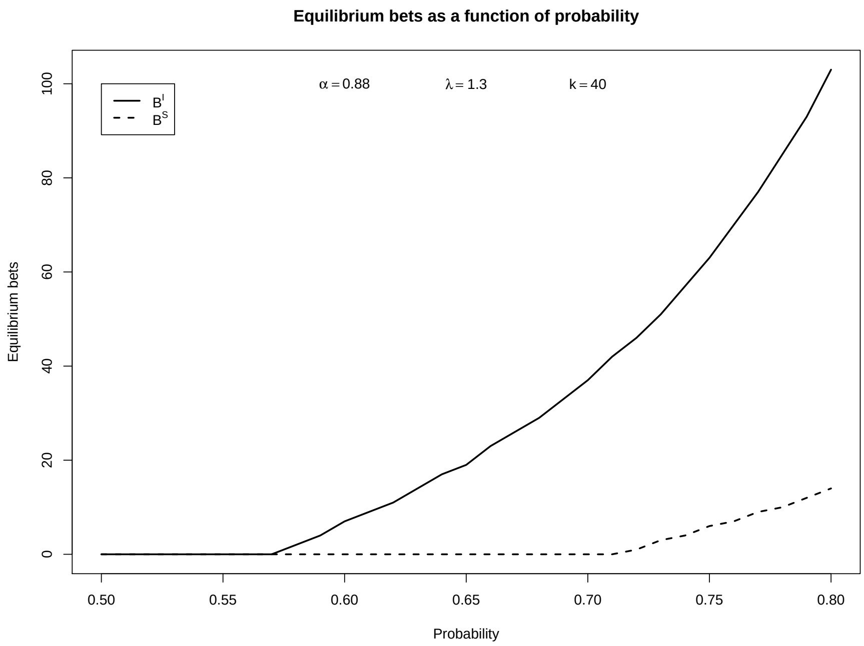

Figure 1: Aggregate equilibrium bets under the integration and the segregation process

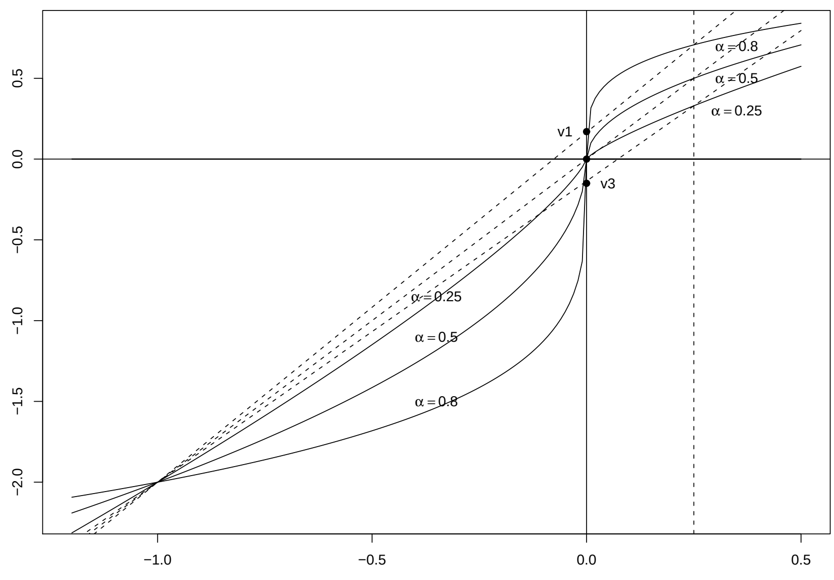

Figure 2: The standard and inverse of the bias under the integration process

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Loss aversion and mental accounting: The favorite-longshot bias in parimutuel betting

Abstract

In this paper it is shown that the combination of mental accounting and loss aversion can fundamentally changes people's way of evaluating risky alternatives. The observation is applied in a market setting: Parimutuel betting markets. In parimutuel betting markets it has been found that for horses with lowest odds (favorites), market estimates of winning probabilities are smaller than objective winning probabilities; for horses with highest odds (longshot) the opposite is observed (the favorite-longshot bias). I build a game theoretical model and show that the favorite-longshot bias is the equilibrium play of the players with loss aversion, and that the degree of the favorite-longshot bias depends on the mental accounting process the players use.

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