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ADDITIVE HEDONIC REGRESSION MODELS WITH SPATIAL SCALING FACTORS: AN APPLICATION FOR RENTS IN VIENNA

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We apply additive mixed regression models (AMM) to estimate hedonic price equations. Non-linear effects of continuous covariates as well as a smooth time trend are modeled non-parametrically through P-splines. Unobserved district-specific heterogeneity is modeled in two ways: First, by location specific intercepts with the postal code serving as a location variable. Second, in order to permit spatial variation in the nonlinear price gradients, we introduce multiplicative scaling factors for nonlinear covariates. This allows highly nonlinear implicit price functions to vary within a regularized framework, accounting for district-specific spatial heterogeneity. Using this model extension, we find substantial spatial variation in house price gradients, leading to a considerable improvement of model quality and predictive power.

Keywords: Hedonic regression, submarkets, multiplicative spatial scaling factors, semiparametric models, P-splines

1. Introduction

This paper is motivated by two common challenges in hedonic price modeling: nonlinear price functions, which require flexible modeling approaches, and the inherent spatial heterogeneity in real estate markets. The purpose of this paper is to address nonlinearity and heterogeneity for rental flats in Vienna simultaneously.

Originally developed for automobiles by Court (1939), hedonic price models have been used extensively in applied economics since the seminal work of Rosen (1974). Often cited classic references are also Lancaster (1966) and Griliches (1971). The theoretical underpinnings are well described e.g. in Follain and Jimenez (1985) and Sheppard (1999). In his 2002 paper, Malpezzi presents a review of the hedonic price literature, and Sirmans et al. (2005) provide a review of specifications and characteristics that have most frequently been used in hedonic pricing studies.

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According to hedonic price theory, differentiated goods like real estate are valued for their utility-bearing characteristics (Rosen, 1974). Since a property is fixed in space, by selecting a specific object, a household implicitly chooses many different goods and services. Therefore, in the concept of implicit markets it is supposed that dwelling characteristics are traded in bundles. The explicit market, with observed prices and transactions, is for the bundles themselves and includes several implicit markets for the property's characteristics (Sheppard, 1999). A hedonic price function describes how the quantity and quality of these characteristics determine its price in a particular market.

Due to the assumption that differentiated goods cannot be easily untied and the resulting impossibility of arbitrage, marginal prices of property characteristics are not constant (Rosen, 1974). Furthermore, the price of one characteristic may depend on the quantity of another. Therefore, we might expect to observe nonlinear relationships between the market price and its measured attributes. A common model specification designed to address this issue takes the log or semi-log form, which furthermore mitigates the problem of heteroscedasticity (Malpezzi, 2002).

Nevertheless, as stated e.g. by Martins-Filho and Bin (2005), a frequent concern in hedonic price literature is the adequacy of parametric specifications. This specification problem arises because economic theory does not provide clear guidance concerning the functional form of the dependence of price on quality (Anglin and Gençay, 1996). As explained e.g. in Wallace (1996), this suggests that functional forms used to estimate hedonic prices should allow for the possibility of nonlinearity in the hedonic price functions. In light of the potentially serious consequences of functional misspecification, there have been some attempts to estimate hedonic price models using semi- or nonparametric methods. The fundamental goal of these approaches is a flexible modeling of the influence of continuous covariates on the dependent variable. Soon after their introduction to hedonic price analysis, they also turned out to be more robust to specification and measurement error (Sheppard, 1999). Semiparametric and nonparametric approaches for real estate can be found e.g. in Pace (1995 and 1998), Anglin and Gençay (1996), Mason and Quigley (1996), Clapp et al. (2002), Clapp (2003, 2004) and Parmeter et al. (2007).

Along with spatial fixation of real estate goes considerable interest in dealing with spatial dependence and variation in hedonic price equations. McMillen (2003) points out that spatial heterogeneity, if modeled inadequately, can lead to spatial dependence. A comprehensive review of various spatial or spatiotemporal econometric models is given in LeSage and Pace

(2004) and Anselin et al. (2004). Very popular in this context is the spatial regression family, which was popularized by Anselin (1988). Spatial autoregressive (SAR) models allow for both spatially lagged dependent variables and spatially lagged disturbance terms. Further reading on related models is provided e.g. by Basu and Thibodeau (1998), LeSage (1999) or Anselin (2003). However, there is a wide range of alternative models, especially semiparametric and nonparametric spatial approaches, which are particularly appropriate to model spatial heterogeneity. Prominent examples are kriging (Diggle and Ribeiro, 2007), 2D tensor product spatial smoothing (e.g. Wood, 2006b; Wood et al., 2008), approaches based on spatial penalization (Besag and Kooperberg, 1995; Fahrmeir et al., 2007) and geographically weighted least squares (Fotheringham et al., 2002).

In this article, non-linear effects of continuous covariates and a time trend are modeled through penalized splines, while discrete spatial effects are implemented as district specific intercepts with spatial regularization in the framework of additive mixed regression models (AMM). An overview concerning additive models and extensions is given e.g. in Fahrmeir et al. (2007) and Wood (2006a). Furthermore, we allow the nonlinear price functions to vary among the districts with spatial scaling factors. Therefore, the basic properties of the nonlinear functions remain constant over all districts (representing submarkets of the Viennese market), while the size of these effects is allowed to vary due to the particular market conditions in a local submarket. Using estimation without spatial scaling factors on the one hand and single districts estimations on the other hand as benchmark models, we find clear advantages of the spatial scaling model. We identify substantial spatial variation in house price gradients, although still in a regularized framework, which leads to a significant improvement of model quality and predictive power.

The remainder of this article is structured as follows: In the next section, the working data set and the particular circumstances in Vienna are described. In section 3, we introduce the models applied in this article, including a discussion of relative (dis-)advantages. We discuss their statistical properties and algorithms for estimation in section 4. In section 5, we present the results, and finally, section 6 concludes.

2. Data Description

The data this article is based on was provided by the ERES NETconsulting-Immobilien.NET GmbH, which operates the largest online real estate platform in Austria. According to a study of the ERES NETconsulting-Immobilien.NET GmbH, more than half of the demand for housing in Vienna is for rental flats. One remarkable characteristic of the rental market in Austria is the Austrian rental law (MRG), which includes the regulation of house rents. Basically, there are four types of rents: free rents ("Freier Mietzins"), adequate rents ("angemessener Mietzins"), reference rents ("Richtwertmieten") and rent categories ("Kategoriemietzins"), depending on the year of construction, renovation, condition, size of the flat and several other criteria. This complicated legal framework makes semiparametric estimation techniques for characteristics like the year of construction and the floor size of the flat even more appropriate, as we might expect that legal restrictions lead to price functions that do not behave typically in a sense that they are monotonically increasing/decreasing.

The data set covers 8767 rental flats in Vienna from the 1st of January 2004 to the 14th of February 2007. It includes offered net rent (in Euro values per month excl. VAT and service charges, which are usually paid by the tenant), continuous variables such as floor size of the flat and indication of the time of letting, integer variables such as the floor the flat is located in, its current condition (captured in 4 categories) and its year of construction (10 categories) as well as discrete variables such as identifying whether the unit has a terrace, a balcony, a garage or a parking lot (Table 1).

While most studies examine the effects of these characteristics on total rents, we follow Fahrmeir et al. (2004) and choose to examine the effects on rents per sq. m. mainly for the following reasons:

- First, using this specification we try to explain the structure of decreasing marginal returns of additional floor size in detail. More specifically, we find substantial decreasing effects of additional floor size in our data.
- Second, rents per sq. m. are especially interesting in the context of the Austrian rental law, which proposes upper limits for this ratio depending on dwelling characteristics. Therefore, the achievable rent per sq. m. is an important benchmark for policy makers, for market participants on the supply as well as on the demand side.

Variable	Description	Number of Obs.	Mean/ frequency	Std.-Dev.	Min	Max
Explained Variables						
rent	rent of apartment [EUR p.m.]	8767	733.1936	471.7273	100	3600
logrent	log of rent of apartment [EUR p.m.]	8767	6.422928	0.5812719	4.60517	8.188689
rent_psqm	rent per sqm of apartment [EUR p.m.]	8767	8.383795	2.515585	2.25	20
logrent_psqm	log of rent rent per sqm of apartment [EUR p.m.]	8767	2.083598	0.2917687	0.81093	2.99573
Explanatory Variables						
area	floor size of the flat [sqm]	8767	85.0649	38.66965	24	200
terr	existence of a terrace	8767	0.1795369	0.3838231	0	1
	0 = no	7,193	82.05 %			
	1= yes	1,574	17.95 %			
balk	existence of a balcony	8767	0.1360785	0.3428914	0	1
	0 = no	7,574	86.39 %			
	1= yes	1,193	13.61 %			
gar	existence of a garage	8767	0.1364207	0.3432543	0	1
	0 = no	7,571	86.36 %			
	1= yes	1,196	13.64 %			
park	existence of parking lot	8767	0.030227	0.1712211	0	1
	0 = no	8,502	96.98 %			
	1= yes	265	3.02 %			
elev	existence of an elevator	8767	0.6951066	0.4603885	0	1
	0 = no	2,673	30.49 %			
	1= yes	6,094	69.51 %			
noelev	no elevator in 3rd floor or higher (interaction variable)	8767	0.0942169	0.292147	0	1
	0 = (floor < 3 or floor >= 3 and exisiting elevator)	7,941	90.58 %			
	1 = (floor >= 3 and no elevator)	826	9.42 %			
cond	condition of the apartment	8767	2.181248	0.5250943	1	4
cond1	First time use of a new/ renovated apartment	379	4.32 %			
cond2	Very good	6,599	75.27 %			
cond3	Good	1,610	18.36 %			
cond4	Fair/ Needs refurbishment	179	2.04 %			
floor	number of floor the apartment is located in	8767	2.814304	1.721735	0	6
floor1	Ground floor	410	4.68 %			
floor2	1st floor	1,923	21.93 %			
floor3	2nd floor	1,960	22.36 %			
floor4	3rd floor	1,782	20.33 %			
floor5	4th floor	1,029	11.74 %			
floor6	5th - 10th floor	610	6.96 %			
floor7	attic floor	1,053	12.01 %			
yearconst	year of construction	8767	1937.96	46.22035	1815	2006
1815	before 1815 (historic)	77	0.88 %			
1830	1815-1850 (Biedermeier)	73	0.83 %			
1870	1851-1889 (Gründerzeit)	169	1.93 %			
1900	1890-1913 (Jugendstil)	4,116	46.95 %			
1930	1914-1945 (interwar time)	276	3.15 %			
1960	1946-1975	1,327	15.14 %			
1980	1976-1984	361	4.12 %			
1990	1985-1994	525	5.99 %			
1997	1995-2000	1,096	12.50 %			
2006	2001 and younger	747	8.52 %			
distr	number of district	8767	10.15718	6.548259	1	23
distr1	Innere Stadt	765	8.73 %			
distr2	Leopoldstadt	425	4.85 %			
distr3	Landstraße	717	8.18 %			
distr4	Wieden	432	4.93 %			
distr5	Margareten	463	5.28 %			
distr6	Mariahilf	475	5.42 %			
distr7	Neubau	387	4.41 %			
distr8	Josefstadt	343	3.91 %			
distr9	Alsergrund	592	6.75 %			
distr10	Favoriten	331	3.78 %			
distr11	Simmering	86	0.98 %			
distr12	Meidling	331	3.78 %			
distr13	Hietzing	453	5.17 %			
distr14	Penzing	258	2.94 %			
distr15	Rudolfsheim-Fünfhaus	217	2.48 %			
distr16	Ottakring	254	2.90 %			
distr17	Hernals	300	3.42 %			
distr18	Währing	655	7.47 %			
distr19	Döbling	712	8.12 %			
distr20	Brigittenau	191	2.18 %			
distr21	Floridsdorf	141	1.61 %			
distr22	Donaustadt	136	1.55 %			
distr23	Liesing	103	1.17 %			
end	last date the object was in the database	8767	7-Dec-05	315.8881	1-Jan-04	14-Feb-07

Table 1: Description of variables

In our application, the district serves as a location variable. Vienna is composed of 23 districts ("Bezirke"), which are numbered for convenience in a roughly clockwise fashion starting in the city center (Figure 1). With a population of about 1.7 million, Vienna is by far the largest city in Austria as well as its capital, its cultural, economic and political centre. It is located in the east of Austria close to the Czech Republic, Slovakia and Hungary. Viennese districts are very heterogeneous, as concerns history, infrastructure, size and density. Until the mid 19th century, the first district used to be the entire city. After several amalgamations, Vienna did not obtain today's borders until the mid 20th century. This historically caused spatial heterogeneity is a strong argument for the existence of submarkets corresponding to districts.

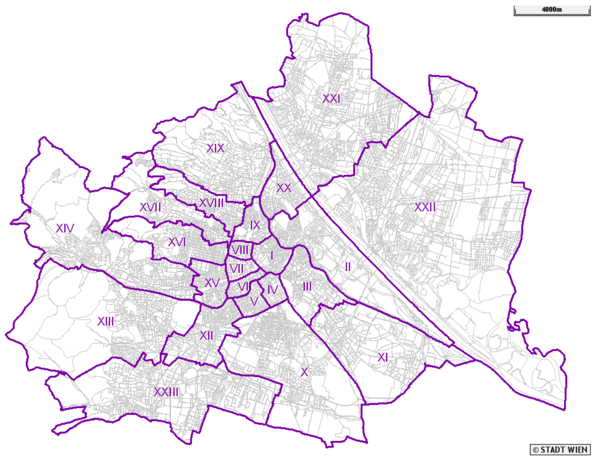


Figure 1: Districts of Vienna

3. Models

In this section, we compare three different model specifications: The first one is a semiparametric model including district specific intercepts, which is also the first step in our two-step spatial scaling procedure. It is therefore called the "base model". In an attempt to account for district specific heterogeneity we estimate additive models for every single district, which is what we call the "single districts model". The last model is the additive mixed model with spatial scaling factors, allowing for district-specific scaling of the nonlinear price functions.

3.1 The Base Model

Although in a parametric approach nonlinear effects of continuous covariates are possible (e.g. through variable transformation, polynomial regression), specification and estimation is tedious and not automated. Hence, we estimate the additive mixed model (AMM)

$$\log(\text{rent\_psqm})_i = \alpha_0 + \gamma_{0d_i} + \alpha_1 \text{park}_i + \alpha_2 \text{balc}_i + \alpha_3 \text{gar}_i + \alpha_4 \text{cond1}_i + \alpha_5 \text{cond3}_i + \alpha_6 \text{cond4}_i + \alpha_7 \text{terr}_i + \alpha_8 \text{elev}_i + \alpha_9 \text{noelev}_i + f_1(\text{area}_i) + f_2(\text{floor}_i) + f_3(\text{yearconst}_i) + f_4(\text{end}_i) + \varepsilon_i \quad \textbf{(1)}$$

AMM replace linear effects by possibly nonlinear functions $f_j(x_{ij})$ of the covariates and may as well include district specific effects. In our specification, $d_i \in \{1,...,23\}$ is the district-specific index of the i-th observation and γ_{0d_i} is its district-specific intercept. The $f_j(x_{ij})$ are possibly nonlinear hedonic price functions of continuous or at least ordinal covariates, and α_k are parameters for dichotomous covariates. The functions f_j are assumed to be reasonably smooth, but no specific functional form is assumed a priori. A number of competing approaches are available for specifying the nonlinear functions f_j and estimating the resulting model. The approach used in our application is based on P(enalized) splines and is described in the next section.

In our application, we find highly nonlinear price functions for some of the covariates, in particular for the year of construction and the floor size of the flat. In a classical linear regression model, these effects could hardly be modeled appropriately. Figure 2 displays the nonparametric functions for the floor size of the flat (*area*, left panel) and the year of construction of the building (*yearconst*, right panel). Shown are estimated price functions as well as the empirical means and standard deviations of the respective partial residuals (note that the price functions are centered about zero for identifiability reasons). This figure illustrates that parametric specifications are likely to lead to wrong specifications and produce autocorrelated residuals.

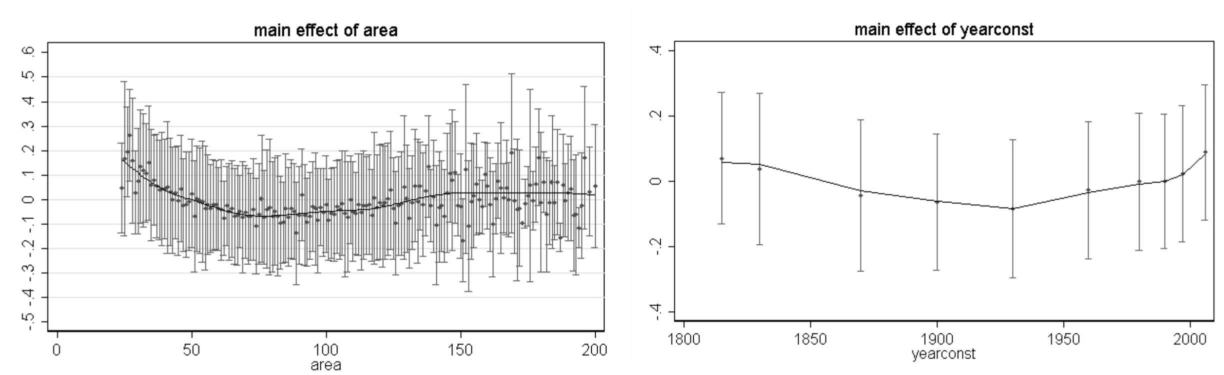


Figure 2: Nonlinear effects of the floor area (left panel) and the year of construction of the building (right panel) along with means and standard deviations of partial residuals

3.2 The Single Districts Model

Although the nonlinear specification fits the data much better than a linear model, building location is only treated as a difference in the intercept in our "base model". A closer inspection of the data shows that in some districts there are still considerable deviations from the estimated price functions. In figure 3, the estimated hedonic price functions for the 1st and the 20th Viennese district are displayed together with partial residuals. In the left panel (1st district), the effect is overestimated, while in the right panel the relationship between the floor size of the dwelling and the expected rent per sq. m. is underestimated, especially for smaller flats. An answer to this district specific heterogeneity could be what we call the "single districts model", where we estimate additive models for each district individually in the same specification as in the "base model" (although without district specific intercept). This approach leads to completely district-specific effects and a maximum of spatial heterogeneity.

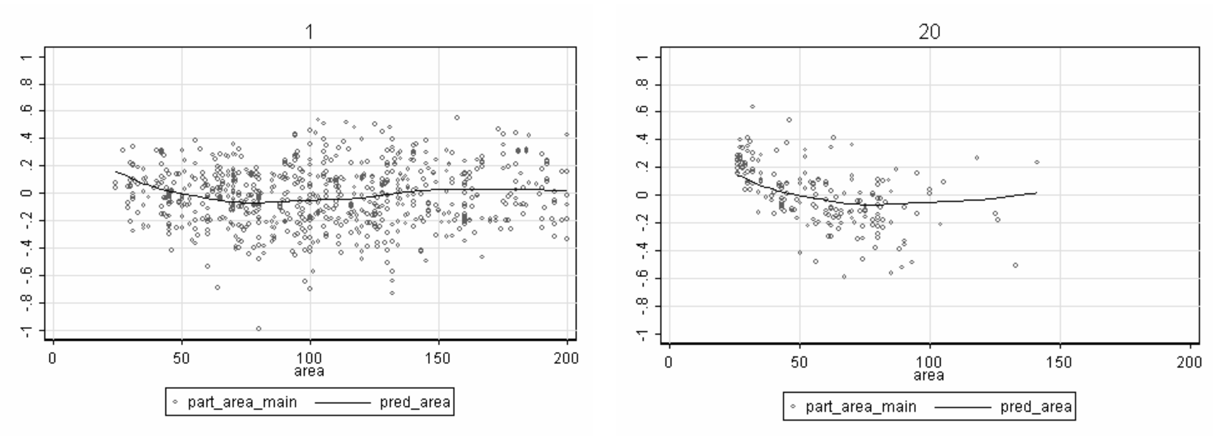


Figure 3: Effect of the base model, evaluated for district 1 (left) and district 20 (right)

Although this model is clearly more flexible than the aforementioned, there are a number of shortcomings. We find extremely heterogeneous effects which cannot be justified from a theoretical point of view, as all the districts are submarkets of the same market. This becomes particularly obvious for the 6th, the 11th and the 21st district, as can be seen in figure 4.

- A very obvious disadvantage of single district estimation is the *low precision* of the estimated price functions due to the relatively small sample size per district. For example, it is noteworthy that in the 21st district credibility intervals are very wide from approximately 120 sq. m. to the largest flat (170 sq. m.). Nevertheless, the function behaves similarly to the price function estimated in the "base model" in the range from 24 to 120 sq. m.

- Another disadvantage of this treatment is that estimation is prone to producing *statistical artifacts*. For example, the price function for the 6th district shows an unexpected effect in the range from 150 to 200 sq. m., pointing down sharply, and the estimated price function for the 21st district goes down and up again in the range from 120 to 170 sq. m. due to a small amount *of influential observations* in this interval.

- In the 11th district, we face a situation where there are hardly any observations of flats larger than 100 sq. m. The effect of floor size is estimated nearly linearly, which is consistent with the fact that the main function is monotonically decreasing in the range of 24 to 80 sq. m. However, according to the shape of the price function estimated in the "base model", the linear estimation is unlikely to hold for flats larger than 100 sq. m. and has therefore no predictive power in this range. In this case, the small range of observations leads to functional *misspecification*.

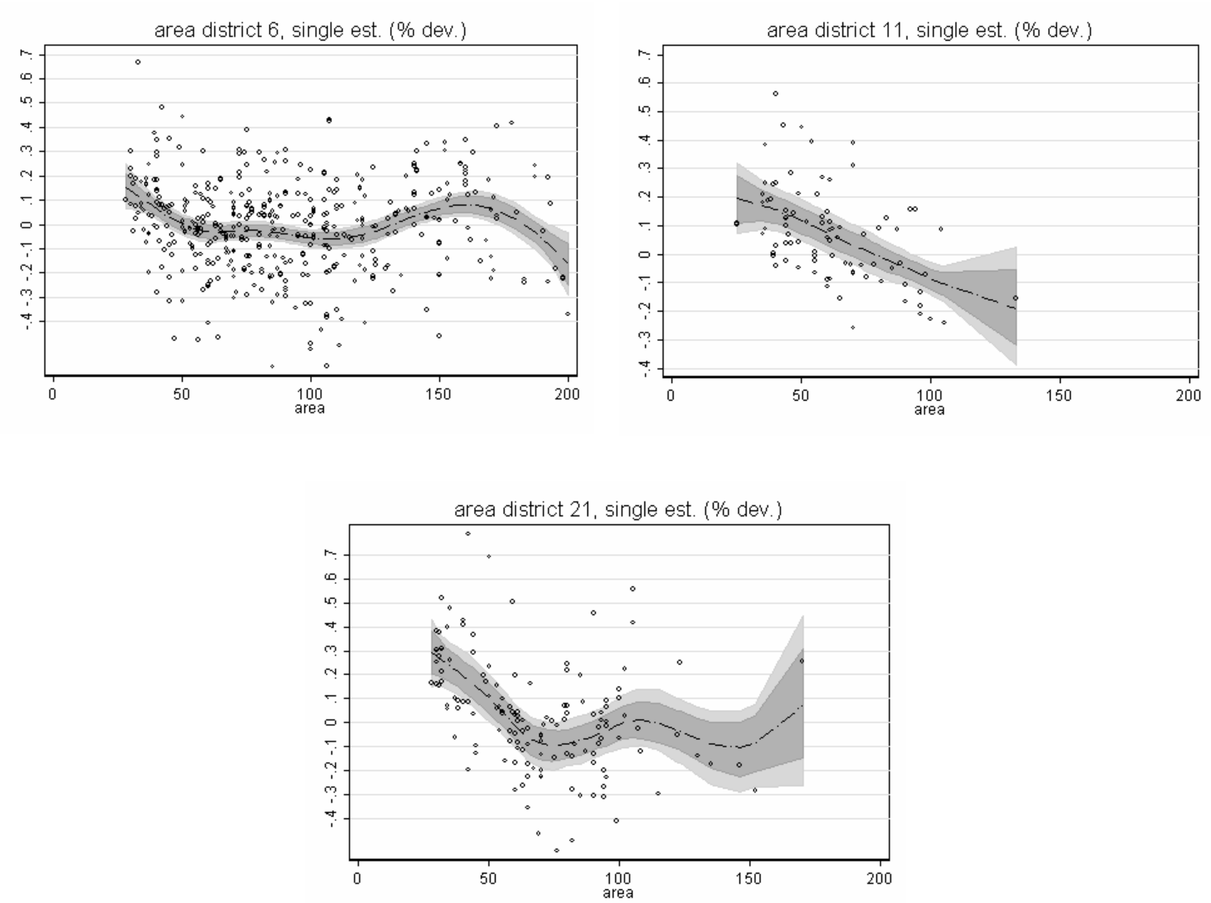


Figure 4: Examples of single districts estimations for covariate *area*: empirical means with pointwise 95% and 80% credible intervals and partial residuals (upper panel: 6th and 11st district; lower panel: 21st district)

To sum up, on the one hand, there seem to be considerable local differences in price functions that are not captured by a model that incorporates district specific heterogeneity only in the intercept. On the other hand, an estimation of models for each district individually produces very unstable estimates and has a very limited capability of forecasting, especially in ranges with few observations. Therefore, we develop what we call a "spatial scaling model".

3.3 The Spatial Scaling Model

With this model, we try to incorporate the existence of submarkets related to districts. The price functions are allowed to vary within a regularized framework but still reflect the basic structure of the price function in the main market. In a two step procedure, scaling factors are estimated that change the slope of the price functions in every district. This leads to the model

$$\begin{aligned}\log(\text{rent\_psqm})_i = {} & \alpha_0 + \gamma_{0d_i} + \alpha_1 \text{park}_i + \alpha_2 \text{balc}_i + \alpha_3 \text{gar}_i + \alpha_4 \text{cond1}_i + \alpha_5 \text{cond3}_i + \alpha_6 \text{cond4}_i + \\ & \alpha_7 \text{terr}_i + \alpha_8 \text{elev}_i + \alpha_9 \text{noelev}_i + (1+\gamma_{1d_i}) f_1(\text{area}_i) + (1+\gamma_{2d_i}) f_2(\text{floor}_i) + \\ & (1+\gamma_{3d_i}) f_3(\text{yearconst}_i) + (1+\gamma_{4d_i}) f_4(\text{end}_i) + \varepsilon_i\end{aligned} \quad , \qquad \textbf{(2)}$$

where equation (1) is expanded by multiplicative district specific effects

$$\gamma_{jd} \qquad d = 1,\ldots,23; \quad j = 1,2,3,4 \,,$$

resulting in the scaling up of the nonlinear function by making it steeper if $(1+\gamma_{jd}) > 1$ and scaling down by making it flatter if $(1+\gamma_{jd}) < 1$.

4. Methodology

We now provide a brief sketch of the statistical methodology used for estimating the models described in section 3. More details are given in the references cited in the text.

4.1 P-splines

A well established approach for modeling nonlinear effects of unknown shape is the use of P(enalized)-splines as first proposed by Eilers and Marx (1996). In a first step the range of a particular covariate z is divided into m equally spaced intervals bounded by the $m+1$ equidistant knots $z_{\min} = \kappa_1 < \kappa_2 < \ldots < \kappa_{m+1} = z_{\max}$. Then, a spline $f(z)$ has the following two properties: in each of the intervals the spline f is a polynomial of degree l, and at the *knots* (the interval boundaries) the spline is $l-1$ times continuously differentiable. The second assumption ensures that the polynomial pieces fit together smoothly (at least for $l > 0$). Typically $l = 3$ is assumed.

Splines of degree l can be represented by a linear combination of a set of $h = m + l$ basis functions $B_j^l(z_i)$, $j = 1,2,\ldots,d$ at a given observation z_i (De Boor, 2001)

$$f_i = \beta_1 B_1^l(z_i) + \beta_2 B_2^l(z_i) + \ldots + \beta_d B_d^l(z_i).$$

For further analysis it is convenient to write the basis functions into a matrix $\mathbf{Z}$ containing elements $\mathbf{Z}[i, j] = B_j(z_i)$, where the value of the j-th basis function at the i-th observation is in i-th row and j-th column. Analogously, the parameters are stacked into a vector $\boldsymbol{\beta}$ and the whole effect of covariate z can be written in matrix notation as $\mathbf{f} = \mathbf{Z}\boldsymbol{\beta}$.

The crucial point in modeling nonlinear relationships through splines is the determination of the number (and position) of knots. Too few result in an overly restrictive spline that might not be able to capture the true variability of the data. Contrariwise, a too large number of knots tends to produce statistical artifacts based on an overfit to the data. In order to overcome these difficulties, Eilers and Marx (1996) have proposed a penalization (P-spline) approach. As a start, a moderately large number of equidistant knots (usually between 20 and 40) is chosen to guarantee enough flexibility. In a second step, a roughness penalty is imposed by punishing large (first or second order) squared differences between two adjacent coefficients β_j and β_{j+1}. This can be accomplished in a frequentist setting by penalized least squares (PLS).

The PLS approach incorporates an additional term that penalizes deviations

$$\mathrm{PLS}(\lambda) = \sum_{i=1}^{n}\left(y_i - \sum_{j=1}^{h} \beta_j B_j^l(u_i)\right)^2 + \lambda \sum_{j=k+1}^{h} (\Delta^k \beta_j)^2 ,$$

where Δ^k is the k-th difference operator and λ governs the trade-off between smoothness and fit to the data. First order differences ($k = 1$) penalize abrupt jumps between successive parameters, second order differences ($k = 2$) penalize deviations from the linear trend. The larger (smaller) λ is the more (less) influence gets the penalization term and the smoother (rougher) is the resulting function. In matrix notation, the penalization term can be rewritten as

$$\lambda \sum_{j=k+1}^{h} (\Delta^k \beta_j)^2 = \lambda \boldsymbol{\beta}' \mathbf{D}'_k \mathbf{D}_k \boldsymbol{\beta} = \lambda \boldsymbol{\beta}' \mathbf{K}_k \boldsymbol{\beta},$$

where $\mathbf{D}_k$ is a difference matrix of order k and $\mathbf{K}_k$ is referred to as a penalty matrix. Therefore, the penalized least squares equation can be rewritten as

$$\mathrm{PLS}(\lambda) = (\mathbf{y} - \mathbf{Z\beta})'(\mathbf{y} - \mathbf{Z\beta}) + \lambda\boldsymbol{\beta}'\mathbf{K}\boldsymbol{\beta}.$$

Minimizing this expression with respect to $\boldsymbol{\beta}$ yields the PLS estimator

$$\hat{\boldsymbol{\beta}} = (\mathbf{Z}'\mathbf{Z} + \lambda\mathbf{K})^{-1}\mathbf{Zy}.$$

The estimated vector of function values $\hat{\mathbf{f}} = (\hat{f}(u_1),\ldots,\hat{f}(u_n))'$ can then be written as

$$\hat{\mathbf{f}} = \mathbf{Z}\hat{\boldsymbol{\beta}} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z} + \lambda\mathbf{K})^{-1}\mathbf{Z}'\mathbf{y}.$$

The choice of the smoothing parameter λ is crucial as we may obtain quite different fits by varying the smoothing parameter. In a frequentist setting the smoothing parameters are either chosen by minimizing some goodness of fit criterion (e.g. AIC, GCV etc.), see e.g. Wood (2006a) for details. Alternatively the model may be rewritten as a linear mixed model. Inference for the smoothing parameters is then based on restricted maximum likelihood; see Fahrmeir et al. (2004) or Ruppert et al. (2003) for details. In this paper inference is based on a fully Bayesian version of P-splines as proposed by Lang and Brezger (2004) and Brezger and Lang (2006). The Bayesian version defines priors for the regression coefficients and the smoothing parameters and therefore allows simultaneous estimation of the function f and the amount of smoothness governed by the smoothing parameter. We used the software package BayesX for estimation, see Brezger et al. (2003a) and Brezger et al. (2003b). The homepage of BayesX (*http://www.stat.uni-muenchen.de/~bayesx/bayesx.html*) contains also a number of tutorials.

In order to illustrate the P-spline approach, we show the construction of a P-spline of degree $l = 3$ for covariate *area* (note that since we perform univariate smoothing in this example, the estimated function differs in shape from the final results in section 5, where multiple regression models are estimated). In a first step, a full spline basis for a given number m of intervals (in this case, $m = 10$, giving a total of $h = m + l = 13$ basis functions) is calculated. As can easily be seen in figure 5, each of these functions has non-zero values in $l + 1$ intervals and overlaps with $2l$ adjacent basis functions. Vertical lines indicate the position of the inner knots. Note that we have to expand the number of knots to $m + l + 1$ in order to define the set of basis functions in every interval of the range of *area* (see Fahrmeir et al., 2007).

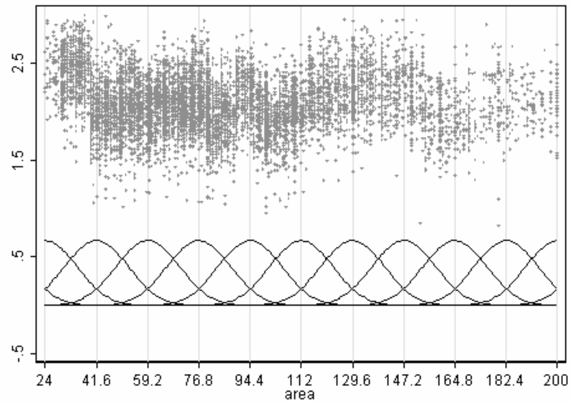


Figure 5: Construction of basis functions for covariate *area*

The basis functions are then scaled by the estimated parameters β_j, $j = 1,2,\ldots,h$, which provide the respective amplitude. Summation of the scaled basis functions leads to the estimated price function (in figure 6, this is represented by a thick line). In a non-penalized approach, this may lead to considerable variability of the function, as can be seen in the left panel of figure 6, especially in the lower range of *area*. In contrast, the P-spline approach leads to a relatively stable fit (right panel of figure 6).

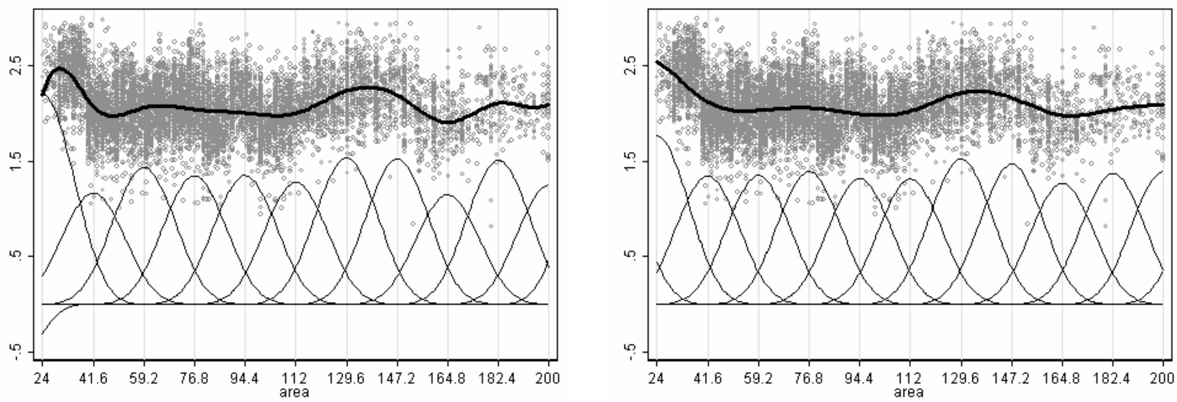


Figure 6: B-spline (left) and P-spline (right) for covariate *area*

The P-spline approach can be extended to modeling more than one nonlinear covariate. Suppose that $z_1,\ldots,z_q$ are continuous covariates to be modeled nonlinearly by P-splines and $x_1,\ldots,x_p$ are further covariates with linear effects. Define design matrices $\mathbf{Z}_1,\ldots,\mathbf{Z}_q$ corresponding to the q continuous covariates and a design matrix $\mathbf{X}$ for the remaining covariates with linear effects. In matrix notation we obtain the model

$$\mathbf{y} = \mathbf{Z}_1\boldsymbol{\beta}_1 + \ldots + \mathbf{Z}_q\boldsymbol{\beta}_q + \mathbf{X}\boldsymbol{\alpha} + \boldsymbol{\varepsilon} = \boldsymbol{\eta} + \boldsymbol{\varepsilon} \quad \textbf{(3)}$$

where $\boldsymbol{\beta}_s$ is the vector of regression coefficients for the s-th nonlinear term of covariate z_s, the vector $\boldsymbol{\alpha}$ contains the regression coefficients of linear effects, and $\boldsymbol{\eta}$ is the predictor vector. The parameters are estimated by maximizing the extended PLS criterion

$$\mathrm{PLS}(\lambda_1, \ldots \lambda_q) = \sum_{i=1}^{n} (y_i - \eta_i)^2 + \sum_{s=1}^{q} \lambda_s \boldsymbol{\beta}_s' \mathbf{K}_s \boldsymbol{\beta}_s \,, \quad \textbf{(4)}$$

where $\mathbf{K}_s$ is the penalty matrix of the s-th nonlinear term. The smoothing parameters involved are again estimated by minimizing a goodness of fit criterion, using the connection to linear mixed models or via a fully Bayesian approach, see the literature cited above.

4.2 Spatial Effects

Spatial heterogeneity may be captured in two different ways, by a non-ordered district-specific intercept or by a smooth spatial term that accounts for the spatial ordering of the information.

District specific intercepts $\boldsymbol{\gamma}_{0d}$ are estimated by adding an additional term $\mathbf{Z}_{\gamma_0}\boldsymbol{\gamma}_0$ in (3) and by adding the ridge type penalty $\lambda_{\gamma_0}\boldsymbol{\gamma}_0'\boldsymbol{\gamma}_0$ in the PLS criterion (4), where the matrix $\mathbf{Z}_{\gamma_0}$ is an incidence matrix with entries 1 in row i and column j if observation i is in district j. The effect of the penalty is to shrink estimated parameters $\boldsymbol{\gamma}_{0d}$ towards zero. Hence the penalty prevents from possibly large variation in the estimates for $\boldsymbol{\gamma}_{0d}$ due to the large number of district specific parameters. For sufficiently large sample sizes within each district the estimated parameters automatically tend to an unconstrained fit.

In our case it is also plausible that objects in neighboring districts (i.e., districts that share a common border) have a higher correlation than two arbitrary objects, which leads us to the notion of spatial dependence. As the spatial ordering of the information is available (i.e., the map including the borders of the districts), we conducted what is called geoadditive regression (see Kamman and Wand, 2003). The penalty is given by

$$\lambda \sum_{d=2}^{23} \sum_{r \in N(d), r<d} (\gamma_r - \gamma_d)^2$$

where $N(d)$ is the set of neighbors of site d. Hence, squared differences of parameters of neighboring sites are penalized.

In our application it turned out that the results based on the two alternative penalties differ only marginally, which indicates that there is a small amount of spatial dependency. In the following we therefore present only results based on the simple ridge type penalty.

4.3 Multiplicative scaling factors

In our application, we allow the nonlinear price functions to vary among districts. For simplicity of presentation consider the model

$$y_i = \gamma_{0d_i} + (1 + \gamma_{1d_1}) f(u_i) + \varepsilon_i \qquad \textbf{(5)}$$

In matrix notation we obtain

$$\mathbf{y} = \mathbf{Z}_{0\gamma}\boldsymbol{\gamma}_0 + \mathbf{DZ}\boldsymbol{\beta} + \boldsymbol{\varepsilon} = \mathbf{Z}_{0\gamma}\boldsymbol{\gamma}_0 + \tilde{\mathbf{Z}}\boldsymbol{\beta} + \boldsymbol{\varepsilon},$$

where $\mathbf{Z}_{0\gamma}$ is the design matrix for the district specific intercept, $\mathbf{D}$ is a diagonal matrix with entries $1 + \gamma_{1d_i}$, $\mathbf{Z}$ is the design matrix for the nonlinear effect of covariate z and $\tilde{\mathbf{Z}} = \mathbf{DZ}$.

For given scaling factors the regression parameters $\boldsymbol{\gamma}_0$ and $\boldsymbol{\beta}$ may be estimated by the estimation techniques outlined in subsections 4.1 and 4.2.

On the other hand, we may rewrite (5) in an alternative way as

$$y_i = \gamma_{0d_i} + f(u_i) + f(u_i)\gamma_{1d_i} + \varepsilon_i.$$

or in matrix notation as

$$\mathbf{y} = \mathbf{Z}_{0\gamma}\boldsymbol{\gamma}_0 + \mathbf{f} + \overline{\mathbf{Z}}\boldsymbol{\gamma}_{1d_i} + \boldsymbol{\varepsilon},$$

where $\mathbf{f} = [f(u_1), \ldots, f(u_n)]'$ and $\overline{\mathbf{Z}} = \text{diag}[f(u_1), \ldots, f(u_n)]\mathbf{Z}_{\gamma_1}$. Hence, for given $\mathbf{f}$, $\boldsymbol{\gamma}_0$ and $\boldsymbol{\gamma}_1$ may again be estimated as described in sections 4.2 and 4.3.

This gives rise to the following two step estimation algorithm (which may be iterated):

1. In the first step we assume a homogeneous function f as in our “base model”, i.e. the district specific intercept γ_{0d} and the scaling factors γ_{1d} are assumed to be identical to zero. Using the PLS estimation we obtain estimates $\hat{\mathbf{f}}$ of the nonlinear function.

2. In the second step we estimate $\boldsymbol{\gamma}_0$ and $\boldsymbol{\gamma}_1$ in an additive mixed model framework by keeping the estimated function $\hat{\mathbf{f}}$ from the first stage fixed.

A generalization to models with more than one covariate is straightforward.

5. Application

In this section we present the estimation results for the models described in section 3. In subsection 5.1, we show the pooled effects of the “base model”. Subsection 5.2 describes the spatial variation of the nonlinear price functions estimated in the “spatial scaling model”.

5.1 Estimation results of the Base Model

In table 2, we show the estimation results of the parameters estimated from “base model” as described in equation (1).

Name	Post. Mean	Std.-Dev.	95% Credibility Interval	
const	2.0537	0.0272	2.0034	2.1073
park	-0.0108	0.0131	-0.0373	0.0141
balk	0.0630	0.0067	0.0503	0.0757
gar	0.0502	0.0076	0.0350	0.0653
elev	0.0367	0.0067	0.0235	0.0492
noelev	-0.0389	0.0106	-0.0606	-0.0173
terr	0.1346	0.0073	0.1204	0.1489
cond1	0.0041	0.0120	-0.0197	0.0278
cond3	-0.1165	0.0060	-0.1283	-0.1053
cond4	-0.2876	0.0163	-0.3203	-0.2559

Table 2: Parametric Effects

Because the model is specified in a semi-log form, the coefficients can be interpreted as (approximately) the percentage deviation from the reference category (actually, a better approximation of the percentage value is given by $\exp(\alpha)-1$, where α is the estimated parameter). In order to get an impression of how these effects influence rents in terms of Euro per sq. m., we transform the functions to natural units, where all other covariates are held constant at mean level of attributes in our sample. The results are presented in table 3, where we display the percentage deviation, the difference evaluated at the mean and the resulting rent per sq. m. if the respective attribute is existent.

	Effect on rents per sq. m.		
Attribute	**Difference (%)**	**Difference (€)**	**Rent per sq. m. (€)**

Terrace	13.46%	1.16 €	8.89 €
Balcony	6.30%	0.53 €	8.26 €
Garage	5.02%	0.42 €	8.15 €
Elevator	3.67%	0.30 €	8.03 €
No elevator (floor > 2)	-3.89%	-0.32 €	7.42 €

Table 3: Effect of attributes as percentage difference (left column), difference in Euro values evaluated at sample mean (center column), resulting rent per sq. m. evaluated at sample mean (right column)

Figure 7 shows the effect of the condition of the flat with category "very good" serving as a reference category in our estimation (left panel) and the district specific intercept (right panel), both evaluated at the sample mean. The condition accounts for a variation of 2.10 Euro per sq. m. evaluated at the sample mean. Also the discrete location effect is very strong, leading to differences in rents per sq. m. of 55% or 5 Euro evaluated at the sample mean, respectively.

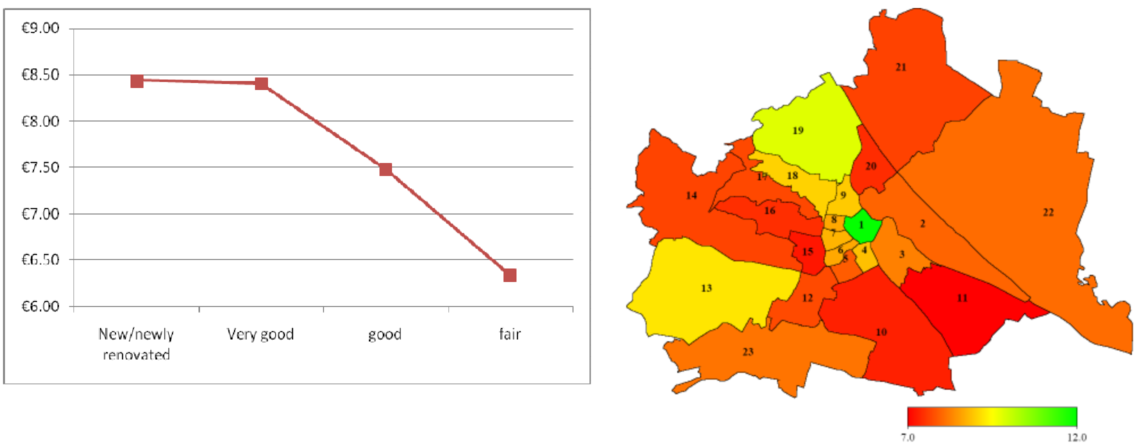


Figure 7: Effect of covariate condition (*cond*, left) and the location of the flat (*distr*, right)

Figures 8 to 10 depict the nonparametric effects of our "base model". Shown are the empirical means with pointwise 95% and 80% credible intervals, with the functions evaluated the at the sample mean.

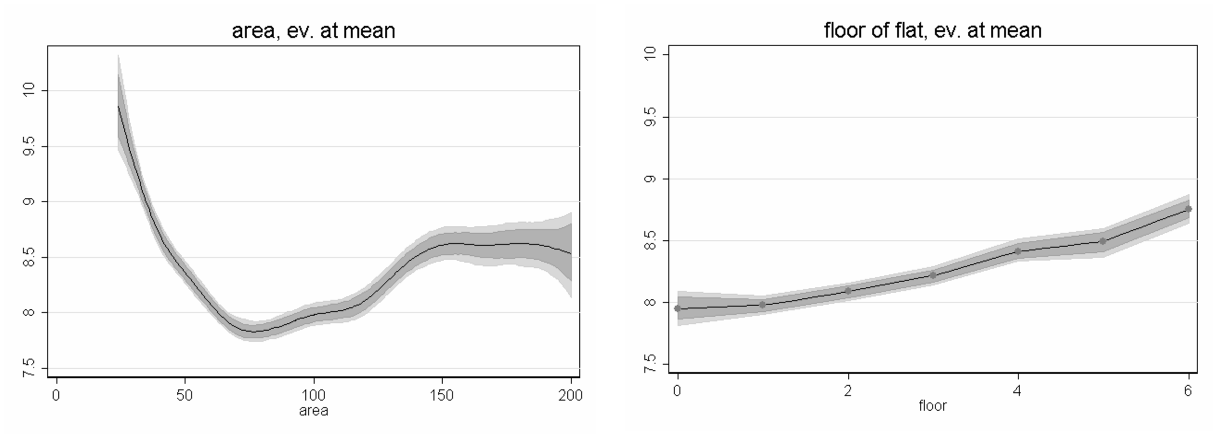


Figure 8: Empirical mean and pointwise 95% and 80% credible intervals of the floor size of the flat (*area*, left) and the floor the flat is located in (*floor*, right)

In the left panel of figure 8, the effect of the floor size of the flat (*area*) is displayed. From hedonic price theory, a decreasing price function over the whole range would be expected for this covariate. However, the shape of this function differs from this assumption and leads to some interesting interpretations: In the low range from 24 to approximately 80 sq. m., there is the expected strong effect of decreasing marginal prices which results in a monotonically decreasing function. However, in the range from 80 sq. m. to 150 rents per sq. m. start to rise and remain nearly constant from 150 sq. m. onwards. This effect can be interpreted in the context of the Austrian rental law (MRG). The MRG, which regulates rents per sq. m. in Austria, is not relevant for flats of more than 130 sq. m. Actually, from about 130 to 150 sq. m., there is a much stronger increase in rents per sq. m. than in the range from 80 to 130 sq. m. This could be interpreted as a kind of convergence to the level of free rents. Furthermore, large flats are rather scarce in Vienna: flats with more than 150 sq. m. make up less than 8% of the sample. Additionally, very large flats are likely to be correlated with a more exclusive interior, which was not measured in this sample. This explains the constant price level in the highest range of flat sizes.

The effect of the floor the flat is located in (*floor*), displayed in the right panel of figure 8, shows the expected increase in price for higher floor levels. It accounts for approximately 10% in expected rent variation and is therefore much weaker than that of the floor size of the flat. Surprisingly, the ground floor and the first floor are valued nearly equally, although it could be expected that flats in the ground floor realize a much lower price. This is due to the fact that many flats are let with a garden, an attribute that was not collected in the data. Additionally to this effect, we introduced an interaction variable that accounted for the nonexistence of elevators if the flat was located in a third floor or higher (see table 1, for estimation results see table 3).

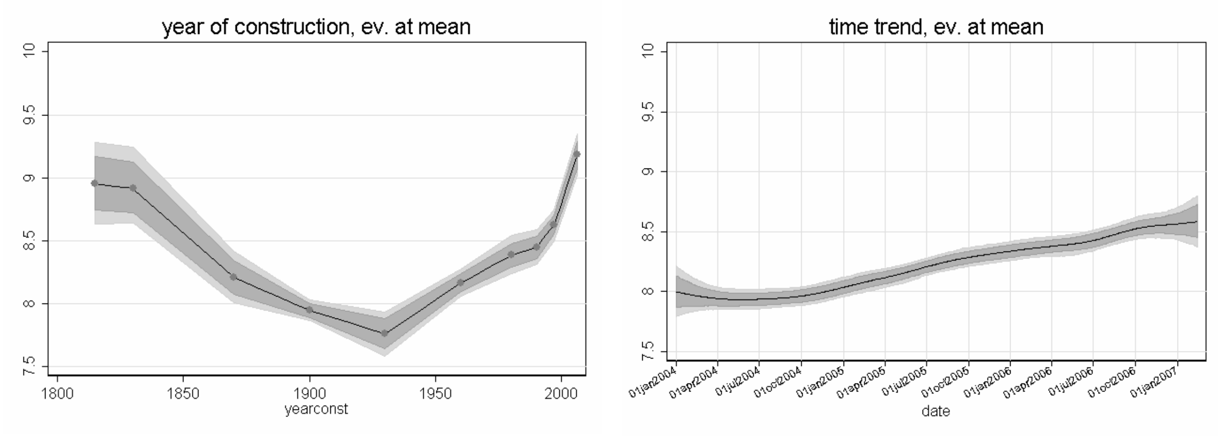


Figure 9: Empirical mean and pointwise 95% and 80% credible intervals of the year of construction of the building the flat is located in (*yearconst*, left) and the time trend (*end*, right)

Concerning the year of construction (*yearconst*) of the building, displayed in the left panel of figure 9, we had two assumptions: We expected a strong increase in rents for buildings constructed after 1945 because for these buildings the MRG is only partly applicable (free rent or appropriate rent instead of reference rent). Furthermore, we thought there might be relatively high rents for new buildings. Actually, our analysis confirmed these two assumptions and showed a strong variation in expected rents due to the year of construction. The lowest expected rent per sq. m. was found for flats in buildings constructed in the interwar time, *ceteris paribus* nearly 20% lower than for flats in buildings constructed after 2001. However, we found that flats in Biedermeier and Gründerzeit buildings had an expected rent per sq. m. that was nearly as high as that of flats in very new buildings. Further research yielded that many of these buildings have been renovated lately and can be found in very desirable locations.

With the variable *end*, which reflects the date of exit from the database, we tried to control for time effects in the sample. As displayed in the right panel of figure 9, it turned out that there is an increase in offered rent of nearly 0.6 Euro per sq. m., evaluated at the sample mean. This is an increase of approximately 8% from the 1^{st} of January 2004 to the 14^{th} of February 2007.

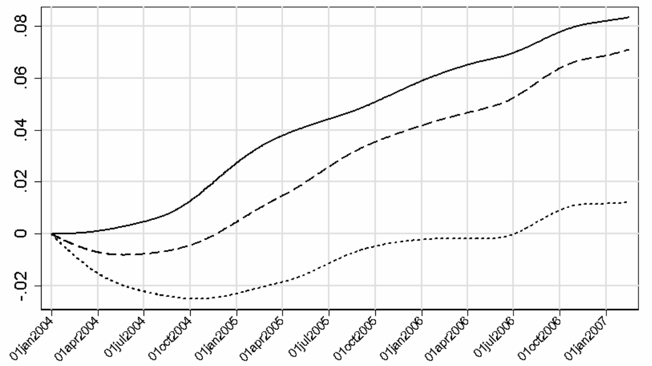


Figure 10: Empirical mean of covariate *end* estimated in different models (highest line: non-quality adjusted; middle line: hedonic price index; lower line: hedonic price index in real 2004 rents)

Yet, the estimated hedonic time index gives some further interesting insights: in figure 10, we compare the quality controlled (hedonic) price index to a nonlinear time trend without quality adjustment. We find that a large amount of the increase in rents is due to higher quality. Furthermore, if the whole model is estimated with 2004 rents (adjusted with a smoothed monthly CPI), we find that there is a decrease in quality adjusted real rents from January to October 2004, an increase to the January 2004 level until January 2006, and real rents start to rise above this level only by July 2006.

5.2 Estimation results of the Spatial Scaling Model

As described in section 3 of this paper, the district specific heterogeneity was the motivation for us to introduce spatial scaling factors as in equation (2), allowing nonlinear price functions to vary among districts. Hence, we demand that in a submarket the price function may differ in scale, but not in its basic structure, which lead us to reject the "single districts estimation" (see section 3).

Applying 10-fold cross-validation, we find that the model extension reduces the mean squared error (MSE) by nearly 17% compared to the base model without spatial scaling factors (see table 4).

MSE Base Model	MSE Spatial Scaling Model	Difference (%)
0.0525	0.0438	16.65%

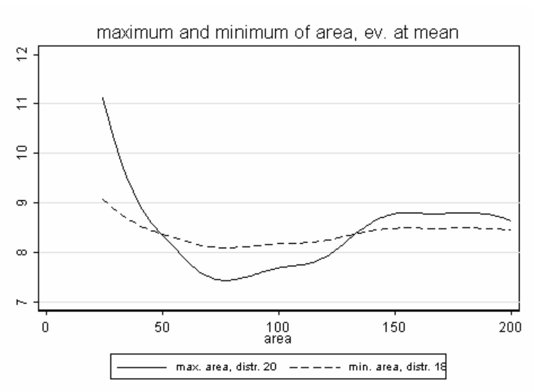
Table 4: MSE of 10-fold cross-validation

In table 5, the range of the estimated scaling parameters is displayed for each of the price functions under consideration. It shows that the spatial variation in scale ranges from 0.76 for *end* (the slope of this effect is scaled down by the factor $(1+\gamma_{jd}) = 0.75$ compared to the "base model" effect in the 10th district and up by the factor $(1+\gamma_{jd}) = 1.51$ in the 4th district) to 2.81 for *floor* (here, the effect in the 9th district is only 0.02, while in the 23rd district it is 2.83 times the effect estimated in the "base model").

	maximum	minimum	difference
area	0.75	-0.51	1.26
yearconst	0.63	-0.36	0.99
floor	1.83	-0.98	2.81
end	0.51	-0.25	0.76

Table 5: Maximum (left column), minimum (middle column) and difference of scaling factors

Figures 11 to 14 show this considerable heterogeneity. In the left panel, the maximum and minimum of the set of scaled functions is displayed for every nonlinear price function, again evaluated at the mean level of attributes in our sample. The right panel shows the spatial distribution of the spatial scaling factors in maps of Vienna, respectively.



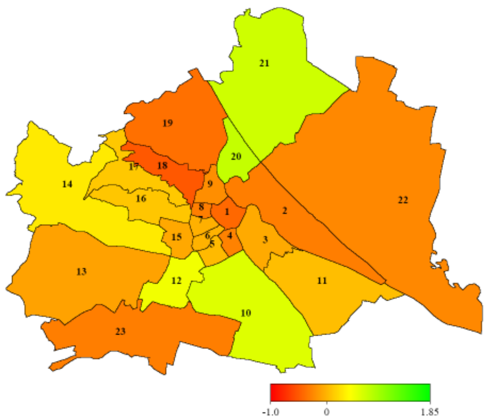
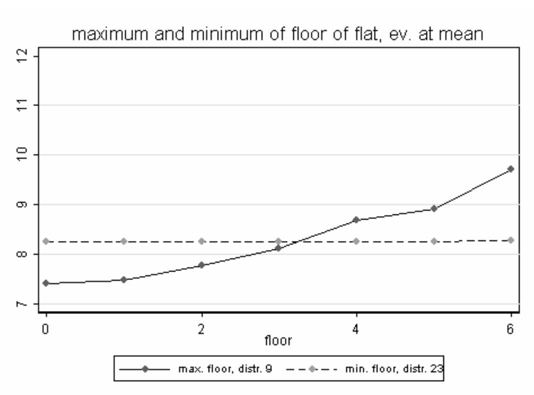


Figure 11: Maximum and minimum of scaled effects (left) and spatial distribution of spatial scaling factors (right) for covariate *area*



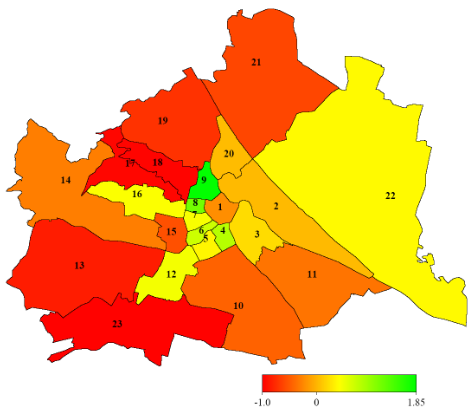
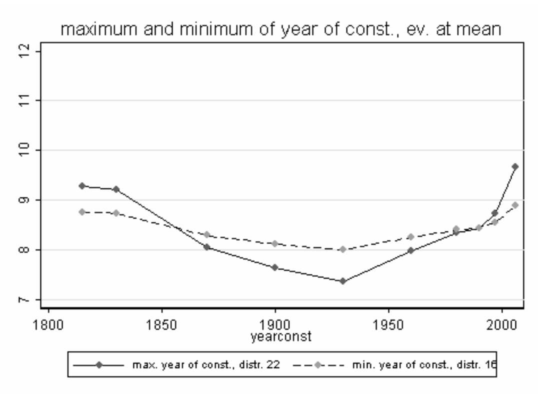


Figure 12: Maximum and minimum of scaled effects (left) and spatial distribution of spatial scaling factors (right) for covariate *floor*



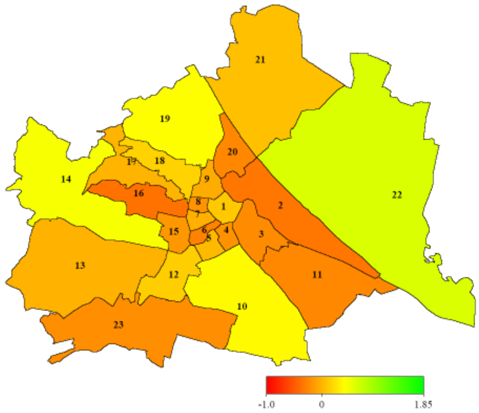
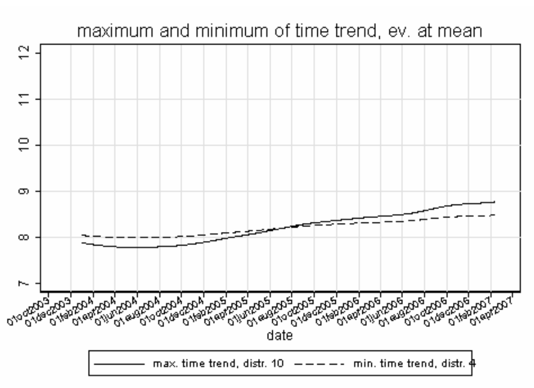


Figure 13: Maximum and minimum of scaled effects (left) and spatial distribution of spatial scaling factors (right) for covariate *yearconst*



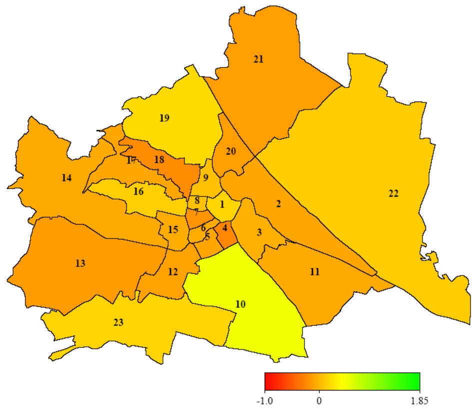


Figure 14: Maximum and minimum of scaled effects (left) and spatial distribution of spatial scaling factors (right) for covariate *end*

6. Conclusion

In this article, we address two major problems in hedonic price modeling: Nonlinearity of hedonic price functions, for which no specific functional form can be derived from theoretical considerations, and spatial heterogeneity, reflecting the existence of submarkets related to districts. For the first problem, we propose an additive mixed model with district specific intercepts. For the second problem, we introduce spatial scaling factors into the additive mixed model. The spatial scaling model treats nonlinearity and spatial heterogeneity in a unified framework, without leading to unstable estimates due to the decrease in sample size as in the single districts estimation. We find considerable improvement in model quality compared to a model without spatial scaling factors on the one hand and a model that estimates hedonic price functions for each district individually on the other hand.

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Additive Hedonic Regression Models with Spatial Scaling Factors: An Application for Rents in Vienna

Abstract

We apply additive mixed regression models (AMM) to estimate hedonic price equations. Non-linear effects of continuous covariates as well as a smooth time trend are modeled non-parametrically through P-splines. Unobserved district-specific heterogeneity is modeled in two ways: First, by location specific intercepts with the postal code serving as a location variable. Second, in order to permit spatial variation in the nonlinear price gradients, we introduce multiplicative scaling factors for nonlinear covariates. This allows highly nonlinear implicit price functions to vary within a regularized framework, accounting for district-specific spatial heterogeneity. Using this model extension, we find substantial spatial variation in house price gradients, leading to a considerable improvement of model quality and predictive power.

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