universität innsbruck

Department of Mathematics

Inn'formal Probability Seminar

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“Delocalisation of height functions.”

Abstract

Take a Simple Random Walk in dimension 1 that starts at 0, makes 2n steps and ends at 0. It is elementary that the variance of the position of the walk at time n is of order $\sqrt{n}$. Same is true for the lazy Random Walk that can stay at the same place with a positive probability.

Now consider a version of this question with a two-dimensional time. Take a square of size n on the hexagonal lattice and place integer heights at the faces, so that the heights at two adjacent faces differ by 0, 1 or -1 and the height at the boundary is fixed to be 0. Assign weight x>0 for every disagreement between adjacent heights.
It was predicted in physics in 70s-80s that a phase transition occurs at $1/\sqrt{2}$:

- *when x<$1/\sqrt{2}$, the variance of the height at the origin is uniformly bounded;*
- *when x≥ $1/\sqrt{2}$, the function is delocalised and the variance diverges as log n.*

We show the second part of this conjecture for all x ∈ [$1/\sqrt{2}$, 1].

Our approach goes through graphical representations of this random Lipschitz function, positive correlation (FKG) inequalities and planar duality. It applies also to the six-vertex (ice-type) model. Note that planarity is crucial: in dimension 3 and higher, the height function is expected to be localised for all x>0.

Based on a joint work with Piet Lammers.

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